INTERMEDIATE PHYSICS

BY

S. C. RAY CHOWDHURY, M.Sc.,

Late Head of the Department of Physics,
Government College, Muzaffarpur

AND

D. B. SINHA, M.Sc, Ph.D. (Lond.), D.I.C.

Reader in Applied Physics, Calcutta University :
Author of "Intermediate Practical Physics" :
Author of "Madhyamik Bhowtik Bijnan" :
Author of "Modern School Physics" :
Author of "Padartha Bijnaner Gorar Katha".

MODERN BOOK AGENCY PRIVATE Ltd.

10, Bankim Chatterjee Street,

CALCUTTA 12

Published by
D. C. Bose,
Modern Book Agency Private Ltd.,
10, Bankim Chatterjee Street, Calcutta 12

Price Rs. 8·00 only

Agent:

BOOK LAND P. LTD.
44, Jonstongunj,
Allahabad,

BOOK LAND P. LTD.
Chowhatta,
Patna,

Printed by
S. B. Mallik, Bani Press,
16, Hemendra Sen Street,
Calcutta 6

PREFACE

"Outlines of Intermediate Physics" written by Prof. S. C. Roy Chowdhury has been thoroughly revised and recast and is being presented to the readers as "Intermediate Physics" under our joint authorship. In preparing the book in its present form, we have been solely guided by the consideration that for the fixing up of new ideas in the minds of the learners it is necessary to indicate the field of their applications also by concrete examples. Additions and alterations have naturally been made here and there. Moreover the book has been rewritten at many a place to bring it in line with modern ideas. Some new topics have also been included in order that it may cover the Syllabus of studies of the various Indian Universities. A large number of new blocks have been added to illustrate the subject matter properly.

The authors will deem their pains amply repaid if this new set up proves to be of greater service to the readers.

Calcutta,
February, 1948.

S. C. Roy Chowdhury,
D. B. Sinha.

CONTENTS

PART I

GENERAL PHYSICS

CHAPTER I

INTRODUCTION

CHAPTER II

MEASUREMENT

CHAPTER III

MOTION AND REST

CHAPTER IV

NEWTON'S LAWS OF MOTION

CHAPTER V

GRAVITATION AND GRAVITY : PENDULUM

CHAPTER VI

WORK, ENERGY AND POWER

CHAPTER VII

PROPERTIES OF MATTER

CHAPTER VIII

PRESSURE IN LIQUIDS

CHAPTER IX

ARCHIMEDES' PRINCIPLE

CHAPTER X

ATMOSPHERIC PRESSURE

CHAPTER XI

APPLICATIONS OF AIR PRESSURE

PART II

HEAT

CHAPTER I

HEAT, TEMPERATURE : THERMOMETRY

CHAPTER II

EXPANSION OF SOLIDS

CHAPTER III

EXPANSION OF LIQUIDS

CHAPTER V

EXPANSION OF GASES

CHAPTER V

CALORIMETRY

CHAPTER VI

CHANGE OF STATE

CHAPTER VII

HYGROMETRY

CHAPTER VIII

TRANSMISSION OF HEAT

CHAPTER IX

MUSICAL INSTRUMENTS : PHYSIOLOGICAL ACOUSTICS

ABBREVIATIONS

The following abbreviations have been used for examination questions.

C. U.—Calcutta University
Pat.—Patna University
Dac.—Dacca University
All.—Allahabad University
Punjab.—Punjab University
Mysore.—Mysore University
L. M.—London Matric
S. C.—School Certificate (Oxford and Cambridge)
C. L.—Cambridge Local
C.M.B.—Cambridge Medical Board
L. U.—London University
M. U.—Madras University

PART I

GENERAL PHYSICS

——(::)——

CHAPTER I

Introduction

1. **Province of Physics.**—Physics may be described as the study of the properties of matter and energy.

Matter is that which occupies space and is perceptible to us through our senses.

Energy is something that exists in nature, and it is, like matter, one of the fundamental realities. But it is unlike matter in other respects, *i.e.*, it is not tangible, it has no weight, it has no extension. One of the objects of Physical Science is to explain the meaning of energy and its various transformations. It is at the expense of energy that work is done.

Energy of a body is its capacity for doing work.

2. **Subdivisions of Physics.**—The subject of Physics is divided into the following branches :—

(1) General Physics.	**(2) Heat.**	**(3) Sound.**
(4) Light.	**(5) Magnetism.**	**(6) Electricity.**

Of these (1) deals with the properties of matter ; (2) to (6) deal with energy in some form or other.

Measurement

3. **Units.**—Every physical quantity is measured in terms of some standard of reference which is called the *unit* of that quantity.

When we say that a pencil is 4 inches long, we take an inch as the unit of length.

Fundamental Units.—All physical quantities are obtained from three fundamental units. These are *Length*, *Mass*, and *Time*.

All other units are based on these three fundamental units and are termed **Derived Units.**

Thus, the unit of area is the area of a square, each side of which is of unit length ; and the unit of volume is the volume of a cube, each

side of which is of unit length. So the unit of area, or that of volume is derived from the unit of length, which is a fundamental unit.

Again, a body has unit velocity when it moves over unit length in unit time. Hence the unit of velocity is derived from the units of length and time. Similarly the unit of force is derived from the units of length, mass, and time, and so on. Thus the *units of area, volume, velocity, force,* etc., are all *derived units.*

4. Systems of Units.—Two systems of units are generally used in scientific measurements.

(i) The French or Metric (C. G. S.) System.
(ii) The British (F. P. S.) System.

In the Metric or Centimetre-Gram-Second System (briefly described as the **C. G. S.** system), the units of **length, mass,** and **time** are the **centimetre (cm.)**, the **gram (gm.)**, and the **second (sec.)** respectively.

This system is preferred throughout the world in scientific work.

In the British or Foot-Pound-Second System (briefly described as the **F. P. S.** system), the units of **length, mass,** and **time** are the **foot,** the **pound**, and the **second** respectively.

This system is used largely by British engineers and in the meteorological observations in the British commonwealth of Nations.

Note.—Another thing should be observed in dealing with units. It is often found that some derived unit is inconveniently large or inconveniently small. In such a case some submultiple (when the derived unit is too large), or multiple (when the derived unit is too small), is used as a unit for the sake of convenience. Such units are termed **Practical Units**, whilst those derived directly from the centimetre, gram, and second (or the foot, pound, and second) are termed **Absolute Units**.

5. Length.—In the **C. G. S. System** the unit of length is the centimetre (cm.), which is one-hundredth part of a metre.

METRIC TABLE OF LENGTH

1 millimetre (1 mm. = ·001 m)		
10 millimetres	= 1 centimetre	(1 cm. = ·01 m.)
10 centimetres	= 1 decimetre	(1 dm. = 0·1 m.)
10 decimetres	= 1 **metre** (m.)	
10 metres	= 1 Decametre	(1 Dm. = 10 m.)
10 Decametres	= 1 Hectometre	(1 Hm. = 100 m.)
10 Hectometres	= 1 Kilometre	(1 Km. = 1000 m.)

Therefore, we find that,

millimetre = 0·1 centimetre = 0·01 decimetre = 0·001 metre.

Hence the units in the Metric system can be altered by moving the decimal (Latin, *Decem* = ten) point. Thus **the Metric is a decimal system**.

In the **F. P. S. System** the unit of length is the **yard,** but the most convenient one for the scientific purpose is the **foot** (ft.)

Remember that—

1 metre	=	1·093 yds. = 39·37 inches.
1 cm.	=	0·3937 inches ; **1 inch = 2·54 cm.**
1 kilometre	=	1000 metres = 0·621 mile.

6. Mass.—The mass of a body is the quantity of matter in it.

The **metric unit of mass** is the kilogram, but, for the sake of convenience, the gram (also written 'gramme')—which is defined as the mass of a cubic centimetre of distilled water at 4°C.—is taken as the unit of mass.

(**Note** that in the metric system there is a connection between the unit of *mass* and the unit of *volume*).

METRIC TABLE OF MASS

1 milligram (1 mgm. = 0·001 gm.)

10 milligrams	= 1 centigram	(1 cgm. = 0·01 gm.)
10 centigrams	= 1 decigram	(1 dgm. = 0·1 gm.)
10 decigrams	= 1 gram.	
10 grams	= 1 Decagram	(1 Dgm. = 10 gm.)
10 Decagrams	= 1 Hectogram	(1 Hgm. = 100 gm.)
10 Hectograms	= 1 Kilogram	(1 Kgm. = 1000 gm.)

Here, also **note** that

1 milligram = 0·1 centigram = 0·01 decigram = 0·001 gram.

That is, the units are altered by moving the decimal point.

The **British** or **F. P. S. unit of mass** is the **pound** (lb.), which is the mass of a cylinder of platinum preserved in London at the Board of Trade Offices.

Prefixes	Meaning
Milli ...	... $\frac{1}{1000}$
Centi ...	... $\frac{1}{100}$
Deci ...	... $\frac{1}{10}$
Unit ...	... 1
Deka ...	... $\frac{10}{1}$
Hecto ...	... $\frac{100}{1}$
Kilo ...	... $\frac{1000}{1}$

Remember that—[For Metric Equivalents vide Physical Tables (2)]

1 gram = 15·432 grains = 0·035274 ounce = 0·0022046 lb.
1 grain = 0·0648 gram. ; 1 ounce = 28·35 gram.
1 pound = 453·6 „ = 16 ounces = 700 grains.

The Indian "tola" has a weight of about 12 grains, so "one seer", or 80 tolas is equivalent to 960 grams, which is nearly equal to one kilogram, or 1000 grains.

7. **Time.**—The unit of time is the mean solar second, which is $\frac{1}{86400}$ of a mean solar day. An ordinary clock or watch keeps mean solar time. The mean solar day is the average value of the actual solar days throughout the year. There are 365·24 mean solar days in the solar year. Each mean solar day is divided into 24 hours, or $(60 \times 60 \times 24) = 86{,}400$ seconds.

8. **Advantages of the Metric (C. G. S.) System.**—(1) Each unit is exactly ten times the next smaller unit. Hence the reduction from one unit to another is effected simply by the proper shift of a decimal point. Thus, 1·234 metres = 123·4 cm. = 1,234 mm.

But, in the British system, cumbersome multiplications and divisions are necessary in reducing a quantity from one unit to another, *i.e.* from feet to inches, or ounces to pounds.

(2) The units of length, volume, and mass are conveniently related. Thus, knowing that the mass of one cubic centimetre of water at 4°C. is one gram, we can write down at once the volume of any amount of water in cubic centimetres, if we know its mass is grams, and *vice versa.*

For example, the mass of 10 litres or 10,000 cubic centimetres of water = 10,000 grams ; and the volume of 10,000 grams of water = 10,000 cubic centimetres (or 10 litres). In the British (F. P. S.) system inconvenient constants have to be remembered, *viz.* the mass of 1 cubic foot of water = 62·5 pounds, 1 quart = 69·278 cubic inches, etc.

(3) This system has been adopted throughout all countries by scientific men.

Questions

Arts. 3, 4 & 8.

1. Name the system of units commonly used for the measurement of physical quantities. Which of them is more scientific, and why?

Explain, with examples, what you mean by Fundamental Units, Derived Units, and Practical Units as distinct from Absolute Units. (Pat. 1938)

Arts. 6 & 8.

2. Name the prefixes employed with metric units, and show how much they increase or decrease the value of the unit. What are the advantages of the metric system?

CHAPTER II

Measurement

9. Measurement of Length.—The length of an object can ordinarily be measured, according to the metric system, by a **metre scale,** where divisions smaller than a millimetre are to be estimated by the eye. If we wish, however, to obtain the measurement with greater precision, it can be done by means of an auxiliary scale, called a **Vernier**, after the name of its inventor, Pierre Vernier, a Belgian mathematician.

10. The Vernier.—The vernier is a short scale by the help of which fractional part of a main scale division can be measured and is so arranged as to slide along the principal or main scale. Verniers may be *straight* or *angular* according to requirements.

Fig. 1 represents a scale in mm. having a movable scale, called the *vernier*, by which readings to one-tenth of a scale division may be read.

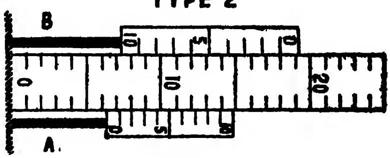


Fig. 1—Vernier.

General Theory.—The vernier is so divided that a certain number n, of its divisions is equal to $(n-1)$ or $(n+1)$ principal scale divisions.

If v = value of one vernier division ; s = value of one scale division,

we have, $(n \mp 1).\ s = nv$; or $v = \frac{n \mp 1}{n}.s.$

$\therefore$ *The least count = Diff. of s & v = 1/n × s.*

So the vernier is said to read *1/n* th of a scale division.

How to use a Vernier.—(1) Find the *value* in fractions of an inch, or centimetre, or degree (if it is a circular vernier), of *the smallest division of the principal scale.* Let it be 1 mm. *i.e.* 0·1 cm. in Fig. 1.

(2) Count the number of divisions on the vernier, and slide the vernier at one end (*i.e.* to the zero position) to see the number of scale divisions to which these are equal.

(a) Vernier Type (1) :—

In Fig. 1 (Type 1), 10 vernier divisions = 9 scale divisions:

(3) Calculate the difference in length between one scale division and one vernier division. This is the smallest amount—called the **least count** (or **vernier constant**)—which can be read with the help of the instrument.

Here, 10 vernier divisions = 9 scale divisions.

$\therefore$ 1 vernier division = $\frac{9}{10}$ scale division.

$\therefore$ Least Count = 1 sc. div. − 1 ver. div.

$= (1 - \frac{9}{10})$ sc. div. $= \frac{1}{10}$ sc. div.

$= \frac{1}{10} \times \frac{1}{10}$ cm. ($\because$ 1 sc. div. = 1 mm.) = 0·01 cm.

(4) Now put the object to be measured on the scale, one of its ends being at zero. The vernier then is pushed along the scale until its zero just touches the opposite end of the object. Read the principal scale just before the zero of the vernier. It is 6 in Fig. 1 (type 1). Then, the length of the object A is greater than 0·6 cm. (but less than 0·7 cm.) by the distance between the 6th division of the principal scale and the vernier zero. To get this length,

(5) Look along the vernier to see which of its divisions coincides with a scale division. In Fig. 1 (type 1), 2nd vernier division coincides with a scale division. *Multiply this number by the least count and add this to the reading obtained from the principal scale.* This is called a **forward reading** (or **positive**) **vernier**.

The value of the fraction of the scale division between the 6th and the vernier zero = (2 × 0·01) cm. = 0·02 cm.

$\therefore$ The length of the object = 0·6 + 0·02 cm. = 0·62 cm.

Verify thus :—

The length of the object A (Fig. 1) = 8 sc. div. − 2 ver. div. = 8 mm. − (2 × $\frac{9}{10}$) sc. div. (for 1 ver. div. = $\frac{9}{10}$ sc. div.) = 8 mm. − $\frac{9}{5}$mm. = $\frac{31}{5}$ mm. = 0·62 cm.

(b). Vernier Type (2) :—In type 1, the vernier division is smaller than the scale division, but sometimes, though very rarely, the vernier division is larger than the scale division in which case $(n+1).s = nv$ (see Art. 10). This form (type 2) has the *advantage* of having its divisions further apart than in the other form, and so easier to read, but this has got the *disadvantage* that the *numbers* on the vernier run the *opposite way* to the number of the principal scale. This is called a **backward reading** (or **negative**) **vernier**.

In the second form (Fig. 1, type 2) we have, 10 ver. div. = 11 sc. div.

$\therefore$ 1 ver. div. $= 1\frac{1}{10}$ sc. div.

$\therefore$ Least Count = 1 ver. div. − 1 sc. div. $= \frac{1}{10}$ sc. div. = 0·01 cm.

For the object *B*, the reading from the principal scale is 0·7 cm., and the 6th division on the vernier coincides with a division on the principal scale ; therefore the length of the object *B* = 0·7 cm. + the length of (10 − 6) or 4 vernier divisions = (0·7 + 4 × 0·01) cm. = 0·74 cm.

Note.—All verniers are not exactly the same as the one described, but by adopting the same rules, as given above, any vernier can be read.

11. The Slide Calipers.—The principle of the vernier is applied to a number of measuring instruments of which the simplest is the **slide calipers** (Fig. 2). A slide calipers consists of two steel jaws : one of them A is fixed at one end, and the other B slides along a scale etched on a thin steel bar R. The movable jaw is provided with a vernier V and a screw nut D for fixing it at any position on the rod. When the two jaws are in contact, the zero of the vernier should coincide with the zero of the fixed scale.

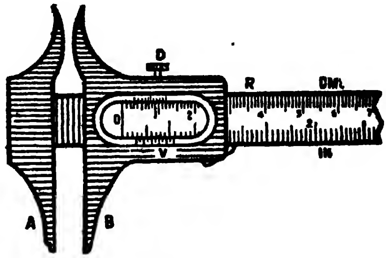


Fig. 2—Slide-Calipers

To use the instrument note whether the fixed scale is divided in millimetres or inches, and then determine the least count, as explained before. The object to be measured is so placed between the two jaws (upper jaws for internal diameters, lower jaws for external diameters) that the jaws just touch two ends of it, and then its length, or diameter is read as usual by noting how much the vernier zero has moved along the fixed scale. *Always take a number of readings and take the mean reading.*

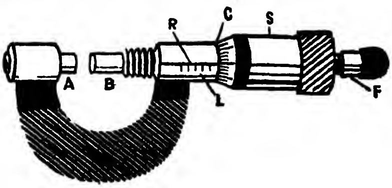


Fig. 3—Screw-Gauge

12. The Screw-Gauge.—The screw-gauze (also called the Micrometer Screw-Gauge) is used for measuring accurately the dimensions of small objects, such as the diameter of a wire, or the thickness of a metal plate. It (Fig. 3) consists of a fixed rod A having a plane end and a movable rod B having also a plane end facing A. The rod B has a screw cut on it (Fig. 4) and the screw works inside a hollow cylinder, called the **hub** having a straight scale L (**linear scale**) etched on it along R, called the reference line. This scale indicates the number of complete turns of the screw. The rod A and the hub are firmly held co-axially at the two ends of a strong metal bar bent in the U form. The screw is worked by means of a large screw-head S which moves over the outside of the hub. Fine adjustment of the screw-head is made by turning the head F, called the *friction clutch.* The edge of the screw-head has etched on it a **head scale** C (circular scale) divided into

a number of equal parts, usually 50 or 100, and is used for determination of the fraction of one complete rotation. One complete turn of the head scale moves the end of the screw through a distance equal to its pitch, which is the distance *P* between the consecutive threads of the screw (Fig. 4). *So pitch is the amount by which the gap between A and B is opened or closed by one complete rotation of the head scale.*

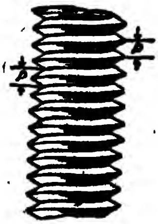


Fig. 4

How to use a screw-gauge.—

(1) Turn the head in order to expose the pitch scale. *Find the value of the smallest division of the pitch (linear) scale.* To do this, compare by dividers the length of, say, 10 divisions of the pitch scale with the subdivisions of a standard scale.

(2) Determine *the pitch of the screw.* To do this bring the zero mark of the head scale into coincidence with the line of graduation of the pitch scale. Now read the position of the graduated edge of the head scale on the pitch scale, and also read the position after giving the head one complete turn. The difference in the two readings is the pitch of the screw.

(3) Count the number of divisions of the head scale. If there are, say, 50 divisions, the screw makes $\frac{1}{50}$th of a turn when a mark on the head scale goes out of coincidence and the next comes into coincidence with the line of graduation. So the gap widens or narrows by $\frac{1}{50}$th of the pitch of the screw. *This quantity*—which is obtained by dividing the pitch by the number of divisions of the head scale—*is the* **least count** *or the smallest reading measured by the instrument.*

(4) *Determine the zero error.* Screw up until the gap between A and B is just closed. Now the zero mark of the head scale should coincide with the zero mark of the pitch scale. If they do not coincide, there is an error in the instrument known as the zero error. The value of it is obtained by oberving what mark of the head scale coincides with the line of graduation when the gap is closed. The zero error is *added* to the readings of the instrument if the gap is slightly open, when the zero mark of the head scale coincides with the zero on R, the line of graduation, and *subtracted* if the gap is closed before the coincidence takes place.

(5) Now place the wire, or any other small object, in the gap between A and B and turn the screw-head until it is just gripped between the jaws. Read the last visible division of the pitch scale and that of the head scale, which is just over R, the line of graduation

of the pitch scale. To this apply the zero error, if any, which may be added or subtracted as the case may be.

N. B. Care must be taken not to hurry the movement of the screw nor to screw up too tightly.

13. Spherometer.—The spherometer is constructed on the same principle as the screw gauge. It is generally used for determining the radius of curvature of spherical surfaces (hence the name) such as those of lenses, and also for measuring the thickness of plates.

The instrument (Fig. 5) rests on three fixed legs, the feet of which are at the same distance from one another—*i.e.* their ends A, B and C occupy the corners of an equilateral triangle. The legs support a a frame-work in which works a micrometer screw having a pointed end O which is equidistant from the three fixed legs. The screw terminates above in a milled head *M*, and a large circular disc *D*, the edge of which is divided into 50, or 100 equal parts. A vertical scale *S* usually divided in millimeters, and having its divisions close to the edge of the disc *D*, is fixed at one end of the frame-work.

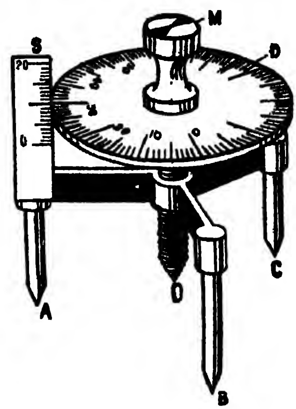


Fig. 5—Spherometer

How to use a Spherometer.—

(1) Examine the vertical scale *S*, and *find the value of the smallest division of it*. To do this, compare by dividers the length of, say, 10 divisions of this scale with the subdivisions of a standard scale.

(2) *Determine the pitch of the screw.* Bring the zero mark of the disc *D* on the edge of the vertical scale S. Read the position of the disc's edge on the vertical scale S, and then rotate the disc through a number of complete turns by means of the milled head *M*, and again read the position on *S*. The **pitch** of the screw is given by the difference in the two readings of the vertical scale divided by the number of turns. If by giving, say, 2 complete turns of the disc the difference in the vertical scale readings is 1 mm., then the pitch of the screw is $\frac{1}{2}$ mm.

(3) Count the number of divisions, say, 100, on the disc *D*. When the disc is turned, each division of its scale, as it passes the vertical scale *S*, shows that the screw has made $\frac{1}{100}$th of one turn, and that the point of the screw has travelled through $\frac{1}{100}$th of its pitch, which is $\frac{1}{2}$ mm. *This quantity*—$(\frac{1}{100} \times \frac{1}{2})$ mm.—*is the* **least count** *of the*

instrument, i.e. it is the smallest amount by which the advance of the screw point can be measured.

(4) *Determine the zero error.* Place the spherometer on a truly horizontal surface, usually the surface of a polished glass plate, called the base plate ; and turn the screw down until its point O just touches the surface. The exact position can be determined by turning the screw steadily and carefully until there is a slight jerk at contact. If the screw point projects forward just a little too far, the whole instrument rotates on the screw as a pivot, but will no longer do so if the screw is raised in the slightest degree. (*This is probably the best test to decide whether the screw is just making contact*).

The four feet of the spherometer are now in one plane. If in the above adjustment the zero (0) of the disc D is against the zero (0) of the vertical scale S, the instrument has no zero error. But, if this is not the case, read the number of divisions between the zero of the disc-scale and the edge of S, which is the **zero error**. Repeat the observation several times and take the mean of the readings as the *zero error. This quantity must be subtracted algebraically from all readings taken with the instrument.*

Note.—If the zero of the disc-scale is **above** the zero of the vartical scale, the difference of the positions of these two zero marks—which is the zero error—is taken as **positive**, and the quantity is to be *subtracted* from the total reading. If the zero of disc-scale is **below** the zero of the vertical scale, the zero error is **negative**, and it should be *added* to the total reading.

For example, let the error be 3 divisions of the disc-scale behind its zero, *i.e.* below the zero of the vertical scale, then the value of the zero error is—$(3 \times 0{\cdot}005) = -0{\cdot}015$ mm. (taking $0{\cdot}005$ mm. as the least count). If now the reading taken with the instrument be, say, $1{\cdot}27$ mm., the corrected reading will be $\{1{\cdot}27 - (-0{\cdot}015)\} = 1{\cdot}285$ mm.

Had the error been 3 divisions of D ahead of its zero, the value of of the zero error would have been $+0{\cdot}015$ mm., and in that case the corrected reading would have been $\{1{\cdot}27 - (+0{\cdot}015) = 1{\cdot}255\}$ mm.

14. To measure the thickness of a plate of glass (by **spherometer**).

(a) First Method :—First determine the zero error of the spherometer. Now raise the screw and place the plate underneath the screw point, and then take the readings of the vertical scale S, and the disc D, when the screw point just touches the top of the plate, while the other three feet of the instrument still stand on the plane surface. Repeat the observation several times at different

places on the surface of the plate and take the mean reading. The mean difference of these two sets of readings gives the average thickness of the plate.

Enter results thus :—

Particulars of the instrument—

(1) Value of 1 smallest division of the vertical scale = 1 mm.

(2) 8 complete turns of the disc move the screw point through 4 mm. Hence pitch of the screw $= \frac{4}{8} = 0{\cdot}5$ mm.

(3) No. of divisions of the disc scale = 100 ; So, least count $= \frac{0{\cdot}5}{100} = 0{\cdot}005$ mm.

(4) No. of divisions between the edge of the vertical scale *S* and the zero of the disc-scale *D* is 6. These divisions are behind zero.

$\therefore$ zero error $= -(6 \times 0{\cdot}005) = -0{\cdot}03$ mm. [So, this error should be added {see (4) page 10}.

Measurements—

Object measured—Thickness of a glass plate

No. of Observations	Reading in mm. Vertical scale	Disc scale	Least Count	Total	Zero error	Corrected thickness	Mean mm.
1	1·5	56	0·005	1·780	0·03	1·810	
2	1·5	54	,,	1·770	,,	1·800	1·801
3	1·5	53	,,	1·765	,,	1·795	

(b) Second Method—It is found with most of the spherometers that two complete turns of the disc are necessary to move the screw point through one division of the vertical scale.

At the time of taking any reading with such an instrument it is often found difficult to judge whether the reading indicated on the disc-scale is a fraction of the first or the second revolution after passing the last division of the verticle scale. For this, and also *to avoid confusion of zero error*, it is convenient not to take any account of the vertical scale reading. Instead of this, the movement of the screw point should be stated in terms of the rotation of the circular divisions only. That is,

(i) first note what division of the circular scale is against the edge of the vertical scale when the four feet of the spherometer stand on the top of the test plate and then, on removing the test plate,

(ii) count from this, the whole and fractional turns of the circular scale until the screw point just touches the base plate. If, for example, 2 whole turns and 56 small divisions of the third turn are necessary for this adjustment, the thickness of the plate = 2 whole turns + 56 = 256 divisions = 256 × 0·005 mm. ($\because$ 0·005 is the least count) = 1·28 mm.

15. To measure the radius of curvature of a spherical surface (by spherometer).—(1) Place the spherometer with the fixed legs resting on the curved surface, and adjust the screw until its point just touches the surface. Read the scales. Repeat the observation several times placing the instrument in different positions of the curved surface. Calculate the mean of the readings.

(2) Place the instrument on the plane glass plate and adjust the screw until its point touches the surface. Read the scales. Repeat it several times, and take the mean reading.

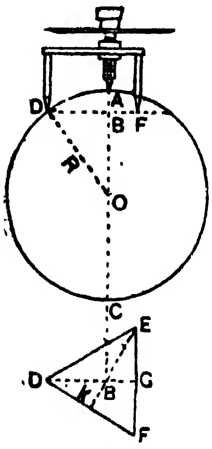


Fig. 6

Find the difference h between any two mean readings. This gives the vertical distance traversed by the screw point.

(3) Measure the distance d between any two fixed legs. To do this, place the instrument on a piece of paper and press gently so as to mark the positions of the three legs D. E. F. (Fig. 6).

Now measure carefully the mean distance d between these marks. Then the radius of curvature R is given by

$$R = \frac{d^2}{6h} + \frac{h}{2}$$

Note.—(1) As d enters as a square in the result, the measurement of d should be made very carefully up to a millimetre, otherwise a small error in this measurement will be magnified in the final result.

(2) *Do not forget to express both d and h in the same unit.*

(3) Here also the second method of measurement, as described above (Art. 14b), may be applied. That is, calculate the value of h from the readings of the *circular scale only* without taking any account of the vertical scale.

(4) When using a spherometer, it should be noted whether there is any slackness between the nut and the screw, because any such slackness will permit of appreciable rotation of the disc without producing any corresponding movement of the screw along its axis. This error, due to lost motion, called

the **back-lash** error, can be corrected for by always turning the screw in the same direction before taking any reading.

(5) The term $h/2$ can often be neglected in comparison with $d^2/6h$.

16. **Proof of the Formula.**—The diagram (Fig. 6) represents a side view of a spherometer resting on a spherical surface. The central leg A and two of the fixed legs of the spherometer are visible. AB represents the vertical distance h through which the central leg must be raised (or lowered) so that it just touches the curved surface. $BD(S)$ is the distance between any of the fixed legs and the central leg, when they are all resting on a plane surface. If R be the radius of curvature, we have $DO^2 = DB^2 + BO^2 = DB^2 + (AO - AB)^2$

$$\text{or } R^2 = S^2 + (R-h)^2 \text{ ; or } R = \frac{S^2+h^2}{2h} = \frac{S^2}{2h} + \frac{h}{2} \quad \ldots\ldots(1)$$

The formula (1) can be put into *another form*. When the central leg just touches the plane of the other three legs, let B be the position of the central leg, and D, E, F, the positions of the other three legs, which form an equilateral triangle (Fig. 6, lower). The angle GDF is 30°, and K is the middle point of DF, the length of which, say, is d.

$DK = DB \cos 30° = S. \sqrt{3}/2.$ *or* $d/2 = S. \sqrt{3}/2$, *or* $d^2 = 3S^2$.

Substituting the value of S^2 in (1) we have

$$\mathbf{R} = \frac{\mathbf{d}^2}{\mathbf{6h}} + \frac{\mathbf{h}}{\mathbf{2}}.$$

N. B. The method of measurement is the same for both convex and concave surfaces.

17. **Measurement of Area.**—In measuring the length of a straight line it is only possible to take *one* observation. So length is said to have one dimension. But in order to measure an area, say the area of a rectangle, two lengths (length and breadth), must be measured. So **an area has two dimensions.**

Units.—The unit of area in the British system is the square foot or the square inch, while in the Metric system, it is the square centimetre.

METRIC TABLE OF AREA

100 sq.	millimetres	=	1 sq. centimetre.
100 sq.	centimetres	=	1 sq. decimetre.
100 sq.	decimetre	=	1 sq. metre.

17 (a). Areas of Regular Figures.—In order to measure the areas of regular geometric figures,

Remember the following.—

Area of rectangle = length × breadth.
,, ,, parallelogram = base × perpendicular height.
,, ,, triangle = $\frac{1}{2}$ × base × perpendicular height.
,, ,, trapezium = $\frac{1}{2}$ × sum of parallel sides × perpendicular height.
,, ,, circle = π × (radius)^2.
,, ,, ellipse = π × semi-major axis × semi-minor axis.

Area of curved surface of cylinder = 2π × radius of base × height.

Area of the surface of a sphere = area of the curved surface of the circumscribing cylinder $= 2\pi r \times 2r = 4\pi r^2$

(where r = radius of the sphere. r is also the radius of the base of the circumscribing cylinder, and the height of the cylinder $= 2r$.)

17 (b). Areas of Irregular Figures.—The area of an irregular figure can be measured,

(*i*) **By squared paper.**—Draw an outline of the figure on the squared paper provided. The boundary of the figure (Fig. 7) passes through a number of small squares on the paper. Now count the total number of complete squares and then count round the boundary line, *ignoring those which are less than half a square, but counting as whole squares those that appear to be more than half squares.* In the case of exact halves, take two to count as one square.

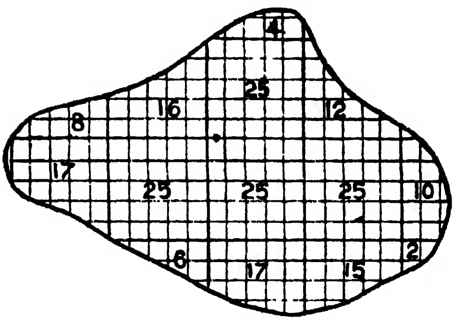


Fig. 7

N. B. This is only a rough method, and the result will not be very accurate. This method, however, can readily be applied to find the area of a country by drawing to scale, on squared paper, a map of its boundary. If the above figure represents the boundary of the map of a country, then the area of the country can be calculated as follows :—

Scale of map = (80 miles = 1 in.) ; *or* 1 sq. in. = 6400 sq. miles.
Hence, area of the country = 6400 × area on graph paper in sq. inches. (sq. miles).

(*ii*) **By weighing.**—Draw the figure on a thin sheet of cardboard, or a thin metal plate, whose thickness should be as uniform as possible. Cut the figure out of it, and weigh it accurately. From the same sheet cut an area, the shape of which may conveniently be a rectangle, or a triangle, and find its weight. Calculate the area of the

rectangle, or the triangle, from its linear dimensions. Then calculate the area of the figure from the relation,

$$\frac{\text{area of figure}}{\text{area of rectangle}} = \frac{\text{weight of figure}}{\text{weight of rectangle}}.$$

18. Measurement of Volume.—The space occupied by a body is called its volume. In order to measure the volume of a body three lengths, *i. e. length, breadth* and *height* or thickness should be considered. Therefore **a volume has three dimensions.**

Units.—The unit of volume in the British (F. P. S.) system is the cubic foot or the cubic inch, while the unit in the Metric (C. G. S.) system is the cubic centimetre (c.c.).

METRIC TABLE OF VOLUME

1000 cubic millimetres = 1 cubic centimetre (c.c.)
1000 cubic centimetres = 1 cubic decimetre (1 litre)
1000 cubic decimetres = 1 cubic metre (c. m.)

BRITISH TABLE OF VOLUME

1 cubic foot = 1728 cubic inches (cu. in.)
1 cubic yard = 27 cubic foot (cu. ft.)

[For Equivalents Vide Physical Tables (2)]

19. Remember the following :—

The litre is the volume of 1 kilogram of cold water.
One gram of cold water fills 1 cubic centimetre.
One fluid-ounce equals 28·35 cubic centimetres.
One cubic foot equals 28·31 litres.
The gallon is the volume of 10 pounds of cold water.
One fluid-ounce is the volume of 1 ounce of cold water.

20. Volumes of Regular Solids.—To calculate the volume of a solid having some regular geometrical figure,

Remember the following :—

Volume of rectangular solid = length × breadth × height.
,, ,, cylinder = area of base × height
$= \pi r^2 h$, (where πr^2 = area of base ; and h = height)
,, ,, pyramid or cone = $\frac{1}{3}$ × area of base × height.
,, ,, sphere = $\frac{4}{3}\pi$ × (radius)^3.

20 (a). Volume of a sphere $= \frac{4}{3}\pi r^3$.

Proof.—The surface of a sphere can be imagined to be divided into an infinite number of small figures (Fig. 8), each of which is practically a plane surface and may be considered to form the base of a pyramid

having a height equal to the radius r of the sphere, *i.e.* with its top at the centre of the sphere. The sum of the bases of all the pyramids is the whole surface of the sphere, and the sum of all these small pyramids is the volume of the sphere.

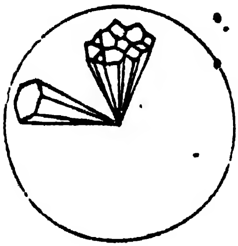


Fig. 8

The volume of a small pyramid = $\frac{1}{3}\times$ area of base $\times$ height.

$\therefore$ The volume of the sphere = $\frac{1}{3}\times$ sum of the bases of all the pyramids $\times$ height

$= \frac{1}{3}\times$ surface of the sphere $\times$ radius

$= \frac{1}{3}\times 4\pi r^2 \times r = \frac{4}{3}\pi r^3$

[$\because$ surface of a sphere $= 4\pi r^2$.]

20 (b). Volumes of Irregular Solids.—The volume of an irregular solid as well as that of a regular one can be determined,

(a) **By Displacement of Water.**—The volume of a small solid may be directly obtained by lowering it carefully into water contained in a graduated vessel, and noting the rise of the surface of the water. The rise of the surface, *i.e* the difference between the first and second positions of the meniscus, gives the volume of water displaced by the solid ; and, as a body immersed in a liquid displaces its own volume of the liquid, the difference between the two positions of the meniscus gives the volume of the solid.

When the body is too big to go inside the measuring vessel, secure a fairly large vessel and attach a narrow piece of gummed paper vertically to the side of it. Put a horizontal pencil mark at a level which will be well above the top of the immersed solid. Pour water in the vessel until its surface is level with the pencil mark.

If now the solid is introduced, an equal volume of water will be displaced or pushed above. Put another mark on the surface of water again. Then take out carefully by a pippette the amount of displaced water—*i.e.* the amount of water between the two pencil marks, and measure it by a graduated vessel. This will give the volume of the solid.

Note.—(i) If the solid floats in water, push it by a needle fixed to the end of a wooden pen-holder until the solid is completely immersed.

(ii) When the solid is soluble in water, use instead of water, some other liquid, say, alcohol or kerosene, in which it is not soluble.

(b) **By weighing.**—Knowing that at ordinary temperatures *one cubic centimetre of water weighs one gram*, the volume of a small solid can be accurately determined by weighing the amount of water displaced by it.

If the weight of the displaced water is, say, 10 gms., then the volume of the displaced water is 10 c.c. (because the volume of 1 gm. of water is equal to 1 c.c.), and so the volume of the solid is also 10 c.c. So, the weight in grams of the displaced water is numerically equal to the volume of the body in cubic centimetres.

21. Measurement of Mass.—The mass of a body is ordinarily measured by means of a common balance (see Art. 62). It can also be measured by a spring balance after calibrating it (see Art. 66.)

22. Measurement of Time—Before the invention of clocks and watches Sun-dial was used for measuring time. This consists of a horizontal board which has graduations from I to XII as those on a clock. A stick is fixed on the board at a definite angle and the particular period of the day is indicated by the position of the shadow of the stick thrown on the graduated board by the sun at that time. Besides this, other methods such as water-clock, hour-glass, etc., were used by the ancient people to measure time.

After the discovery of the laws of pendulum by Galileo, it was possible to measure time by clocks and watches. The length of the rod in a second's pendulm can be so chosen as to take one second to swing from one extremity to the other (see Art. 76).

Metronome is an instrument to mark time. It has a mechanism run by clock-work to move the pendulum by which 'ticks' can be heard at the end of each swing. The 'ticking time' can be altered by adjusting the position of a sliding weight on the pendulum rod.

Stop-watch.—It is a type of watch often used in the laboratory for measuring fraction of a second. It has a large second's hand which moves once over the dial in one minute. The watch can be started or stopped by pressing a knob.

Questions

Art. 17 (a).

1. The area of the surface of a sphere is 154 sq. cm. Find its radius.

[*Ans* : 3·5 cm.]

2. Assuming the earth to be spherical, calculate its surface area in square miles, taking its diameter to be about 8000 miles.

[*Ans* : $2{\cdot}0114 \times 10^8$ sq. miles.]

3. A circular ring is enclosed between two concentric circles whose radii are 119 ft. and 167 ft. long respectively. Find the length of the radius of a third concentric circle which will divide the ring into two rings whose areas shall be equal to one another. (C. P.)

[*Ans* : 145 ft.]

Art. 17 (b).

4. How would you measure the area of an irregular figure drawn on a sheet of paper ?

Art. 20.

5. Calculate the quantity of gas in cubic feet contained in a cylindrical gasometer having a height of 150 ft. and diameter 150 ft.

[*Ans* : 2,650,000 cu. ft.]

Art. 20 (b).

6. How will you find the volume of a solid of irregular shape ?

(C. U. 1917, '29 ; Dac. 1932.)

MECHANICS

CHAPTER III

Motion and Rest

23. Vector and Scalar Quantities.—Any physical quantity, which requires both magnitude and direction for its complete specification, is called a **vector quantity,** and other quantities having magnitudes only are called **scalar quantities.**

Thus, if a man moves 5 yards away from a place, he may be anywhere on the circumference of a circle of 5 yards radius about his first position, *i.e.* there is no particular direction. But if it is stated that he moves 5 yards *south*, his new position is completely specified. Therefore, to describe a displacement, it is necessary to give both its *magnitude* and *direction*. So, **displacement is a vector quantity**.

Any vector quantity can be represented by a straight line whose length is proportional to the magnitude of the vector, and whose direction, shown by an arrow sign, represents the direction of the vector.

Similarly, *velocity*, *force*, etc., which involve the idea of magnitude as well as direction, are examples of *vector* quantities ; and *speed*, *time*, *mass*, *volume*, *density*, which have got magnitudes only, are *scalar* quantities.

24. Motion and Rest.—A body is in motion when it changes its position with time, and it is at rest when it does not change its position for any length of time. Motion and rest may be *absolute* or *relative*.

Absolute motion is the motion of a body with respect to a fixed point in space, but as no such fixed point is known in the universe, absolute motion cannot be realised in nature. **Absolute rest** is also impossible in nature, for the earth and other heavenly bodies being always in motion, every body in the universe is in motion, and the term absolute rest also has got no significance.

If a body does not change its position with respect to its surrounding objects, it is said to be at **relative rest**, and when a body changes its position with respect to certain objects, it is said to be in **relative motion**,

A passenger in a railway train is in relative rest with respect to other passengers and to the train as well ; but he is in a state of relative motion with respect to trees and houses on the roadside.

25. Force, Velocity, Speed and Acceleration.

(a) **Force** is that which, acting on a body, changes or tends to change the state of rest or of motion of the body. (See Chap. IV).

Force is a vector quantity, so it can be represented by a straight line with an arrow.

(b) The **velocity** of a moving body is its rate of change of position in a given direction.

(c) **Speed** is simply the distance covered in a given time. It is a scalar quantity having got *no direction.*

Velocity is a vector quantity. So, to specify a velocity completely, its *magnitude* as well as its *direction* must be stated ; but to specify speed completely, it is necessary to state only its magnitude.

Hence *velocity is speed in some particular direction.* If a moving body traverses equal distances in equal times, the velocity is said to be *uniform.* In other cases, the velocity is said to be *variable.*

To understand clearly the difference between speed and velocity, take, for example, the case of a motor bicycle travelling round a circular track at a constant rate. In this case, the speed of the bicycle is constant, but its velocity is variable.

When the velocity is variable, its average value can be taken by dividing the total distance travelled by the total time taken. So, in the case of variable velocity, the value obtained as above does not represent the actual velocity, but the average velocity during that interval.

Units of Velocity.—A body has unit velocity when it traverses unit distance in unit time. So,

the **F.P.S.** unit is **one foot per second.**	the **C. G. S.** unit is **one centimetre per second.**

(d) **Acceleration** is the rate of change of velocity. Acceleration is *uniform*, when equal changes in velocity occur in equal intervals of time. In other cases, it is *variable*.

Suppose a body starts with a velocity of 10 ft. per sec., and at the end of the first second the velocity becomes 12 ft. per second, then during the interval of one second the velocity of the body has increased by 2 ft. per second. If, again, at the end of successive seconds, the velocity becomes 14, 16, 18, etc., ft. per sec., then the change of velocity of the body is uniform, and is effected at the rate of 2 ft. per sec. in each second ; so the rate of change of velocity, *i.e.* the acceleration of the body, is 2 ft. per sec. per sec. In this case, the velocity is increased by equal amounts in equal intervals of time. So, it is a case of *uniform* acceleration.

Acceleration, like velocity or force, is **a vector quantity,** and has both direction and magnitude. So, like velocity or force, it can be represented by a straight line with an arrow.

In acceleration, the **unit of time** comes **twice,** because it involves both change of velocity, and also the time in which the change has been effected.

A falling stone is gradually increased in velocity vertically downwards by 32 ft. per second in every second ; so, the acceleration of the stone will be expressed as 32 ft. per second per second, or 981 cm. per sec. per sec. (or cm. per sec.^{2}).

Units of acceleration.—A body has unit acceleration, if its velocity changes by unity in unit time.

The **F. P. S.** unit is **one foot per sec. per sec.**	The **C. G. S.** unit is **one centimetre per sec. per. sec.**

Retardation.—When a moving body gradually comes to rest, its velocity diminishes, and the rate of diminution is known as **retardation**. *A retardation is a negative acceleration.* A stone thrown vertically upwards has negative acceleration, or retardation, till it stops and begins to fall. If the velocity of a train entering a station decreases 2 ft. per sec. in a second, we say its acceleration is—2 ft. per sec. per sec. or retardation is 2 ft./sec.^{2}.

26. Angular velocity.—If a line revolves about a fixed point in it in a fixed plane, the rate of describing the angle is known as its *angular velocity*.

If in a time t, the angle described be θ (pronounced "*theta*"), then the angular velocity ω (pronounced "*omega*")

$$\omega = \theta/t \text{ degrees per second.}$$

But the angular velocity is generally expressed in the circular measure, *i.e.* in *radians** per second.

In one complete revolution, four right angles are described and the circular measure of four right angles is 2π (radians). Hence, if t be the time for one revolution, $\omega t = 2\pi$ or $\omega = 2\pi/t$ radians per sec.

If any body makes n revolutions per minute, the time for one revolution $= t = 60/n$ sec.

$\therefore$ The angular velocity of the body $\omega = 2\pi \div 60/n$

$= 2\pi \times n/60 = \pi n/30$ radians per sec.

27. Linear and Angular velocity.—Let ω be the uniform angular velocity of a particle moving round the circumference of a circle of radius r. If t seconds be the time for one complete revolution,

$t = 2\pi/\omega$ sec. ($\because$ the angle turned through is 2π radians.)

Again, if v be the linear velocity of the particle,

$$t = \frac{\text{circumference}}{v} = \frac{2\pi r}{v} \text{ sec.}$$

Hence, $\quad 2\pi r/v = 2\pi/\omega$; $\qquad$ or $v = \omega r$............(1)

Thus, the linear velocity of any particle of a rotating body is directly proportional to its distance from the axis of rotation ; and is obtained by the product of the angular velocity and this distance.

Example.—*A circus horse trots round a circular path at a speed of 8 miles an hour, being held by a rope 20 ft. long. Find the angular velocity of the rope.*

$$8 \text{ miles an hour} = \frac{5280}{60 \times 60} \times 8 \text{ ft. per sec.}$$

Here $\qquad v = \frac{5280}{60 \times 60} \times 8$; $r = 20$;

$$\therefore \quad \frac{5280}{60 \times 60} \times 8 = 20\omega.; \text{ from eq. (1), Art. 27.}$$

or $\qquad \omega = 0{\cdot}58$ radians per sec.

28. Equations of Motion.—

(i) Distance traversed in t secs. by a body moving with uniform velocity v.

If the body moves with a uniform velocity v, then, by definition, v is the distance traversed by the body in each unit of time.

---

*One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

Hence, in 2 units of time the total distance traversed is $2v$;
,, 3 ,, ,, ,, ,, ,, $3v$;
and so, ,, t ,, ,, ,, ,, ,, tv ;
Therefore, if s be the distance traversed in time t,

$$\mathbf{s = vt.}$$

Example.—*A train moves at the rate of 60 miles an hour. Express its velocity in feet per second.*

1 mile = 5280 ft. ; ∴ 60 miles = 60 × 5280 ft. ; and 1 hour = (60 × 60) secs.
So, the train moves (60 × 5280) ft. in (60 × 60) secs.

or $$v = \frac{60 \times 5280}{60 \times 60} = 88 \text{ ft. per sec.}$$

✓(**Remember** that **60 miles per hour = 88 feet per second).**

(ii) Velocity v acquired in time t secs. by a body moving with a uniform acceleration of f ft. per sec. per sec.

Suppose that the body starts with a velocity u. Since the acceleration of the body is f, the velocity of the body is increased in each second by a velocity of f ft. per sec.

∴ *at the end of 1 sec. the velocity is* $u+f$;
,, ,, 2 *secs.* ,, ,, $u+2f$;
,, ,, 3 ,, ,, ,, $u+3f$;
and so ,, ,, t ,, ,, ,, $u+tf$.

Hence $$\mathbf{v = u + ft.}$$

or $$v - u = ft.$$

or *Increase of velocity = acceleration × time* :

and $$f = \frac{v-u}{t}.$$

or *Acceleration = increase of velocity ÷ time.*

Examples.—*(1) A body starts from rest and acquires a velocity of 8 kilometres in 2 minutes. What is its acceleration ?*

8 kilometres per sec. = 800000 cms. per sec. ; 2 minutes = 120 secs.

Here $v = 800000$; $u = 0$; $t = 120$; $f = ?$

$$v = u + ft \text{ ; or } 800000 = 0 + f.\,120 \text{ ;}$$

or $$f = 6666{\cdot}66 \text{ cms. per sec. per sec.}$$

✓*(2) A body starts with a velocity of 144 ft. per sec., and is subject to a retardation of 32 ft. per sec. What is its velocity after 10 seconds ?*

Here $u = 144$; $f = -32$; $t = 10$; $v = ?$

We have $$v = u + ft = 144 + (-32) \times 10 = 144 - 320 = -176.$$

Hence the body is moving with a speed of 176 ft. per sec. in the opposite direction to that in which it started.

(iii) Distance traversed in t secs. by a body moving with a uniform acceleration of f ft. per sec. per sec.

The velocity of the body at the beginning is u, and at the end of the time t is $u+ft$.

Therefore, the average velocity during this time,

$$v=\frac{u+(u+ft)}{2}=u+\tfrac{1}{2}ft.$$

But since the velocity increases uniformly, the distance s traversed in time t must be the same, as if the body had moved for time t with this average velocity.

$$\therefore\ s=\textit{average velocity}\times \textit{time}=(u+\tfrac{1}{2}ft)\times t=\mathbf{ut}+\tfrac{1}{2}\mathbf{ft^2}.$$

✓**Example.**—*Calculate the initial velocity of a train which runs down 325 feet of incline in 10 seconds with a uniform acceleration of 2 ft. per sec. per sec.*

Here $\quad s=325\ ;\ t=10\ ;\ f=2\ ;\ u=\ ?$

$\therefore \quad 325=10u+\tfrac{1}{2}\times 2\times 10^2=10u+100\ ;$

or $\quad 10u=325-100=225.$ Hence $u=22{\cdot}5$ ft. per sec.

(iv) Velocity of a body, starting with a velocity u, acquired in distance s.

From Eq. in (*ii*), $v^2=(u+ft)^2=u^2+2uft+f^2t^2=u^2+2f(ut+\tfrac{1}{2}ft^2)$
$=\mathbf{u^2+2fs}$ ... from Eq. (*iii*).

✓**Example.**—*A train runs at a speed of 30 miles per hour. The brakes are then applied so as to produce a uniform acceleration of -2 ft./sec^2. Find how far the train will go before it is brought to rest.*

30 miles per hour $=\dfrac{88\times 30}{60}=44$ ft./sec.

Here $\quad u=44$ ft./sec. ; $v=0$; $f=-2$ ft./sec.^{2} ; $s=\ ?$

we have $\quad v^2=u^2+2fs\ ;\quad$ *or* $\quad 0=44^2+2(-2\times s)=44^2-4s$

$$\therefore \quad s=\frac{44\times 44}{4}=484\ ft.$$

Special cases—When the body starts from rest, we have $u=0$, and the above formula takes the simple forms ;

$$(i)\quad v=ft\ ;\ (ii)\ s=\tfrac{1}{2}ft^2=\tfrac{1}{2}vt.\ ;\ (iii)\ v^2=2fs.$$

29. To calculate the distance traversed in any particular second.

The distance traversed in the nth sec. = the distance traversed in n seconds − the distance traversed in $(n-1)$ seconds =

$(un+\frac{1}{2}fn^2)-\{u(n-1)+\frac{1}{2}f(n-1)^2\}$...from Eq. (iii), $\qquad = u+\frac{2n-1}{2}f.$

General Hints.—In working out problems,—

(i) Set down all the values of the given quantities and the symbol for the quantity required, and then consider what equation, out of those given above, connects them. From this equation, find the unknown quantity.

(ii) Remember that all the symbols involved in the above equations are algebraic, *i.e. may represent either positive or negative quantities.*

Example.—(1) *A body is thrown up with a velocity of 32 feet per second. Find how high it will rise.*

The body will rise till its velocity is zero, after which it begins to fall and its velocity becomes negative.

Here $u=32$ *ft./sec.* ; $v=0$; $f=g=$ accel. due to gravity $=-32 ft/sec.^2$; $s=$?
We have $\qquad v^2=u^2+2fs.$ or $0=(32)^2+2\times(-32)\,s$;

$$\therefore\ s=\frac{32\times 32}{2\times 32}=16\ ;\ \therefore\ \text{The body will rise 16 ft.}$$

(2) *A body travels 100 ft. in the first two seconds and 104 ft. in the next four seconds. How far will it move in the next four seconds, if the acceleration is uniform ?*

Here $\qquad s=100\,ft$; $t=2$ *sec.* ; $u=$? $f=$?

we have $s=ut+\frac{1}{2}ft^2$; *or* $100=2u+\frac{1}{2}f\times 4$; *or* $u+f=50$......(1)

Motion during the first six seconds,

$s=100+104=204\,ft.$; $t=6$ *sec.* ; $u=$? ; $f=$?

$204=6u+\frac{1}{2}f\times 36$; $204=6u+18f$; *or* $34=u+3f$..............(2)

From (1) and (2), $f=-8\,ft./sec.^2$, $u=58\,ft./sec.$

Considering the motion during the total time (10 secs.),

$u=58\,ft./sec.$; $t=10$ *sec* ; $f=-8\,ft./sec.^2$: $s=$?

$s=ut+\frac{1}{2}ft^2$ $\therefore$ $s=58\times 10+\frac{1}{2}(-8)\times 10^2=580-400=180\,ft.$

Thus the distance travelled in the last four seconds
$=180-100-104=-24$ ft., *i.e.* it travels 24 ft. in the opposite direction.

30. Representation of a Force by a Straight line.—Every force has a certain magnitude and acts in a certain direction. A force is completely known if we know its (*i*) *Point of application, i. e.* the point at which the force acts ; (*ii*) *Direction ;* and (*iii*) *Magnitude.*

All these can be represented by a straight line provided that
(*i*) The line is drawn from the point of application of the force ;
(*ii*) The line is drawn pointing in the direction of the force ;
(*iii*) The length of the line is proportional to the magnitude of the force.

31. Composition of Forces—When two or more forces act simultaneously on a particle at rest, the particle may remain at rest, in which case the forces are said to be in equilibrium, or the particle may begin to move in a definite direction by the combined effect of the forces. The same effect can also be produced by the action of a single force and this force is called the **resultant** of the original forces. The original forces are themselves known as **components** ; and the process of finding out the resultant is known as the **composition of forces.**

(The composition of velocities can also be effected in a similar manner.)

32. Parallelogram of Forces.—*If a particle is acted on simultaneously by two forces, represented in magnitude and direction, by the two adjacent sides of a parallelogram drawn from a point, these forces are equivalent to a single resultant force, represented in magnitude and direction, by the diagonal of the parallelogram passing through the same point.*

Let the sides OA and OB of the parallelogram $OACB$ represent two forces P and Q in magnitude and direction inclined at an acute angle BOA (Fig. 9), and let the diagonal OC represent their resultant (R) in magnitude and direction. Produce OA to D, and drop CD perpendicular on OD.

Let $\angle BOA = \theta = \angle CAD$. Then we have,

$$OC^2 = (OA + AD)^2 + DC^2 = OA^2 + AD^2 + 2OA.\,AD + DC^2$$
$$= OA^2 + AC^2 + 2OA.\,AD \quad (\because AC^2 = AD^2 + DC^2)$$
$$= OA^2 + AC^2 + 2OA.\,AC \cos\theta \quad (\because AD = AC\cos\theta)$$

or $\mathbf{R^2 = P^2 + Q^2 + 2PQ\cos\theta}$.

If $\theta = 90^\circ$, $R^2 = P^2 + Q^2$ ($\because \cos 90^\circ = 0$.)

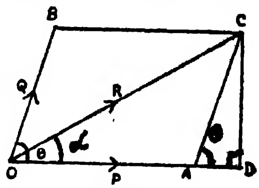


Fig. 9

The direction of the resultant is obtained as follows :—

Let the resultant make an angle a ($\angle COA$) with one of the component forces, say OA. Then $\tan a = \dfrac{CD}{OD} = \dfrac{CD}{OA + AD} = \dfrac{Q\sin\theta}{P + Q\cos\theta}$.

[**Note.**—If the angle θ be obtuse, D falls between O and A, but the value of R^2 remains unaltered.

The Parallelogram Law is also applicable in the case of velocity and acceleration.]

Experimental Verification.—Take a wooden board fitted with two frictionless pulleys (Fig. 10), and fix it vertically. Fasten a sheet of paper on the board. Take three strings and knot them together in a point O, and to their ends attach three weights $P(=3$ lbs.), $Q(=4$ lbs.), and $R(=5$ lbs.), any two of which are together greater than the third. Pass the two strings carrying the weights P and Q over the pulleys, and allow the third to hang vertically downwards with its weight R. Now the point O is in equilibrium under the action of these three forces.

Mark on the paper, by means of your pencil point, the direction of the forces, and, taking a convenient scale, say, an inch per pound weight, measure off along OK and OH, lengths OA and OB, containing 3 and 4 units to represent P and Q respectively. Complete the parallelogram $OACB$ and join OC. It will be observed that (i) the diagonal OC is vertical, and that (ii) OC is in the same straight line with OR, and contains R units (*i. e.* 5 units) of length in the same scale.

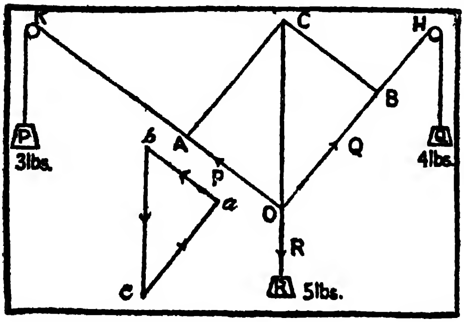


Fig. 10

Conclusion.—The knot O is in equilibrium under the action of three forces P, Q, and R. So the resultant of P and Q is equal and opposite to the force R (*i.e.* 5 lbs.) acting vertically upwards. But OC is vertical and it contains R units (*i.e.* 5 units) of length. Therefore OC represents the resultant in magnitude and direction of the forces P and Q represented by OA and OB respectively. Hence we find that *the diagonal of a parallelogram represents the resultant of the two forces which are represented separately by its sides.*

N. B. (i) The downward force R represented by OR at O, which is equal and opposite to the resultant of the forces represented by OA and OB and by which the system is kept in equilibrium, is called the **equilibrant** of those two forces.

(ii) The above experiment will be found to be true whatever be the relative magnitudes of P, Q, and R, provided that any one of them is not greater than the sum of the other two.

Illustrations.—(*i*) If a boat is pulled by two tugs in two different directions, and the forces exerted on the tugs are represented in magnitude and direction by two lines OA and OB respectively (Fig. 11),

then the boat, instead of moving in the direction of either of the forces OA or OB, will move along OD, the diagonal of the parallelogram constructed with OA and OB as adjacent sides. OD represents the resultant of those two forces.

(*ii*) If a man walks across the floor of a compartment of a railway train with a velocity represented by OA (Fig. 11) while the train itself is running with a velocity OB, the resultant velocity OD of the man can be obtained graphically in the same way.

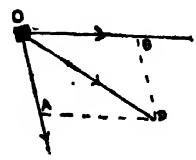


fig. 11

33. The Triangle of Forces.—On the same sheet of paper used in the above experiment (Fig. 10) draw a line parallel to the force OA, and, from this, measure off a length ab to represent, to a convenient scale, the magnitude of OA. From b draw bc parallel to the force OR and make its length represent, to the same scale, the magnitude of OR. In the same way draw another line ca parallel to the force OB and containing OB units of length. If the whole work has been accurately done, the end of the last line will coincide with the starting point a, and this line closes the triangle abc. Mark, by means of arrow heads, the directions of the forces on the sides of the triangle, and it will be found that the arrows point round the sides of the triangle in order. It is evident, therefore, that *if three forces, acting at a point, are in equilibrium, they can be represented in magnitude and direction by the sides of a triangle taken in order*. This is known as the principle of the **Triangle of Forces.**

[**Note** the expression '*taken in order*', which means that the direction of the forces must go round the sides of the triangle all in the same direction, *i.e.* either *clockwise* or *anti-clockwise*.]

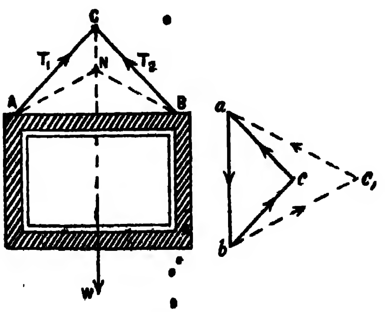


Fig. 12

The converse of this is also true. If three forces, acting at a point, can be represented in magnitude and direction by the sides of a triangle taken in order, they are in equilibrium.

Practical Problem—A Hanging picture. In Fig. 12, a picture is suspended by a string from a nail C. It is in equilibrium under the action of the following forces, (*i*) the weight W of the picture ; (*ii*) the tension T_1 of the string represented by AC ; and (*iii*) the tension T_2

represented by BC. The three forces meet in one point C, and are in equilibrium, so, by the principle of triange of forces, draw three lines ab, bc, ca representing in direction and magnitude the three forces W, T_1, and T_2 respectively. If the value of W is known, the values of T_1 and T_2, which are evidently equal, and which are represented by the lengths bc and ca, are also known, because they are drawn to the same scale.

If the string is shortened, as shown by the dotted line ANB, it will be seen, by applying the principle of triangle of forces, that the tensions of the string now will be represented by the sides ac_1 and bc_1, which are evidently greater than ac and bc respectively. That is, the tensions are increased. It is clear from this that *the string is more likely to break when it is shortened.*

34. Resolution of Velocities and Forces.—As we have seen that two velocities or forces can be compounded into a single resultant velocity or force, so, conversely, a single velocity or force can be resolved into two components ; because with a given velocity or force as diagonal we can construct a parallelogram, the two adjacent sides of which would give the two components. But as with a given diagonal any number of parallelograms can be constructed, so with a given resultant velocity or force, an infinite number of components can be obtained unless any particular direction is mentioned.

The most important case in practice is where the two components are taken at *right angles.*

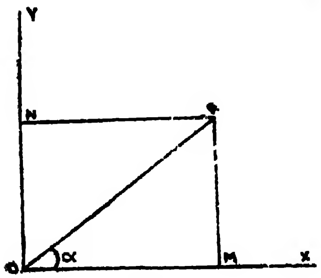


Fig. 13

Let OX and OY be two axes at right angles, and OP represent a velocity or force in direction and magnitude (Fig. 13), which is to be resolved into two components. From P draw perpendiculars PM and PN on OX and OY respectively. Then OM and ON are the required *components.*

If the angle POX be denoted by a, we have $OM = OP \cos a$;

$ON = OP \sin a$.

35. Resultant of a number of Forces acting at a point.—

Let P_1, P_2, P_3 denote several forces acting at any point O (Fig. 14). Take any direction OX in the plane of the forces, and draw OY, perpendicular to OX,

Resolve each force into two components, one along the direction OX, and the other along OY.

Let the components of P_1, P_2, etc., along OX be X_1, X_2, etc., and components along OY be Y_1, Y_2, etc.

Now, if X be the resultant of all the forces along OX,

$X = X_1 + X_2 + X_3 + \ldots\ldots\ldots\ldots$

Similarly, if Y be the resultant of all the forces along OY,

Fig. 14

$Y = Y_1 + Y_2 + Y_3 + \ldots\ldots\ldots\ldots$

The whole system of forces is then reduced to two forces, X and Y; and, if R be the resultant, we have

$$R^2 = X^2 + Y^2 = (X_1 + X_2 + X_3 + \ldots\ldots)^2 + (Y_1 + Y_2 + Y_3 + \ldots\ldots)^2$$

36. Relative Velocity.—The velocity of a body is usually given with respect to some object which may be regarded as fixed. For example, by saying that a railway train is travelling at the rate of 60 miles an hour we mean that it is the velocity of the train relative to the earth. So when two bodies are in motion one has got the velocity *relative* to the other, and *the relative velocity of any body A with respect to another body B is the rate of change of the position of A relative to B.*

When those two bodies are travelling in the same direction with uniform velocities u and v respectively, the velocity of B relative to A is $(v-u)$; and the relative velocity will be zero when they travel with the same velocity. If they are travelling in opposite direction, the relative velocity is $(v+u)$.

Generally the relative velocity of a body B with respect to another body A is obtained in direction and magnitude by compounding with the velocity of B a velocity which is equal and opposite to that of A.

37. Some Practical Problems,—(i) **Why it is easier to pull a lawn-roller on soft turf than to push it**—When *pulling* the roller by the handle, the force OA (Fig. 15), representing the force exerted by the hand, is resolved into two components, one OB, acting horizontally, is effective in pulling the roller, and the other OC, which is vertically

upwards, acts in a direction opposite to the weight of the roller, and so reduces the pressure exerted on the ground.

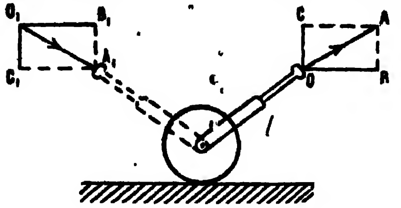


Fig. 15

When *pushing* the roller, the force O_1A_1 is resolved into two components O_1B_1 and O_1C_1, of which O_1B_1 is effective in pushing the roller and O_1C_1, acting downwards, adds to the weight of the roller, and so increases its pressure on the ground.

(ii) **The sailing of a boat against the wind**—Let the line PL represent the sail and let the force due to the wind be represented in direction and magnitude by WK. Resolve the force WK into two compotents, one LK parallel to, and the other NK perpendicular to, the surface of the sail (Fig. 16).

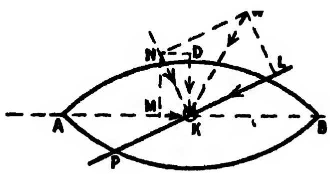


Fig. 16

The force LK acting along the surface of the sail is ineffective and the effective component of the wind pressure is measured by NK.

Now resolve NK into MK along, and DK perpendicular to, the length BA of the boat. The component MK drives the boat forward, while the component DK tends to make the boat move at right angles to its length, *i. e.* sideways.

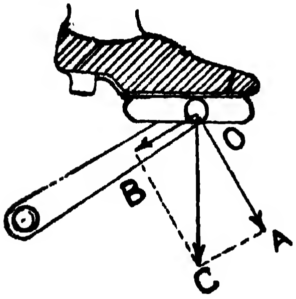


Fig. 17

It should be noted, however, that the component DK moves the boat very slowly at right angles to its length, the resistance to motion in that direction being very great. A rudder applied at A seeks to neutralise this component.

(ii) **The effective pressure of the foot on a Bicycle crank.**—In cycling, the effect of the pressure applied by the foot on the crank changes according to the position of the crank. In (Fig. 17) when the pressure of the foot is applied vertically downwards, with a force represented by OC, the component OB of it along the crank is lost, and the component OA, acting perpendicular to the crank, is *only effective* in driving the cycle. It is evident that when the pressure of the foot

acts perpendicularly to the crank, the pedalling becomes most effective, because in that position no component of it is lost.

(iv) **Flying of a Kite.**—Let AB be the surface of the kite [Fig. 18(a)]. Though the wind pressure acts on all parts on the under-surface of the flying kite, the total effect of it may be taken to be equivalent to a single force CO acting at a point O. The force CO may be resolved into two components, one OD acting along the surface, and the other OE acting at right angles to it. For the steadiness of the kite, OD is not effective, and the component OE is the effective part of

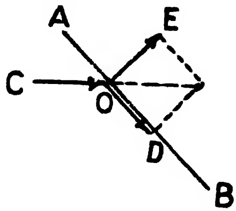


Fig. 18(a).

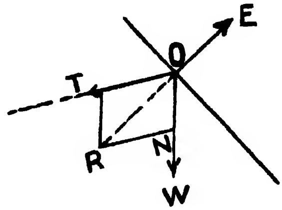


Fig. 18(b).

the wind pressure. Besides the force OE due to the wind pressure, there are two other forces, the tension T (represented by OT) of the string, and the weight W (represented by ON) of the kite acting vertically downwards [Fig. 18 (b)]. The kite is in equilibrium under the action of these three forces. For the kite to be at rest, OE must be equal and opposite to the resultant of OT and ON, which is represented by OR. If OE increases, the kite will rise until OR is again equal and opposite to OE. The weight W being constant, OR may increase due to any increase in the tension T. Again, when the kite rises, the angle between T and W becomes less, and, due to this also, their resultant OR may increase. Similarly, when the wind pressure decreases, the kite will fall. When the magnitude and direction of the wind pressure suddenly change, the kite moves irregularly, but this is diminished by attaching a tail to the kite, which, under the action of wind pressure, checks the sudden irregular movements.

(v) **Flying of an Aeroplane.**—It is seen, in the case of the flying of a kite, that wind pressure is an absolute factor. For this reason, when, at starting, the kite is near the ground, where there may be very little wind, the boy trying to fly the kite has got to run fast by holding the string. This produces sufficient wind pressure upon the surface, due to which the kite may rise to a considerable height where there may be sufficient air current, and the running may no longer be be necessary. The faster the running the better the flight. In other

words, there must be sufficient wind pressure on the surface of the kite to make it rise to a considerable height into the air. This may be obtained by the movement of the air or the movement of the kite.

If the string of a flying kite breaks, the equilibrium of the forces is destroyed, and the kite either trembles, or glides down to earth backwards. If it were possible to attach to the kite, just at the moment of rupture, a weightless engine and propeller, exerting a pull equal to the tension of the string, the kite would remain stationary. Besides this, if the wind prosure drops suddenly and the engine gives the kite a motion in a suitable direction so as to produce the wind pressure equivalent to the original one, the kite will again be stationary. If the magnitude of this wind pressure be increased by faster motion of the engine, the component OE (Fig. 19) will increase and the kite will move forward, and be an aeroplane, *i.e.* a self-supporting **heavier-than-air machine**. The boy, running with his kite in order to produce a sufficient wind pressure on the surface of the kite, resembles very closely an aeroplane in which an engine and propeller take the place of the boy, and, like the action of the boy, the action of the engine and propeller produces sufficient wind pressure on the wings of the aeroplane.

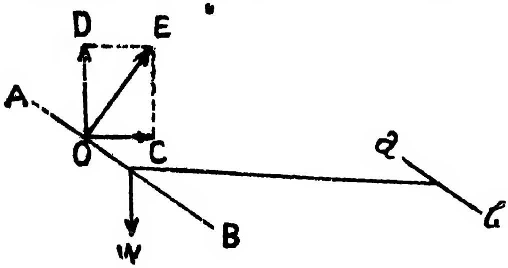


Fig. 19.

An aeroplane must have a minimum velocity of 50 miles an hour in order to maintain its flight in the air, and if, any how, this

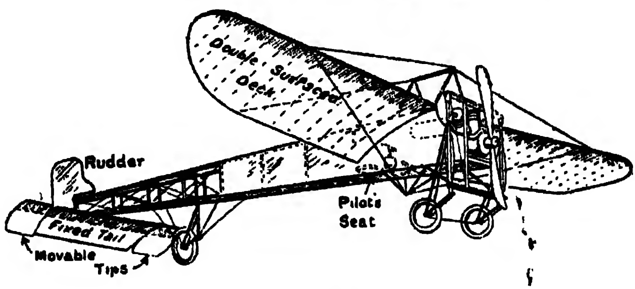


Fig. 20—Aeroplane

speed be lost the machine cannot be controlled and this becomes

Let AB represent the surface of the main wing of an aeroplane and OE the total wind pressure acting at O (Fig. 19). OE may be resolved into two components, one OC acting horizontally and the other OD acting vertically upwards. Besides these two component forces, the weight W of the aeroplane acts vertically downwards at the centre of gravity of the aeroplane. At the time of starting, the engine makes the propeller (Fig. 20) rotate swiftly due to the action of which the aeroplane runs forward on the ground, and, when the speed of the aeroplane becomes rapid enough to make the vertical component of the wind pressure, namely OD (Fig. 19), slightly greater than the weight W, the aeroplane leaves the ground and rises. The forward motion of the aeroplane, besides creating wind pressure on its wings, as described above, also overcomes the horizontal component OC.

Now, the action of the two forces OD and W (Fig. 19) would tend to turn the wing into a vertical position to prevent which there is a tail-plane ab, like the tail of the kite. The wind pressure acting on the tail-plane, the angle of which is controlled by the pilot, keeps the inclination of the wing constant.

The movable parts of the tail of the aeroplane modify the wind pressure so that the machine can ascend or descend according to the will of the pilot. The pilot also controls the rudder (Fig. 20) which is attached to the tail, and which works exactly like the rudder of a boat.

Remember.—

Equations of Motion—

(*i*) $s = vt$.

(*ii*) $v = u + ft$.

(*iii*) $s = ut + \frac{1}{2} ft^2$.

(*iv*) $v^2 = u^2 + 2fs$.

Special cases—When $u = o$,

(*a*) $v = ft$.

(*b*) $s = \frac{1}{2} ft^2$.

(*c*) $v^2 = 2fs$.

Questions

Art. 24.

1. Explain the terms 'absolute motion' and 'relative motion'. Which of them is more important to man, and why? [Pat. 1932.]

Arts. 25 & 26.

2. Distinguish between speed and velocity. What is angular velocity? (C. U. 1930.)

3. Explain what is meant by acceleration of a point moving in a straight line. Show that when a body moves with uniform velocity in a straight line, the velocities at the ends of successive seconds are in arithmetic progression. (Pat. 1927)

Art. 28.

4. A stone is thrown vertically upwards with a velocity of 160 ft. per second from the top of a cliff 120 ft. high. How high will the stone rise above the cliff, and after how long will it fall to the foot of the cliff? What will be the velocity of the stone when it is 80 ft. above the point of projection? (*M.U.*)

[*Ans* : (i) 400 ft. ; (ii) 5·70 secs. ; (iii) 143·1 ft. per sec.].

5. A velocity of one foot per second is changed uniformly in one minute to a velocity of one mile per hour. Express numerically the acceleration when a yard and a minute are the units of space and time. (Pat. 1923)

[*Ans* : $9\frac{1}{2}$ yds. per min^2.]

Arts. 32 & 33.

6. State, explain, and prove the principle of the parallelogram of forces. The wind blows from a point intermediate between North and East. The northerly component of its velocity is 5 miles per hour, and the easterly component is 12 miles per hour. Find the total velocity. (C. U. 1934)

[*Ans* : 13 miles per hour.]

7. (a) Define the terms 'resultant' and 'equilibrant' of forces. Explain each by means of an example. (b) State the law of Triangle of forces and describe an experiment to verify it. (c) The three forces of 4, 5 and 6 grms. weight respectively act at a point and are in equilibrium. What are the angles between their lines of action?

[*Ans* : Angle between 4 and 5, 98° ; between 5 and 6, 139° ; between 6 and 4, 123°].

8. The following forces act at a point—18 lbs. weight due East, 16 lbs. wt. 60° North of East, 25 lbs. wt. North-west, 40 lbs. wt. 75° South of West. Find graphically the resultant force at the point. (*Mysore*)

[*Ans* : 7·1 lb. due South].

9. Enunciate and give theoretical and experimental verification of the proposition known as the triangle of forces. (Pat. 1932, '34)

Art. 35.

9. (a) How would you find the resultant of a number of forces acting at a point? (Pat. 1917)

Art. 36.

10. What is meant by relative velocity, and show how it is determined. Give examples to illustrate your answer. (Pat. 1946)

A man walking on a road with a velocity of 3 miles per hour encounters rain falling vertically with a velocity of 22 ft/sec. At what angle should he hold his umbrella now in order to protect himself from the rain? (Pat. 1946)

11. A railway passenger observes that rain appears to him to be falling vertically when the train is at rest, but that when the train is in motion the rain-splashes on the window are not vertical. Explain this and show how the relative velocities of the train and the raindrops may be determined.

Explain also why a passenger is thrown forward in the direction of motion of a train when the velocity of the train is suddenly reduced. (*C. L.*)

12. A man in a boat rows at 2 m. p. h. relative to the water at right angles to the direction of the current of a river flowing at 2 m. p. h. Another man starting from the same point walks along the bank upstream at 3 m. p. h. How far apart will the two men be after six minutes ? (*L. M. B.*)

[*Ans* : 0·5335. ml.]

13. A man walks across the compartment of a railway carriage at right angles to the direction of motion of the train, when the train is travelling at 10 m. p. h. ; and walks back, with the same velocity relative to the train, when the train is travelling at 21 m. p. h. His resultant velocity in the latter case is twice that in the former case. Prove that his velocity relative to the train is very nearly 3·7 m. p. h. (*C. L.*)

Art. 47.

14. Explain with the aid of a diagram the flight of a kite.

(Pat. 1927 ; '31)

CHAPTER IV

Newton's Laws of Motion

38.* First Law.—Every body continues in its state of rest or of uniform motion in a straight line, except when it is compelled by impressed forces to change that state.

This law is very often known as the **Law of Inertia,** as *it is the virtue of every inert body to preserve its state of rest or of uniform motion in a straight line.*

A person sitting on a horse back will experience the effect of inertia if the horse suddenly starts galloping, when the upper part of the body will lean backwards. Similarly, if the horse suddenly stops, when at full speed, the rider will be shot forward. If a cyclist, when travelling quickly, is suddenly stopped, he will fall over the handle-bar of his cycle.

The *second part* of the law provides with the definition of *force.* It states that unless impressed forces act on a body, its state of motion, or of rest, will remain unchanged. From this we get that *force is that which tends to set a body in motion, or tends to change the motion of the body on which it acts.*

39. Second Law.—The rate of change of momentum is proportional to the impressed force and takes place in the direction of the force.

Momentum.—*Momentum* of a moving body corresponds to what may be called the *quantity of motion*. It is a property, the moving body possesses, by virtue of its mass and velocity conjointly, and is measured by the product of its mass and velocity.

For instance, the momentum possessed by a 400-ton train moving with a velocity of $\frac{1}{2}$ mile per minute is equal to the momentum possessed by another 200-ton train moving with a velocity of 1 mile per minute. For, $400 \times \frac{1}{2} = 200 \times 1$.

The great havoc sometimes done by a cyclone is due to the great momentum of the moving mass of air. The mass of air may be small, but its velocity is very great, and so the momentum (*i.e.* mass × velocity) is large.

By taking the hammer at a distance before striking a nail in order to drive it into a piece of wood, a greater velocity of the hammer is acquired and consequently a greater momentum is obtained.

[N. B. It must be noticed that **momentum = mass × velocity,** (and not mass × speed), *i.e.* momentum is a vector quantity.]

Units of momentum.—Unit momentum is the momentum possessed by unit mass moving with unit velocity. Hence

The F. P. S. unit *is the* momentum *possessed by a mass of 1 lb. moving with a velocity of 1 ft. per sec.*

The C. G. S. unit *is the* momentum *possessed by a mass of 1 gm. moving with a velocity of 1 cm. per sec.*

40. Measurement of Force.—The second law of motion gives us a method of *measuring* force.

Let a body of mass m be moving with a velocity u under the action of a constant force P, and let v be the velocity after t seconds, then the initial momentum of the body is mu, the final momentum of the body is mv, and the change of momentum in time $t = m(v-u)$;

∴ The rate of change of momentum $= \dfrac{m(v-u)}{t}$; and this, according to the Second Law, is proportional to the impressed force P ;

$$i.e.\ P \propto \frac{m(v-u)}{t} \propto (m \times \text{acceleration})$$

Denoting acceleration by f, we have

$$P \propto mf = Kmf, \text{ where } K \text{ is a constant.}$$

Now, if we choose our unit of force in such a way that it produces unit acceleration in unit mass, we have $m=1$, $f=1$, when $P=1$. Hence K must be equal to 1 and we get

$$\mathbf{P=mf} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Hence, we may write, Force = mass × acceleration.

Again, $P=\frac{mv-mu}{t}$; or $Pt=mv-mu$.........(2)

The product (**force × time**) is called the **Impulse** of the force during that time.

For a body originally at rest, $u=0$ and therefore equation (2) becomes $\mathbf{Pt=mv}$........................(3)

41. Unit Force.—From what has been shown above, the **unit of Force** may be defined as—(i) *That force which acting on a unit mass produces unit acceleration* [see equation (*1*)].

(*ii*) *That force which acting for unit time on unit mass initially at rest creates in it unit velocity* [see equation (*2*)].

(*iii*) *That force which acting on any mass for unit time produces in it unit momentum in the direction of the force* [see equation (*3*)].

Units of Force :—

There are *two systems* of force units, (a) the absolute and (b) the gravitational. The absolute units do not vary throughout the universe, but the *disadvantage* of the gravitational units of force is that they are not constant, because they depend upon the pull of the earth, *i.e.* the value of the acceleration due to gravity (g), which varies, though slightly, at different places on the earth's surface (see Chapter V).

(a) Absolute or Dynamical Units of Force.

The F. P. S. absolute unit of force is called one poundal, which is the force that creates an acceleration of one foot per second per second when acting on a mass of one pound.

The C. G. S. absolute unit of force is called one dyne, which is the force that creates an acceleration of one centimetre per second per second when acting on a mass of one gram.

(b) Gravitational unit.—The weight of a body is the force with which it is attracted by the earth. The acceleration with which a body falls freely is denoted by 'g', the value of which in the F. P. S. system is 32·2 ft. per sec. per sec., and in the C. G. S. system the value is 981 cm. per sec. per sec. So—

(*i*) The weight, or force, of 1 lb. acting on a mass of 1 lb. produces an acceleration of 32·2 ft. per sec. per sec.

But the force of 1 poundal acting on a mass of 1 lb. produces an acceleration of 1 ft. per sec. per sec.

∴ **Weight of a pound** (also called a **pound-weight**) = **32·2** (*i.e.* **g**) **poundals.**

∴ m **pounds-weight** = mg **poundals.**

Hence, a force of 1 poundal = 1/32·2 weight of one pound.
= wt. of 16/32·2 oz.
= wt. of half an ounce nearly.

(*ii*) Again, the weight, or force, of one gram acting on a mass of 1 gram produces an acceleration of 981 cm. per sec. per sec.

But the force of 1 dyne acting on a mass of 1 gram produces an acceleration of 1 cm. per sec. per sec.

∴ **weight of a gram** (also called a **gram-weight**) = **981** (*i.e.* **g**) **dynes.**

∴ m **grams-weight** = mg **dynes.**

Hence, a force of 1 dyne = 1/981 of a gram-weight.

Generally, if m lbs. be the mass of a body, the only force acting on it is its weight, W. So, by substituting W for P, and g for f in the formula $P = mf$, we get, $\mathbf{W = mg}$;

i.e. **Weight of a body (in poundals) = mass (in lbs) × g ;** where **(g = 32·2)**

and **Weight of a body (in dynes) = mass (in grams) × g ;** (where **g = 981**)

[**Note.**—A force of 1 dyne can be practically realised by the weight W of one milligram ; for $W = mg = 1/1000 \times 981 = 1$ dyne (nearly)].

The **gravitational unit of force** is the weight of unit mass. Hence—

The F. P. S. gravitational unit of force is a force equal to the weight of a pound.

The C. G. S. gravitational unit of force is a force equal to the weight of a gram.

So, from (*i*) and (*ii*) above

the gravitational unit of force = g × absolute unit of force.

[**Note.**—(i) The weight of a pound has different values at different places of the earth due to the difference in the values of g.

(ii) The formula $P = mf$ is true only when all the forces are

expressed in absolute units, *i.e.* in poundals or dynes, and not in pounds-weight or grams-weight.

(iii) In solving problems using the above formula, (*a*) reduce all the forces into absolute units (if they are given in gravitational units, *i.e.* in lbs.-wt. or gms.-wt) by multiplying by the corresponding value of g.

(iv) Finally, if necessary, reduce the forces to gravitational units by dividing by 'g'.]

42. Relation between a Dyne and a Poundal.

1 Poundal $= 1/32{\cdot}2$ wt. of a pound
$= 1/32{\cdot}2 \times 453{\cdot}6$ wt. of a gram, ($\because$ 1 pound $= 453{\cdot}6$ grams)
$= 981/32{\cdot}2 \times 453{\cdot}6$ dynes ($\because$ 1 gm. wt. $= 981$ dynes)
$= 13{,}800$ dynes.

Examples.—(1) *Express in dynes the force due to 1 ton weight,* ($g = 981{\cdot}4$ cms. per sec.^{2})

1 ton weight $= 2240$ lbs. wt. $= 2240 \times 453{\cdot}6$ gms. wt.
$= 2240 \times 453{\cdot}6 \times 981{\cdot}4$ dynes. $= 9{\cdot}97 \times 10^8$ dynes.

(2) *A force equal to the weight of 10 lbs. acting on a body generates an acceleration of 4 ft. per sec. per sec. Find out the mass of the body.*

Here $P =$ wt. of 10 lbs. $= 10 \times 32$ poundals : $f = 4$ ft. per sec. per sec.

$\therefore$ by the formula $P = mf$, we have $10 \times 32 = m \times 4$; *or* $m = 80$ lbs.

(3) *A train weighing 400 tons is travelling at the rate of 60 miles an hour. The speed of the train is reduced to 15 miles per hour in 30 seconds. Find the average retarding force on the train.*

400 tons $= 400 \times 2240$ lbs. ; 60 miles an hour $= 88$ ft. per sec.
15 miles an hour $= 22$ ft. per sec.
We have, by equation (2), p. 37, $Pt = mv - mu$
or $P \times 30 = (400 \times 2240 \times 22) - (400 \times 2240 \times 88)$

$$\therefore \quad P = -\frac{400 \times 2240 \times 66}{30} = -1{,}971{,}200 \text{ poundals.}$$

(4) *On turning a corner a motorist rushing at 45 miles an hour finds a child on the road 100 ft. ahead. He instantly stops the engine and applies brakes so as to stop within 1 ft. of the child (supposed stationary). Calculate the time required to stop the car, and the retarding force. (Car and the passenger weigh 2000 lbs.)* (Pat. 1939.)

Here $u = 45$ miles per hour $= 66$ ft. per sec.

The final velocity $v = 0$, and the distance travelled before the car stops $= 100 - 1 = 99$ ft.

If f be the acceleration generated by the force, we have $v^2 = u^2 + 2fs$;

or $0 = 66^2 + 2f \times 99$; when $f = \dfrac{-66^2}{2 \times 99} = -22$ ft. per sec^2.

Again, $v=u+ft$; or $0=66-22\,t$.; whence $t=66\div22=3$ sec.

or the time required to stop the car $=3$ sec.

$\therefore$ The retarding force $P=mf=2000\times22=44{,}000$ poundals.

(5) *A constant force acts for 3 secs. on a mass of 16 lbs. and then ceases to act. During the next 3 secs. the body describes 81 ft. Find out the magnitude of the force in lbs.-wt. and poundals. (Acceleration due to gravity = 32 ft. per sec. per sec.)* (Pat. 1947).

If the force P acts for t secs., the impulse $P\times t=m(v-u)$.

Here $u=0$, so we have $P\times3=16\,v$.........(1)

After the force ceases to act, the body describes 81 ft. in 3 secs. So the uniform velocity during this period $v=81/3=27$ ft.

$$\therefore \text{ from (1) } P=\frac{16\times27}{3}=144 \text{ poundals. } \left(or\ \frac{144}{32}=4\cdot5 \text{ lbs.-wt.}\right)$$

Otherwise thus :—

The uniform velocity during the last 3 secs. $=81/3=27$ ft.

So, 27 ft. is the final velocity of the first 3 secs.

Hence, considering the first period of 3 secs., we have

$u=0$; $v=27$; $f=?$

$v=u+ft$. or $27=0+f\times3$; $\therefore$ $f=27/3=9$ *ft. per sec*^{2}.

Hence $P=mf=16\times9=144$ poundals; (or $4\cdot5$ lbs.-wt.)

43. Physical Independence of Forces.—The latter part of the **Second Law states that change of motion produced by a force takes place in the direction of the force.**

If two or more forces act simultaneously on a body, each force will produce the same effect independently of others. Hence their combined effect is found by considering the effect of each force on the body independently of others and then compounding their effects. This principle is known as the *Physical Independence of Forces*.

Illustrations.—(a) A stone dropped from the top of the mast of a ship, which is travelling without rolling, falls at the foot of the mast, whether the ship be in motion or not, and the time taken by the stone to fall is the same in the two cases.

This shows that whether the stone be initially in motion due to the ship's motion, or at rest, when the ship is not moving, the attraction of the earth produces the same result upon the stone.

(b) A circus rider is another good illustration. When in the course of running the rider jumps in a vertical direction from the horse's back, his horizontal velocity, which is the same as that of the horse, remains unchanged and independent of the vertical force due to his weight. For this reason he is able to alight again on the horse's back and does not fall behind.

46. Circular Motion.—Suppose a particle A moves round a circular path of radius r with a uniform velocity v (Fig. 22). If the particle were free to move, it would travel along AB in a tangential direction. The particle is, however, constrained to move in a circular path, and, since the velocity is constant, any such force must always act perpendicularly to the path of the particle at any instant, *i. e.* towards the centre.

Let AC represent a small arc discribed in time t. Had there been no constraining force in the direction BC, the particle would be moved through a distance AB in a tangential direction, where $AB = vt$.

During the same time t, the particle has been deflected through BC under the action of the constraining force. Hence, if f be the acceleration due to the force, we have $BC = \frac{1}{2}ft^2$.

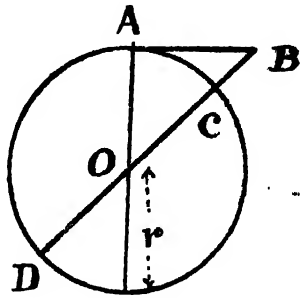


Fig 22.

From the figure, $$OB^2 = OA^2 + AB^2$$

$$\therefore \quad (r + BC)^2 = r^2 + AB^2$$

or $$r^2 + 2.\, r.\, BC. + BC^2 = r^2 + AB^2$$

or $$2r.\, BC + BC^2 = AB^2$$

But, as BC is very small, BC^2 may be neglected.

Hence, $2r.\, BC = AB^2$, *or* $2r \times \frac{1}{2}ft^2 = v^2t^2$; *or* $fr = v^2$;

or $$f = \frac{v^2}{r}.$$

Hence, when a particle is moved round a circular path of radius r with a uniform velocity v, it is always subject to a normal acceleration towards the centre. If m be the mass of the particle, the corresponding force $= mv^2/r$ (in absolute units.)

47. Centripetal and Centrifugal Forces.—The normal force (mv^2/r.) which acts upon a rotating particle in a direction *towards* the centre of the path in order to constrain it to move in the circular path, is known as the **Centripetal Force.** In the absence of this force, the particle might have pursued a rectilinear path. This force is resisted by *an equal and opposite force* (Newton's Third Law of Motion), the tendency of which is to make the body *fly outwards.* This is known as the **Centrifugal Force.** This force is exerted by the rotating particle on the centre of its circular path.

Every cyclist must have noticed that the mud from a bicycle tyre flies off tangentially when there is not sufficient adhesive force between it and the tyre to keep it moving in a circle.

Some Illustrations.—(i) If we tie a piece of stone with a string and swing it round in a circle by holding the other end, we feel that we have got to exert a force along the string towards the centre of the path. This force is the centripetal force. Due to reaction there is also a force, of magnitude mv^2/r which is *directed away from the centre of the path.* This force is exerted *by the stone upon the string* due to which the string is kept tight. This is the centrifugal force. If the centripetal force vanishes, the centrifugal force does also vanish.

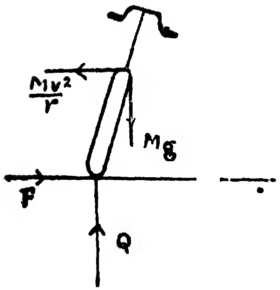


Fig. 23

(ii) Motion of a bicycle rider on a circular path is also an example of the *centrifugal force.* A cyclist turning a corner instinctively inclines his body inwards towards the centre of the curved path (Fig. 23).

At that time the forces acting are, (a) Mg, the total weight of the machine and rider ; (b) the centrifugal force Mv^2/r, where r is the radius of the curved path and v the velocity ; (c) a frictional force F, and (d) the vertical component Q of the reaction of the ground. The couple formed by Q and Mg is balanced by the couple formed by F and Mv^2/r.

The higher the speed, the greater will be the centrifugal force, and the rider will have to bend his body *inwards* at a greater angle. Since Mv^2/r and F are equal there will be a slip when the limiting value of the friction will be attained.

(iii) **The Banking of Tracks**—A racing track for a motor car is constructed in such a way that it is banked inwards, so a stationary car would have a tendency of slipping towards the centre of the track.

In the case of a railway train taking a bend the tendency of the train to move in a straight line produces a pressure on the curved rails and the reaction of which at the flanges of the wheel supplies the necessary centripetal force. But for a high speed of the train there is always a danger of derailment, to avoid which the outer rail is placed a little higher than the inner one, so that the upward reaction of the rails due to the part of the weight of the van may supply the necessary centripetal force.

A bucket containing water may be rapidly swung round in a

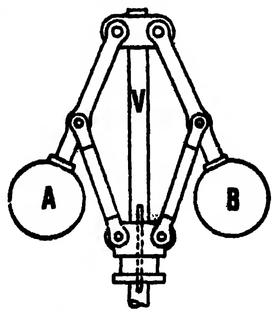
vertical circle without the water falling out even when it is upside down.

(iv) **Centrifugal drier.**—This affords another example of the centrifugal force. This is used in laundries. The wet clothing which is to be dried is placed in a cylindrical wire cage which is caused to rotate at a high speed. The water becomes separated from the clothing and flies out, as the adhesive force between it and the material of the clothing is not sufficient to keep it moving uniformly in the circle.

(v) **Cream separator.**—A given volume of cream has smaller mass than the same volume of skimmed milk, and so a smaller force is required to hold it in a circle of a given radius. Hence if cream particles and milk particles are set in rapid rotation, the milk particles will have greater tendency to move to the outside of the vessel, the cream particles remaining nearer the centre.

(vi) **Flattening of the Earth.**—Initially the earth consisted of a mass of fused matter. Due to the revolution of the earth about its axis centrifugal force is generated, which is greatest at the equator and which decreases gradually towards the pole. As a result of this the earth is bulged at the equator and has got flattened at the poles.

(vii) **Watt's Speed-governor.**—This device was originated by James Watt for automatic regulation of the maximum speed of a steam engine. It consists of two heavy metal balls A and B at the end of two rods hinged at the top of a vertical rod V [Fig. 23 (a)]. The balls are again supported by two other rods resting upon the safety valve of the engine. Ordinarily the valve remains closed by the weight of the balls partially communicated through the rods. The vertical rod, being geared with the engine shaft, takes up the speed of the shaft. When the speed of the engine goes high the centrifugal force acts on the balls which then gradually rise up and thus partially diminish the weight on the valve. This opens the valve and reduces the pressure of steam which slows down the speed of the engine. The balls now drop down and close the valve again. Thus the speed of the engine is automatically regulated to a constant value.



Watt's governor
Fig. 23 (a)

(viii) **A body loses weight due to the Earth's rotation.**—As the earth rotates, everybody on its surface also rotates with the same angular velocity which requires a centrifugal force acting on

it, the magnitude of which is mv^2/r (Art. 47). The only force acting on these bodies is the attraction of the earth due to gravity ; so the apparent weight of each body at the equator should be $(mg - mv^2/r)$ or $m(g - v^2/r)$.

The value of v, the linear velocity of a particle is greatest at the equator and gradually diminishes towards the pole, where the value is zero. So the centripetal force is also greatest at the equator and it is zero at the poles. Hence the reduction in weight of a body is also greatest at the equator, that is, the gravitational attraction on a body (and so the apparent weight) is less at the equator than at the poles. This is the reason of the bulging of the earth at the equator.

Examples.—*A stone whose weight is 1 lb. swings round in a circle at the end of a string 4 ft. long and takes ½ second for every complete revolution. Calculate the stretching force in the string.*

The magnitude of the stretching force $= mv^2/r$.

Velocity of the stone $v = \dfrac{\text{distance}}{\text{time}} = \dfrac{2\pi r}{\text{time}}$.

$$\therefore \frac{mv^2}{r} = \frac{m \times 4\pi^2 r^2}{t^2 \times r} = \frac{m \times 4\pi^2 r}{t^2} = \frac{1 \times 4 \times 9{\cdot}87 \times 4}{\frac{1}{4}}$$

$$= 631{\cdot}68 \text{ poundals ; or } \frac{631{\cdot}68}{32{\cdot}2} = 19{\cdot}6 \text{ lbs.-wt.}$$

2. Calculate the apparent weight of a body of one-ton mass at the equator, the radius of the earth being 4000 miles.

The apparent weight $= m\,(g - v^2/r)$; 4000 miles $= 4000 \times 5280$ ft. ; and 1 ton $= 2240$ lbs.

$$\text{As in Ex. 1, } mv^2/r = \frac{2240 \times 4 \times 9{\cdot}87 \times 4000 \times 5280}{(24 \times 60 \times 60)^2} \text{ (Here } t = 24 \text{ hrs.)}$$

$$= 250{\cdot}2 \text{ poundals} = \frac{250{\cdot}2}{32{\cdot}2} \text{ lbs. wt.} = 7{\cdot}77 \text{ lbs. wt.}$$

Hence the apparent wt. of the body $= 2240 - 7{\cdot}77 = 2232{\cdot}23$ lbs. wt.

Questions

Art. 38.

1. Explain clearly how the idea of 'inertia' of a body is deduced from Newton's First Law of Motion. (Pat. 1921).

1 (a). What do you understand by force ? (Pat. 1947)

Arts. 39, 40, 41.

2. State Newton's Second Law of Motion and explain how it enables you to measure forces. (Pat. 1918, '25, '26, '30, '47 ; C. U. 1934)

3. State the laws of motion which are associated with the name of Newton in their final form, and add explanatory notes leading to the definition of a force and of the unit for its measurement. (Pat. 1940)

4. State Newton's Second Law of Motion and show how starting from the law of Parallelogram of velocities, we can arrive at the law of Parallelogram of forces. What is the relation between a poundal and a pound-weight? (Pat. 1926)

5. State and explain the laws of motion. (C. U. 1930)

6. The speed of a train of mass 200 tons is reduced from 45 m. p. h. to 30 m. p. h. in 2 min. Find (a) change in momentum, (b) average value of the retarding force. (S. C.)

[*Ans* : 9,856,000 f. p. s. units ; 55/48 tons wt.]

7. What definition for force would you give? What is momentum?

A train of mass 175 tons has its velocity reduced from 40 miles per hour to zero in 5 minutes. Calculate the value of the retarding force assuming that it is uniform. What has been the change in momentum? (P. U.)

[*Ans* : Retarding force = 1·07 tons-wt.
Change in momentum = 10270 tons-feet/sec.]

7 (a). What are the units in common use for expressing a force? (Pat. 1947)

8. Find the uniform force required to stop in a distance of 10 yards a 3 ton lorry running on a level road at a speed of 15 m. p. h. Find also the time during which the force acts. (S. C.)

[*Ans* : 1,694 lbs. wt. ; 30/11 sec.] ? 3388 lbs. wt ; 15/11 secs.

8. (a) State Newton's Law of Motion and discuss the bearing of the second law on the use of the word 'force'.

A velocity of one foot per second is changed uniformly in one minute to a velocity of one mile per hour. Express numerically the acceleration when a yard and a minute are the units of space and time. (Pat. 1935)

[*Ans* : $9\frac{1}{3}$ yds. per min^2.]

Art 45

9. (a) State Newton's third law of motion and explain it by examples of its application. (Pat. 1946)

(b) A man who weighs 100 lbs. slides down a rope hanging freely, with a uniform speed of 3 ft/sec. What pull does he exert upon the rope, and what would happen if at a given instant he would reduce his pull by one third?

10. When a man drags a heavy body along the ground by means of a rope, the rope drags the man back with a force equal to that with which it drags the body forward. Why then does motion ensue? (Pat. 1928)

11. Explain clearly how a horse produces motion in a carriage to which it is tied by means of a rope by pulling it, in the face of the 3rd law of motion. (Pat. 1933)

12. Explain with the aid of a diagram the flight of a bird. (Pat. 1931)

Fig. 24.

Hints.—At the time of flying, the bird strikes against the air with wings, but as every action has its opposite reaction (Newton's 3rd Law of Motion), the forces *OA*, *OB*, due to reaction act in opposite directions (Fig. 24). If *OA*, *OB* represent these reactions in magnitude and direction, then, by the Parallelogram of forces, the resultant force with which the bird advances is represented in magnitude and direction by *OC*.]

Art. 46.

13. Explain why a force is needed to keep a body moving uniformly in a circle. Calculate this force in terms of the mass of the body, its uniform speed, and the radius of the circle. (Part. 1944)

Art. 47.

14. What are centripetal and centrifugal forces, and what are their relations with a body moving on a circular orbit ? Discuss in detail their importance to man. (Pat. 1932)

15. Find the pressure exerted on a vertical wall by the water from a fire-hose, which delivers water with a horizontal velocity of 18 metres per sec. from a circular nozzle of 5 cm. diameter. The water is assumed not to rebound. (L. M. B.)

[*Ans* : 6.37×10^7 dynes.]

means (force used) ——
here

CHAPTER V

Gravitation and Gravity : Pendulum

48. Laws of Gravitation.—

(1) *In nature every material body attracts every other material body towards itself.*

(2) *The force of attraction between any two bodies varies directly as the product of their masses and inversely as the square of the distance between them.*

If m_1 and m_2 be the masses of two bodies and d the distance between them, the force of attraction F, which each exerts on the other, $F \propto m_1 m_2$, and also $F \propto 1/d^2$;

$$\therefore \quad F \propto \frac{m_1 m_2}{d^2}, \quad \text{or} \quad F = G\frac{m_1 m_2}{d^2},$$

where G is known as the *Universal Gravitation constant.* Its value, as given by Boys, is $6{\cdot}65 \times 10^{-8}$ in C. G. S. units.

Let m_1 and m_2 in the above equation be each equal to 1 gram and d equal to one centimetre, then $G = F$, which means that *G is numerically equal to the force of attraction between two masses, each of one gram, when separated by a distance of one centimetre.*

49. **Gravitation and Gravity.**—The force of attraction between any two material bodies is called *gravitation.* The term is more specially applied to the attraction exerted between two heavenly bodies.

Gravity is the force of attraction that the earth exerts on other bodies on or near its surface towards its centre. If the mass of the earth is taken to be M and the mass of any object on its surface is m, the force of attraction due to gravity $= G\frac{Mm}{R^2}$, where R is the radius of the earth. So **Gravity** is a particular case of gravitation and may be called earth's gravitation. *The weight of a body is due to this force,* and as this force is not the same at all points on the earth's surface, *the weight of a body is not constant all over the globe.*

The earth is a sphere and for a body external to it, it acts as if its whole mass were concentrated at its centre ; and as this mass (about $5{\cdot}98 \times 10^{27}$ grams) is so large in comparison with that of a body attracted by it, the effect of their mutual attraction is seen in the movement of the body towards the earth along one of its radial or *vertical* lines.

50. **The acceleration due to gravity (g).**—We know that any force acting on a body produces acceleration in it, so a body falling freely due to the force of attraction exerted by the earth, *i.e.* the gravity, experiences a uniform acceleration, which is denoted by the letter 'g' and is called the acceleration due to gravity.

We have seen in Art. 48 that the force of attraction due to gravity varies inversely as the square of the distance from the centre of the earth. Therefore the acceleration due to gravity 'g' also varies in the same way according to Newton's second Law of motion, mass being constant. It also varies due to rotation of the earth on its axis.

The value of g, the acceleration due to gravity, at sea-level and in latitude 45° is generally taken as the standard for reference. The value of g at any place depends upon its height above the sea-level,

being less at the top of a high mountain. *The value of g is constant at the same place,* and varies with the latitude. It is minimum at the Equator and increases gradually to attain the maximum value at either of the Poles. At the Equator, the value of g is about 978 cm. per sec. per sec., and at the Poles, it is about 983 cm. per sec. per sec., and so the mean value of g is taken to be 981 cm. per sec. per sec., or 32·2 ft. per sec. per sec.

VARIATION OF 'g' WITH LATITUDE

Place	Latitude	Value in ft./sec.^{2}	Value in cm./sec.^{2}
Equator	0°0′	32·09	978·10
Madras	13°4′	32·10	978·36
Bombay	18°53′	32·12	978·63
Calcutta	22°32′	32·13	978·76
New York	40°43′	32·16	980·19
Paris	48°50′	32·18	980·94
London	51°29′	32·19	981·17
Poles	90°	32·25	983·11

51. Variation of 'g' (i.e. also wt. of a body) **from place to place.**—Let R be the radius of the earth and D its mean density, then the mass of the earth,

$$M = \tfrac{4}{3}\pi R^3 D.$$

(1) The value of 'g' or wt. of a body above or below the earth's surface,—

(*a*) **Above earth's surface.**—At a height h above the surface of the earth, the force of attraction on a body of mass m, according to the second law of gravitation $= G\dfrac{Mm}{(R+h)^2}$.

So, the force of attraction, and hence the acceleration due to gravity, on a body above the surface of the earth is *inversely proportional* to the square of the distance of the body from the centre of the earth. So 'g' will be less as the distance increases from the surface of the earth.

(b) Below the surface of the earth.—Again, consider a body at a depth 'h' below the surface of the earth. Imagine a sphere concentric with the earth and with its surface passing through points at a distance 'h' below the surface of the earth. It is known that the gravitational force of attraction inside a hollow spherical shell is zero. Here the given body is on the surface of the inner sphere, but it is *inside* with respect to the portion of the earth outside the smaller sphere ; so the latter portion has no attractive force on the body. Therefore, the force of attraction on the body will be due to the inner solid sphere of radius $(R-h)$ towards the centre of the earth and is equal to,

$$\frac{\frac{4}{3}\pi(R-h)^3 . D. m}{(R-h)^2} = \frac{4}{3}\pi(R-h). D.m,$$

where $\frac{4}{3}\pi(R-h)^3 . D$ is the mass of the inner sphere.

$\therefore$ The force of attraction, and hence g inside the earth, is *directly proportional* to $(R-h)$, that is, to the distance of the body from the centre of the earth.

So g will be less inside the earth's surface, the greater the depth the less is the value of g. Hence the *maximum value* of 'g' is *on the surface of the earth*, and *the value of g is minimum* (*i.e. zero*) *at the centre of the earth.*

(2) The value of 'g' or wt. of a body on the earth's surface.—It varies due to two reasons.

(a) **The peculiarity in the shape of the earth.**—As force of gravity is inversely proportional to the sq. of the radius of the earth at a place on it, it is greatest at the poles and least at the equator, since the polar radius is the least and the equatorial radius the greatest, the difference being about 13·5 miles. So the value of 'g' or wt. of a body decreases from the poles to the equator.

(*b*). **The rotation of the earth about its axis.**—Owing to diurnal rotation of the earth about its axis, everybody on it revolves and its wt. (= the force of gravity) is opposed by the centrifugal force generated and therefore the observed wt. of a body is less than its true wt. But *this loss in wt.* is least at the pole and increases to a maximum at the equator, for during diurnal rotation a point near the pole has a smaller linear velocity than one at the equator, the former having to describe a smaller circle than the latter in the same time and is hence subjected to a smaller centrifugal force too $\left(\frac{mv^2}{r}\right)$. Thus due to this reason also a body should weigh less at the equator than at the poles.

52. Centre of Gravity.—It follows from the law of gravitation that every particle of a body upon the earth's surface is attracted towards the centre of the earth. The sum of all the attractive forces on the particles of the body is equal to the total attractive force of the earth upon the body, *i.e.* to the weight of the body. So the weights of all the small particles of the body can be regarded as a system of forces, which, considering the length (about 4000 miles) of the radius of the earth, may be considered to be parallel, all acting vertically downwards. These parallel forces can be replaced by a single resultant force—which is equal in magnitude to the weight of the body—also acting vertically downwards at a single point G in the body (Fig. 25). This point is called the **centre of mass** or **centre of gravity** (written, C. G.) of the body. *Hence the centre of gravity of a body may be defined as the point through which the line of action of the weight of the body always passes, in whatever manner the body may be placed.* So, when a rigid body is supported at its centre of gravity it will remain in equilibrium. If the body is freely suspended by a string, the centre of gravity of the body lies vertically below the point of attachment of the string.

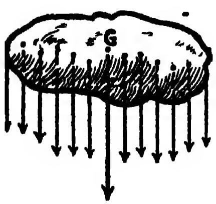


Fig. 25.

Experimental Determination of Centre of Gravity. (a) The centres of gravity of regular geometrical figures can be determined very easily. The centre of gravity of a rod having the same breadth, thickness, and density throughout, is at its middle point. The centre of gravity of a parallelogram is at the intersection of its diagonals. The centre of gravity of a triangular lamina is found by bisecting any two sides and joining the middle points so obtained to the opposite vertices. The centre of gravity is at the intersection of the lines so drawn. The centre of gravity of a circular lamina is obviously at its centre, and also the centres of gravity of solid homogeneous spheres and solid cubes are at their centres.

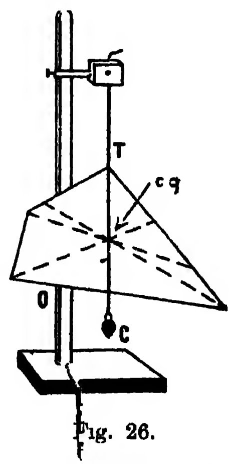


Fig. 26.

(b) The centre of gravity of an irregular lamina, say an irregular sheet of cardboard, can be determined by suspending it with the help of the stand O from different corners of the lamina. (Fig. 26). When it is suspended from one corner T by a string, the centre of gravity lies on the vertical line given by the plumb line TC

through the point of suspension. This line is marked in chalk on one face of the lamina. The operation is repeated by suspending the lamina from another corner. The intersection of the two chalk lines gives the centre of gravity. On suspending the body from other corners, the other vertical lines so obtained will also pass through the common point of intersection, called the Centre of Gravity.

53. **Stable, Unstable, and Neutral Equilibrium.**—A body is in equilibrium if the resultant of the forces is zero, and also if there is no moment tending to turn the body about any axis.

Suppose that a body is displaced slightly from its position of equilibrium. It may happen that the forces tend to restore the body to its original equilibrium condition, or the forces may tend to increase the displacement. In the former case the equilibrium is called **stable**, and in the latter case **unstable**. If, however, the forces have no tendency to increase or diminish the displacement, the equilibrium is called **neutral**.

A body is, hence, said to be in **stable equilibrium** if it returns to the original position, when slightly displaced from it. A cube resting on its side, a glass funnel resting on its mouth, a pendulum are examples of stable equilibrium.

A body is said to be in **unstable equilibrium** if, after a slight displacement, it moves further from its original position. A cone standing on its points, a glass funnel standing on the end of its neck, an egg standing on its end are examples of unstable equilibrium.

A body is said to be in **neutral equilibrium** when, after a slight displacement, it neither returns to the original position nor moves further from it. A spherical ball resting on a horizontal plane, a cone or a funnel lying on its side are examples of neutral equilibrium.

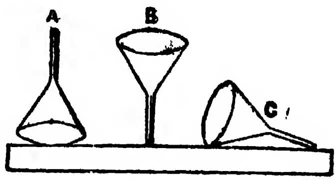


Fig. 27

In stable equilibrium the centre of gravity of a body is at its lowest position, and a slight displacement tends to raise it. When the glass funnel A (Fig. 27) is slightly tilted, its C. G. is elevated and so the body returns to the original position as soon as it is allowed to do it.

In unstable equilibrium the C. G. is at its highest point, and a slight displacement tends to lower it. When the glass funnel *B* (Fig. 27) is slightly tilted, the C. G. at once occupies a lower point, and comes outside the base, and so can easily be overturned. Remember

that the centre of gravity of a body always tends to occupy the lowest possible position.

In neutral equilibrium the C. G. is neither raised nor lowered by a slight displacement. This is evident in the case of the funnel C in Fig. 27.

54. Friction.—Whenever two bodies are in contact, any attempt to slide either over the surface of the other will set up forces which oppose the motion. Such forces are called **forces of friction** between the two surfaces in contact.

Friction is important in our daily life. If there were no friction, walking would be impossible, nails and screws would not remain in the wood, fibres of a rope would not hold together, a ladder would not rest on the ground, and locomotive engines would not be able to draw a train.

Let a rectangular block of wood rest on a horizontal table BC (Fig. 28). The forces acting on the block are its weight W acting vertically downwards, and the re-action R acting normally upwards at the surface of contact. So long as the block is at rest, R is equal to W. Now, suppose a small force P is applied to the block parallel to the surface BC. As the applied force P is gradually increased, the opposing force of friction F also increases at the same rate until a certain maximum is reached. If the applied force be increased beyond this value, the block begins to move. The magnitude of this maximum force when the block is just on the point of sliding, measures what is called the **Limiting friction** (or **Static friction**).

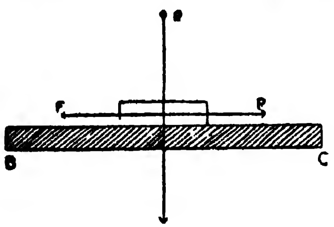


Fig. 28

When the block has once started to move, a *smaller* force would be found to be sufficient to keep the block moving with a constant velocity; this smaller force is called **Sliding friction** (or **Kinetic friction**).

54 (a). Laws of Friction.—Friction obeys the following laws :—

(i) It always opposes motion.

(ii) It is proportional to the total pressure between the surfaces in contact..

(iii) It is independent of the extent of the areas of the surfaces in contact, but it depends on the nature of the surfaces in contact.

(b). Co-efficient of Friction.—If the normal reaction acting across two solid surfaces in contact be equal to R, and F denotes the limiting friction, the constant ratio, F/R, is called the **co-efficient of friction,** which is generally denoted by μ. So, we have $F=\mu R$.

Expts.—(i) **Horizontal Plane.**—Place on a horizontal wooden table a rectangular block of wood [fig. 28(a)] to act as a slider. The surfaces in contact of both these pieces of wood should be as smooth as possible. The slider is attached to a light string, which is passed over a light pulley fixed at the end of the table. A scale pan is attached to the end of the string passing over the pulley. The pulley should be so fixed that the position of the string above the table should be horizontal.

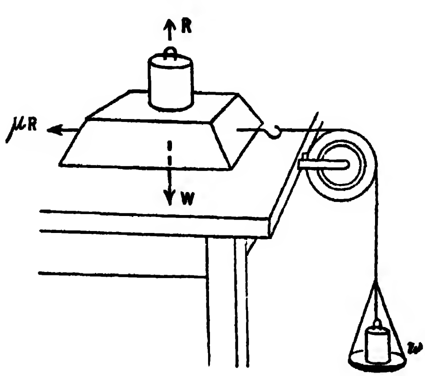


Fig. 28 (a)

Weigh the slider and put a known weight on it. Now put weights on the scale pan until the slider is just on the point of motion. Near about the slipping point gently tap the table to ascertain the required weight to be placed on the scale pan. If W be the total weight of the slider and the weight placed on it, and W' the total weight of the scale pan including the weights w placed on it, the value of the limiting friction $= W'$, and that of the normal reaction $= W$. So, we have, $\mu W = W'$; or $\mu = W'/W$.

Repeat the experiment several times with different weights on the slider, and again on inversing the block and thus find out the mean value of μ.

(ii) **Inclined Plane.**—Place a rectangular slab of wood D on an inclined plane AB (Fig. 29), and gradually increase the inclination of the plane to θ, when D just begins to slide down the plane. Ascertain this by gentle tapping as in the last method. When this is the case, the friction $F\ (=\mu R)$ acts up the plane and balances the component ($= W \sin\theta$) of the weight acting down the plane. The normal reaction R acts at right angles to the plane.

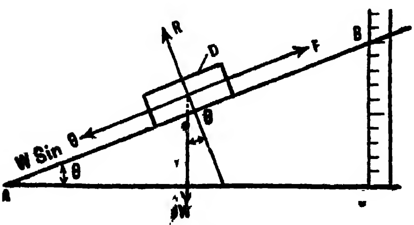


Fig. 29

Resolving W in directions perpendicular and parallel to the plane, we have,

$$W \cos \theta = R, \text{ and } W \sin \theta = F,$$

Hence, $\therefore \dfrac{F}{R} = \dfrac{W \sin \theta}{W \cos \theta} = \tan \theta.$ But $\dfrac{F}{R} = \mu;$

$\therefore$ $\mu = \tan \theta$; or, the co-efficient of friction is simply the tangent of the angle at which sliding begins. Again, $\tan \theta = \dfrac{height}{base}$ of the plane $= \dfrac{BC}{AC}$.

Hence the **co-efficent of friction** is obtained by taking the height of the plane and dividing it by the base.

Repeat the experiment several times and calculate the mean value of μ.

55. Machine.—A machine is a contrivance by which a force applied at a given point is made to overcome a *resistance* (*force*) at some other point with an alteration in its direction and magnitude or both. The first force (*i.e.* the force applied) is called the **effort** (or *power*), and the resisting force overcome by the machine is called the **load** (or *resistance or weight*). The ratio $\dfrac{Load}{Effort}$ is called the **mechanical advantage** of the machine.

Efficiency.—In any machine the useful work done by the machine is always less than the energy supplied to it, due to work done against friction. The ratio $\dfrac{useful\ work\ done}{total\ enery\ supplied}$ is called the **efficiency** of the machine. A *perfect machine* utilises the entire energy supplied to it, and so its efficiency is unity.

The ratio of the displacement through which the effort works to the displacement of the load is called the **velocity ratio** of the machine.

It can be shown that *mechanical advantage* = *efficiency* × *velocity ratio.*

56. The Lever.—A lever is a rigid bar (straight or bent) which turns about a fixed point called the **fulcrum**. The working force exerted to use the lever is called the **effort** or power, and the resistance overcome is known as the **load** or weight. The perpendicular distances from the fulcrum to the lines of action of the forces acting on a lever are known as the **arms** of the lever.

Expt.—Let a metre stick $A\ B$ be balanced on a sharp edge of a wedge-

shaped piece of wood (Fig. 30), and let a weight, say 200 gms., be suspended by a string from a point 20 cms. from the fulcrum P. Now, a point is found on the other side of P at which a weight of 100 gms. will just support the load of 200 gms. This point will be found to be at 40 cms. from the fulcrum. It is seen at once that the product of (200×20) is equal to the product of (100×40). If,

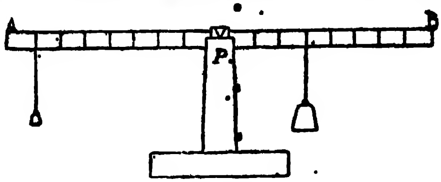


Fig. 30

instead of 100 gms., a weight of 400 gms. is taken to balance the weight of 200 gms., it must be hung at a point at a distance of 10 cms., instead of 40 cms., from the fulcrum. Again, it is seen that the product of (200×20) is equal to the product of (400×10). From this, the principle of the simple lever is learnt as follows :—

Weight on one side of fulcrum × *its perp. distance from fulcrum* = *weight on other side of fulcrum* × *its perp. distance from fulcrum*

Or Load × its arm = Effort × its arm.

57. Moment of a Force.—Each of the above products, *i.e. the product of any force and its perpendicular distance from the fulcrum* (or **force × arm**), *is termed the* **Moment** or **turning power** *of the force.* So the moment of a force represents the tendency of the force to produce rotation.

Whenever a body is restrained at any point by means of a hinge or nail, the body is usually capable of turning about its point or line of restraint. For example, a door can only swing about its hinge, a pendulum swings about its point of support, and so on.

Now, taking the case of a door, it will be seen that the **moment** or **turning power** of the force applied to the door depends upon (*i*) *the magnitude of the force*,—that is, a force of 20 lbs. can cause twice the amount of turning compared to a force of 10 lbs ; and (*ii*) *the perpendicular distance from the point or line of restraint*. The second effect will be evident at the time of opening the door, if the force be applied at right angles to the door, first near the hinge and then near the edge. It will be seen that—*the nearer the force is to the hinge the smaller the perpendicular distance and the greater the force must be to succeed in opening the door.*

58. Parallel Forces.—Forces whose lines of action are parallel are called parallel forces. Parallel forces are said to be **like** when they act in the same direction ; they are said to be **unlike** when they act in opposite directions.

In Fig. 30, the two forces—a force of 100 gms.-wt. and a force of

200 gms.-wt.—are acting downwards. In this case, they are like parallel forces. With one of the forces acting upwards, they would have been unlike parallel forces. The direction of the **resultant** of like parallel forces will be the same as that of its component forces, and, in the case of unlike parallel forces, the *direction* of the resultant is the same as that of the greater component. The *magnitude* of the resultant is the sum of the component forces in the case of like forces, and difference of the components when the forces are unlike. The resultant of two parallel forces acts at a point about which the sum of their moments is zero.

Couple.—If a body is acted upon by two equal but unlike parallel forces they are said to constitute a *couple*. A couple acting on a body exerts a turning effect even if the body is not pivoted.

The **moment of a couple** is the product of one of the forces and the perpendicular distance between them, and it is independent of the point about which the moments are taken.

59. Three classes of Levers.—There are three classes of levers; (a) that in which the fulcrum P is between the load (or resistance) L at B, and the effort E at A [Fig. 31 (a)] :—a crowbar used for moving a weight at its end, a spade used in digging, a *common balance*, a pump handle are examples of this class, and a pair of scissors, forms a double lever of this class ; (b) that in which the load L is between the fulcrum P and the effort E [(Fig. 31(b)] ;—a wheel barrow, an oar are examples of this class, and a pair of common nut-crakers is a double lever of of this kind ; (c) that in which the effort E is between the fulcrum P and the load L [Fig. 31(c)] ;—a man's fore-arm, the treadle of a lathe, the safety valve of a steam engine are examples of this class, and a pair of fire-tongs forms a double lever of this kind.

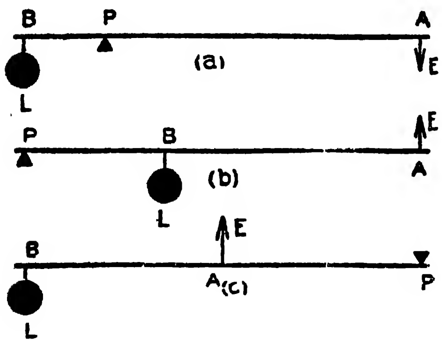


Fig. 31

Considering the lever to be in equilibrium, we have, from Fig. 31, by the *law of moments*, $L \times BP = E \times AP$ (if the lever is without weight) ; or, $\frac{\text{Load}}{\text{Effort}} = \frac{AP}{BP}$.

The ratio L/E is called the *mechanical advantage* of lever.

59 (a). Wheel and Axle.—It is a modification of the lever. It

consists of two cylinders of different diameters capable of turning about a common fixed axis, the larger of which is called the *wheel* and the smaller the *axle* [Fig. 32(a)]. The load W to be raised is attached to a rope coiled round the axle and the effort E is applied to a rope coiled round the wheel in the opposite direction, so that when the rope round the wheel is uncoiled the rope round the axle is coiled round, and thereby the weight is raised. Fig. 32(b) shows a section where OB is the radius (r) of the axle and OA the radius (R) of the wheel. Taking moments about O, the axis, $E \times OA = W \times OB$.

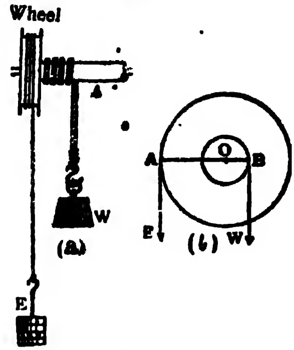


Fig. 32

$$\text{The mechanical advantage} = \frac{W}{E} :: \frac{OA}{OB} :: \frac{\text{Radius of wheel } (R)}{\text{Radius of axle } (r)}.$$

The **windlass** by which water is drawn from a well is of the same class as the wheel and axle, the crank-handle of which serving the purpose of the wheel. The **capstan** used on board a ship for raising the anchor is also of this class.

60. The Pulley.—A pulley is also another simple machine which consists of a grooved wheel, called the **sheave**, over which a string can pass. The wheel is capable of turning freely about an axle passing through its centre. The axle is fixed to a framework, called the **block**. The pulley is termed **fixed** or **movable** according as its block is fixed or movable.

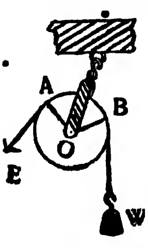


Fig. 33

(1) The Single Fixed Pulley.—In this (Fig. 33), the load W is attached to one end of the string and the effort E is at the other end. With a perfectly smooth pulley and weightless string the tension of the string will be the same throughout. Hence the distance through which the load is raised is equal to the distance through which the effort descends. For equilibrium, the moments of E and W about O, the centre of the wheel, must be equal and opposite ; or $E \times AO = W \times OB$, but $AO = OB$ being radii. Hence $E = W$.

$$\therefore \quad \text{The mechanical advantage} = W/E = 1.$$

In practice, pulleys are not perfectly frictionless, and W is always less than E, that is, the mechanical advantage is always less than 1, but inspite of this the arrangement is useful as the operator can use

the weight of his body for raising the load. It is generally used for raising weights, drawing curtains, pulling punkhas, etc.

(2) **The Movable Pulley.**—Here one end of the string passes round the pulley A and is attached to a fixed support (Fig. 34), and the effort E is applied at the other end. The load W to be raised is attached to the movable pulley. For a frictionless pulley the tension of the string is the same in any point of it and is equal to E. When the strings are vertical the total upward force is $2E$ and, neglecting the weight of the pulley, the downward force is W. So for equilibrium $W=2E$. So the mechanical advantage $=W/E=2$; *i.e.* a given effort can raise twice its weight.

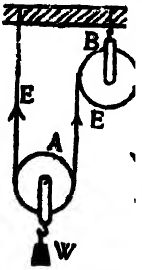


Fig. 34

If the weight of the pulley is ω and cannot be neglected, we have $W+\omega=2E$; or the mechanical advantage $=W/E=2-\omega/E$. Thus the mechanical advantage is slightly less than 2.

When the direction of the force is to be changed, another fixed pulley (B in Fig. 34) should be used, which does not in any way affect the mechanical advantage.

(3) **Combination of Pulleys.**—A combination of pulleys is very often used inorder to secure mechanical advantages greater than two. Different systems having different mechanical advantages are used for different purposes, but the important combination, which is in general use, is given here.

Pulley Block.—This system (often known as the *2nd System of Pulleys*) consists of two blocks, each containing 2 or 3 pulleys, the upper one being fixed to a support and the lower one movable to which the load W is attached (Fig. 35). The string is attached to the upper, or to the lower block, and is then passed round a movable and a fixed pulley in turn, finally passing over a fixed pulley, the effort E being applied at the free end. It should be noted that, when the string is attached to the upper block, the number of wheels in the two blocks must be the same, but when it is attached to the lower block, the number of wheels in it will be one less than that in the upper one.

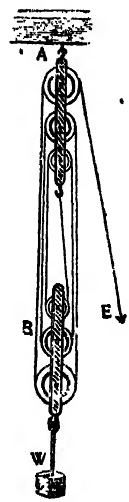


Fig. 35

The tension everywhere round the string is the same and is equal to the effort E. If the number of portions of the string in the lower block be n, the total upward force on it is nE and this must be equal to the

load W supported. Thus, we have, $nE = W + \omega$, when ω is the weight of the lower block.

Hence the mechanical advantage $= \frac{W}{E} = n - \frac{\omega}{E}$.

61. Inclined Plane.—An inclined plane is a smooth rigid flat surface inclined at any angle to the horizontal. By means of this a heavy body can be raised to a certain height by the application of a force (or effort) which is less than the weight of the body.

Let AB be the inclined plane inclined at an angle θ to the horizontal line AC, and BC the height of the plane (Fig. 29).

The body D is acted upon by three forces, (i) W, its weight acting vertically downwards along DE, (ii) F, the force or effort, and (iii) R, the reaction of the plane. Let the normal to the plane at D meet AC in P. Then $\angle PDE = 90° - ADE = DAE = \theta$.

Case I.—Let the force F act upwards along the plane (see Fig. 29).

In order that the body may be in equilibrium, the sums of resolved parts of the forces parallel and perpendicular to the plane AB are separately equal to zero. Resolving W parallel and perpendicular to AB, we have $W \sin\theta$ along DA and $W\cos\theta$ perp. to DA.

Hence, $\quad W\sin\theta - F = 0$; $W\cos\theta - R = 0$;

$\therefore \quad F = W\sin\theta$; $R = W\cos\theta$.

The mechanical advantage, $\frac{W}{F} = \frac{1}{\sin\theta}$, *i. e.* $= \frac{\text{length}}{\text{height}}$ of the plane.

When $\theta = 30°$, the mechanical advantage $= 1 \div \frac{1}{2} = 2$; that is, a body of weight W can be supported by another force $P = W/2$, acting up the plane.

Case II.—Let the force F act horizontally, *i.e.* parallel to the base AC (Fig. 36).

The vertical and horizontal components of R are $R\cos\theta$ along ED and $R\sin\theta$ along FD.

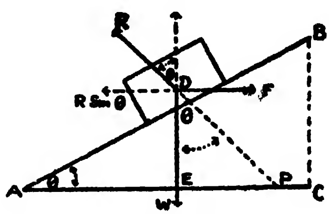


Fig. 36

$\therefore \quad R\cos\theta = W$; and $R\sin\theta = F$

$\therefore$ The mechanical advantage, $\frac{W}{F} = \frac{R\cos\theta}{R\sin\theta} = \cot\theta = \frac{\text{base}}{\text{height}}$ of the plane.

62. Balance.—If in Fig. 31(a), equally heavy scale pans are hung from A and B, where $AP = BP$, then if a weight W_1 placed in the pan

at A is balanced by a weight W_2 placed in the pan at B, we have, by the law of moments (Art. 57). $W_1 \times AP = W_2 \times BP$.

But . $AP = BP$; $\therefore$ $W_1 = W_2$.

Or, when the arms of the balance are equal, a weight placed in one pan is balanced only by an equal weight placed in the other pan. **This is the principle of the balance.**

The balance is an example of the first kind of lever in which the effort and load are equal.

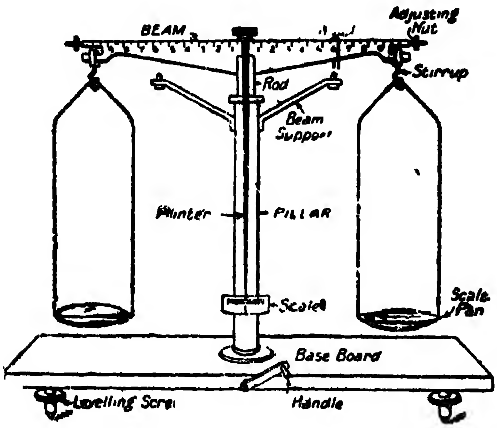


Fig. 37

It has been shown (Art. 41) that the weight of a body is directly proportional to its mass (see also p. 63). The balance is an instrument for determining the weight of any body by comparing its weight with the combined weight of a number of standard masses.

The balance consists of a horizontal rigid **beam** balanced at its centre on a knife-edge which rests on an agate plane attached on the top of a vertical **pillar** (Fig. 37).

Two **scale pans** of equal weight are suspended from stirrups (or hangers) carried by knife-edges at the two extremities of the beam. The distances between the central knife-edge and those at the extremities are called the **arms** of the balance, which should be equal. A long **pointer** attached to the centre of the beam moves over a **graduated arc** (scale) fixed on the pillar. For accurate weighing the pointer should swing evenly to equal distances on each side of the middle mark of the scale. There is a **lever arrangement** (handle) by which the pillar, which supports the beam, can be lowered and the beam arrested in order to preserve the sharpness of the knife-edges.

The use.—*To use the balance,* it is first of all levelled by levelling screws at the base board and then adjusted by means of two *screws* (adjusting nuts) at the two extremities of the beam, until the pointer oscillates equally on both sides of the middle division. The *body* to be weighed is then placed on the *left-hand pan,* and *weights* from the

weight box are added on the ***right-hand pan*** until the pointer oscillates in the same way, *i.e.* until equilibrium is restored. As the arms are equal, the two weights on the two pans are also equal. So the weight on the right-hand pan is the weight of the body.

Weight box.—The weight box (Fig. 38) with which a balance is provided contains the following weights :—100, 50, 20, 20, 10, 5, 2, 2, 1 grams. Besides these, the box contains a few fractional weights,—from 500 mgm. (*i.e.* 0·5 gm.) up to 10 mgm (*i.e.* 0·01 gm.).

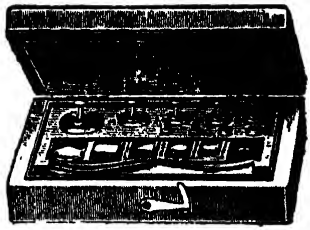


Fig. 38—Weight box.

Rider.—For very accurate weighings by means of a good balance, a bent piece of wire of mass 10 mgm. (*i.e.* a centigram), called a *rider*, is often used. Each arm of a good balance is divided into 10 equal parts (Fig. 37), and the rider can be placed on the right arm at any one of these points by means of a sliding rod from outside the case in addition to the weights from the weight box already placed on the pan until the pointer swings equally on both sides. When the rider is placed on the 10th division, *i.e.* at the end of the arm, it is equivalent to adding 10 mgm. on the corresponding pan of the balance If the rider is placed on any other division, say the 1st, the equivalent weight on the pan becomes $(\frac{1}{10} \times 10)$, *i.e.* 1 mgm., and so the rider placed on the nth division is equivalent to adding n mgm. on the corresponding pan.

Note—(i) *Weighing* by a balance means determination of a known mass which has the same weight as that of the unknown mass ; and mass being proportional to weight, a common balance is used only to compare the masses ; for,

Let W, W' be the weights of two bodies in poundals or dynes, as the case may be ; and let their masses be m and m' respectively. Then, we have, $W=mg$, and $W'=m'g$, where g is the acceleration due to gravity at that particular place. $\therefore \frac{W}{W'}=\frac{mg}{m'g}=\frac{m}{m'}$.

Thus, **the weights of two bodies at a given place are proportional to their masses.**

(ii) The position of equilibrium between any two masses is unaltered by taking the balance to another place where the value of g is different, when weighing is done by a common balance.

62(a). Requisites of a Good Balance—A good balance must be (a) **accurate,** (b) **sensitive,** (c) **stable,** and (d) **rigid.**

(a) **For accuracy.**—

(i) The two arms must be of equal length.

(ii) The centre of gravity of the beam must be vertically below the fulcrum, when the beam is horizontal.

(iii) The scale pans must be of equal weight so that the beam must be horizontal when the scale pans are empty.

(b) **For sensitiveness.**—

The beam of a good balance should appreciably deviate from its horizontal position for a very small difference between the weights in the scale pans.

The following **conditions** are to be satisfied in order that a balance may be sensitive :—

(i) The beam must be light. (ii) The arms of the beam must be large. (iii) The centre of gravity of the beam must be very near the fulcrum.

(c) **For stability.**—

It should return to its original position of equilibrium quickly after disturbance.

This **condition** is satisfied *if the centre of gravity of the beam is below the fulcrum.* It should be noted that this condition is opposed to the condition (iii) for sensitiveness, so *for extreme accuracy* in weighing, as in chemical balances, *the condition (iii) is sacrificed.*

(d) **For rigidity.**—

The beam of a good balance must be sufficiently strong not to bend under the weights it has got to carry.

62(b). Test of Accuracy.—Let a and b be the lengths of the arms of the balance, and S and S' the weights of the scale pans. Now, if the beam is horizontal with empty pans, we have, by taking moments about the fulcrum, $S.\,a = S'.\,b$............(1)

Again, the beam will be horizontal if equal weights W, W are placed on the pans. We have then, $(W+S).\,a = (W+S').\,b$.........(2). From (1) and (2) we get, $Wa = Wb$ $\therefore$ $a = b$, *i.e.* the arms are of equal length ; and since $S.\,a = S'.\,b$, we have $S = S'$;

i.e. the scale pans must be of equal weight. So, to test the accuracy of the balance, put a body in one of the scale pans, and put weights in the other pan to balance it. Next, interchange the positions of the body and the weights. If the beam of the balance is still horizontal, the balance is true, otherwise the arms are of unequal length, if scale pans are of equal weight.

63 Weighing by the Method of Oscillation.—The operation of weighing by a sensitive balance takes a very long time before its beam comes to rest. It is, however, unnecessary to wait till the pointer comes to rest, for we can calculate the position, which the pointer would occupy if the balance comes to rest. This can be done by observing the readings of the scale corresponding to the turning points of the pointer while the balance is swinging. The position so determined is called the "**resting point**" (written, R. P.) for a particular adjustment of weights and load, or for empty pans. It is more accurate and much quicker to perform the weighing by this method, which is called the **method of oscillation.** *This method is suitable when the weight to be taken is small.*

Procedure.—(1) Imagine the scale divisions over which the pointer moves to be numbered from left to right, as shown in the figure (Fig. 39). Slowly raise and lower the beam two or three times so that the pointer swings over about $\frac{3}{4}$ of the scale divisions. When after two or three oscillations the motion becomes regular, take a reading of the turning point of the pointer, by avoiding parallax, as it swings to the left (say 4). Then read the extreme position of the subsequent swing to the right (say 14). Again read the next swing to the left (say 5).

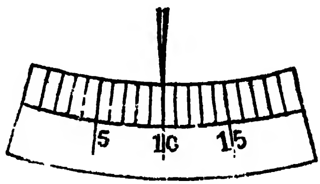


Fig. 39

Thus three consecutive readings, one to the right and two to the left, have been taken from which the R. P. for empty pans can be calculated in the following way. Take the mean of the two left hand readings, *i.e.*, the first and the third readings. Then the mean of this mean and the right-hand reading, *i.e.*, the second reading, will give the mean R. P. for empty pans.

For greater accuracy five consecutive readings (two to the right and three to the left) should be taken. The R. P. for the empty pans is found as above a number of times and therefrom the mean R. P. (x) for the empty pans obtained.

The reason for taking an odd number of observations is that the arc over which the pointer is swinging is continually growing less, due to friction and air resistance, and thus if only two observations, say, one to the left and then another to the right, or two to the left and two to the right, are taken, the position of rest obtained by taking the arithmetic mean of these two will be too far to the left. The mean of any odd number of observations, obtained as above, will, represent the true position of rest more approximately.

Next place the body to be weighed on the left hand pan and try to get its weight (w) by adding wts. on the right hand pan until the pointer oscillates within the scale. Let the mean R. P. for the loaded pans be y.

Next find out the mean R. P. (z) when the wt. w on the right-hand pan is increased by 1 milligram or any such small wt.

Now it is necessary to calculate out what wt. must be added to or subtracted from w in order to reduce the R. P. from y to x.

Calculation :—

True wt. $= w + \frac{0 \cdot 001}{y - z} \times (y - x)$ gm. when $y > x$.

If y is less than x, true wt. $= w - \frac{0 \cdot 001}{y - z} \times (x - y)$ gm.

Note.—As the sensibility of a balance varies with the load it should be calculated everytime a body is weighed. *The sensibility of a balance is defined as the change of the resting point due to a change of some definite weight, usually one milligram, in one of the pans.*

64. The true weight of a body can be determined with the help of a false balance in the following two ways :—

(i) **The Method of Substitution** *(Borda's Method)*.—Place the body to be weighed on the left-hand pan and counterpoise it by sand (or other convenient substance) in the right-hand pan. Then remove the body and replace it by known weights to balance the sand. Since the body and the weights both balance the sand under exactly the same conditions, they must be equal.

(ii) **The Method of Double Weighing** *(Gauss's Method)*.—Let a and b be the lengths of the arms. Place a body of true weight W in the left-hand pan, and let its apparent weight be W_1.

Then taking moments of the force on each side, $W \times a = W_1 \times b$ ...(1)

Now put the body in the right-hand pan, and let W_2 be its apparent weight, then $W_2 \times a = W \times b$. ...(2)

From (1) and (2), $\frac{W}{W_2} = \frac{W_1}{W}$; *or* $W^2 = W_1 . W_2$; *or* $W = \sqrt{W_1 . W_2}$. Thus, *the true weight is the geometrical mean of the two apparent weights.* ...(3)

Ratio of the Arms.—

From eq. (1), $\frac{a}{b} = \frac{W_1}{W}$, and from eq. (2), $\frac{a}{b} = \frac{W}{W_2}$

$$\therefore \frac{a^2}{b^2} = \frac{W_1}{W} \times \frac{W}{W_2}\text{; or } \frac{a}{b} = \sqrt{\frac{W_1}{W_2}}. \qquad \ldots(4)$$

64(a). A False Balance.—By using a false balance with unequal arms a tradesman will defraud himself if he weighs out a substance, to be given to a customer, in equal quantities by using alternately each of the scale pans. Let W be the true weight of the quantity of a substance which appears to weigh W_1 and W_2 successively by the two scale pans of a balance of which a and b are the lengths of the arms. Here, the customer gets $(W_1 + W_2)$ instead of $(W + W)$ *i.e.* $2W$ and we have, $W_1 + W_2 - 2W = W\frac{a}{b} + W\frac{b}{a} - 2W$ (from eqs. 1 and 2).

$$= W\left(\frac{a^2 + b^2 - 2ab}{ab}\right) = W\frac{(a-b)^2}{ab}.$$

The right-hand side of the equation is always positive whatever be the values of a and b, and so $(W_1 + W_2)$ is always greater than $2W$. Thus the tradesman defrauds himself by the amount $W\frac{(a-b)^2}{ab}$.

Hence, at the time of purchasing a substance, *a customer should always insist on having half of the substance weighed on one pan and the other half on the other.*

Example.—*An object is placed in one scale pan and it is balanced by 20 lbs. The object is then put into the other scale pan, and now it takes 21 lbs. to balance it. When both scale pans are empty the scales balance. What is the matter with the balance, and what is the true weight of the object?* (*Pat. 1934*)

Two different weights are required to balance the same object when placed on different pans of the balance because the arms of the balance are unequal [see Art. 64 (ii). The true wt. $= \sqrt{20 \times 21} = 20{\cdot}494$ lbs.

65. Common (or Roman) Steelyard.—This is a form of balance with unequal arms and is used for rough and quick weighing. It consists of a graduated beam AB (fig. 40) movable about a fixed fulcrum F very near one of its ends. A known sliding weight E slides over the arm AB. The object W to be weighed is suspended at a hook A and then the beam is made horizontal, that is, the body is balanced, by changing the position of E. It should be noted that the graduations are

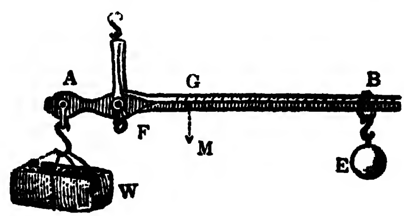


Fig. 40

correct with only a constant weight E, and if this weight is changed the graduations must be changed correspondingly. If M be the weight of the beam acting at its centre of gravity G, we have, for equilibrium, $W \times AF = (M \times GF) + (E \times BF)$

The **platform balances** often used in Railway stations work on the principle of common steelyard.

66. Spring Balance.—Spring balance is essentially an instrument for measuring a force such as the wt. of a body. It consists of a *spiral spring*, the lower end of which is attached to a hook for supporting the body to be weighed (Fig. 41). An *index* or a pointer, attached to the spring moves along a metal scale which is graduated in grams or pounds with the help of known wts. The body to be weighed is suspended from the hook, and *the spring is elongated due to the force with which the body is attracted by the earth.* The position of the pointer on the metal scale indicates the weight of the body which is the measure of the force with which it is attracted towards the centre of the earth. So by a spring balance the weight of a body at a given place is directly obtained, and this weight will differ at different places as the pull on the spring due to the force of attraction of the earth changes from place to place. So a spring balance can give a true weight of a body only at the particular place where it was graduated. By a *spring balance* we compare *different weights* while by a *common balance* we compare *different masses* and not weights (see note, Art. 62).

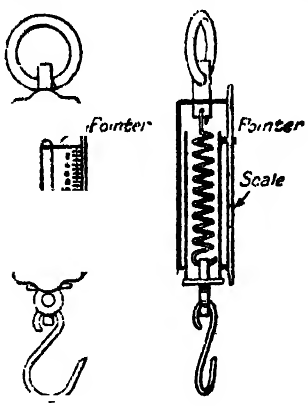


Fig. 41—Spring balance.

66(a). The principle of the spring balance may be learnt by arranging a spiral, made of thin steel wire, to move in a groove between two strips of wood. The upper end of the spring is clamped and the lower end carries a scale pan and a pointer, or index, which moves over a millimetre scale attached to the side of the spring. This forms a **spring balance** or a **spring dynamometer** (force-measurer).

Experiment.—*To graduate a dynamometer*, fix it vertically and mark the initial position of the pointer. Add known weights, say 10 gms., at a time, and read the position of the pointer after each addition. Repeat these observations until the spring is extended to nearly twice its original length. Then reverse the process, *i.e.*, remove the weights, step by step, and note the readings as before. Tabulate the readings.

Now plot a curve (Fig. 42) taking weights as abscissæ and index readings as ordinates. *The graph is a straight line.*

Conclusion.—The result of the above experiment shows that (i) the amount of elongation is proportional to the load applied, and that (ii) the spring used is a very elastic material, because, on the removal of various loads, the index returns to the original position. The first of these is known a **Hooke's Law**, after its discoverer, which states that *the strain is proportional to the stress*, where *strain* signifies the fractional elongation produced by a load, and *stress*, the internal forces equal to the distorting force tending to bring the body back to its original form. Hooke's law is applicable to any kind of deformation or distortion of an elastic body, such as bending, twisting, or stretching.

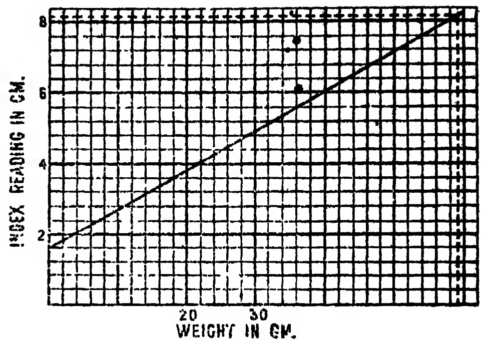


Fig. 42

Experiment.—*Determination of an unknown weight.*—Place a small object on the scale pan of the dynamometer, and note the position of the index, which is, say, 8·2 cm. Now by means of the graph, as obtained above, deduce the weight of the object. The weight, as indicated by the graph (Fig. 42), is 59 gms.

Note that, unlike common balance, the spring balance not only measures the absolute weights of bodies, but it can also measure other forces, after its scale has been calibrated to a given unit.

67. Distinction between Mass and Weight.—The **mass** of a body is the quantity of matter in the body, and the **weight** of a mass is the force with which it is attracted by the earth.

If g be the acceleration due to gravity at any place, the weight of a body of mass m grams at that place is mg dynes (p. 38). So, at different places the weight of the same body will be different, if the value of g be different, but the mass, or the quantity of matter in the body remains constant. The value of g differs from place to place on the surface of the earth due to (a) *the peculiar flattened shape of the earth*, the polar diameter being less than the equatorial diameter (see Art. 51). Thus, the value of g being greater at the Poles, the same body would weigh greater on the Poles than on the Equator that is, *the weight of the body increases from the Equator to the Poles.*

For examples, *the absolute weight of a pound mass varies from 32·091 poundals at the Equator to 32·255 poundals at the Poles;* and *the absolute weight of a gram mass varies from 978·10 dynes at the Equator to 983·11 dynes at the Poles.* (b) As the value of g decreases with the increase of altitude [being *inversely* proportional to the square of the distance of the body from the centre of the earth [*vide* Art. 51], the weight of a body decreases in higher altitudes, the mass remaining constant. For instance, the weight of a 10 lb. body, at a height of 50,000 miles, would be about as much as an ounce at the surface of the earth. Again, the value of g is less inside the earth (being *directly* proportional to the distance of a body from the centre of the earth [see Art. 51]), so the weight of a body decreases as it is taken down inside the earth, say at the bottom of a mine, the greater the depth the less is the weight. Thus the *weight is not the essential property of matter,* as a body taken to *the centre of the earth,* where the value of g is zero, would have *no weight.*

The value of g also differs owing to (c) *the diurnal rotation of the earth about its axis,* by which every body on the earth's surface revolves, and in order to keep the body in the circular path a certain fraction of the true weight of the body is lost. So the observed weight becomes less than its true weight.

At any one place the mass of a body is proportional to its weight. This means that if a piece of iron weighs five times as much as a piece of lead, the mass of iron is five times that of the lead piece. Hence when we obtain the weights of various bodies, we also measure their respective masses.

Mass is measured in *grams* or *pounds,* while weight should be measured in units of force, *i.e., dynes* or *poundals.* Ordinarily, however, the two words, mass and weight, are used as synonymous, because, as stated above, we get only a comparison of the masses by weighing any body by an ordinary balance, and so the weight of a body *at a given place* may be regarded as a measure of its mass, and this has led to the use of units of mass as units of weight. But we must **beware of this double meaning of "weight."**

67(a). Detection of Variation of wt. of a body on change of Place.—The difference in wt. of a body at different places cannot be detected by an ordinary balance, because the body as well as the 'wts.' that are used to weigh it are both equally affected by the variation of g, and so we get comparison of their masses only. It can, however, be detected by means of a delicate spring balance, where the body depresses the pointer to different distances, as the force on it becomes different for different values of g. For instance, if a cubic foot of

water is weighed in a spring balance in London and also at the equator, the indicated weight would be $3\frac{1}{5}$ ozs. greater in London than at the equator. Similarly it would be $\frac{1}{5}$ oz. greater in Manchester than in London (see the table in p. 50).

Examples.—(1) *A body is weighed in a spring balance at a place where $g=980{\cdot}94$, and the reading indicated by the balance is 50 grams. What will the reading be if the body be taken at a place where $g=981{\cdot}54$?*

Let W_1 and W_2 be the readings of the spring balance, g_1 and g_2 the corresponding values of g; then, if m be the mass of the body, which is the same everywhere, we have,

$$\frac{W_2}{W_1}=\frac{mg_2}{mg_1};\quad \therefore\quad W_2=\frac{g_2}{g_1}\times W_1=\frac{981{\cdot}54\times 50}{980{\cdot}94}=50{\cdot}031 \text{ gms. (nearly)}$$

(2) *If the weight of a man is 160 lbs. on a beam balance at a place where $g=980{\cdot}665$ cm./sec^2., how much he would weigh on an accurate spring balance at the Equator ($g=978{\cdot}1$), and at the Pole ($g=983{\cdot}1$)?*

The mass of the man $=\dfrac{160}{980{\cdot}665}$, which remains constant both at the Equator and at the Pole.

If w' be the weight of the man at the Equator and g the value of acceleration due to gravity, we have, $w'=mg'=\dfrac{160}{980{\cdot}665}\times 978{\cdot}1=159{\cdot}56$ lb.

Similarly, if w'' be the weight at the Pole, $w''=\dfrac{160}{980{\cdot}665}\times 983{\cdot}1=160{\cdot}39$ lb.

(3) *If the mass of the earth is 81·53 times that of the moon, and the diameter of the earth is 3·673 times that of the moon, compare the weight of a body on the surface of the moon with its weight on the surface of the earth.*

We know from the law of gravitation that the forces of attraction between two bodies are directly proportional to their masses and inversely proportional to the square of the distance between them.

Let m be the mass of the body, M the mass of the moon, M' the mass of the earth, and d the distance between the moon and the body, when it is on the surface of the moon, *i.e.*, $d=$ the radius of the moon; and d' the radius of the earth, *i.e.* distance between the body and the earth, when the body is on the surface of the earth. Then, we have the attraction of the moon $F\propto\dfrac{mM}{d^2}$ and the attraction of the earth $F'\propto\dfrac{mM'}{d'^2}$. $\therefore\ \dfrac{F}{F'}=\dfrac{M}{M'}\times\dfrac{d'^2}{d^2}=\dfrac{1}{81{\cdot}53}\times\dfrac{(3{\cdot}673)^2}{1}=0{\cdot}1655$

$$\left(\because\ \frac{d'}{d}=\frac{\text{radius of earth}}{\text{radius of moon}}=\frac{\text{diameter of earth}}{\text{diameter of moon}}\right).$$

This problem shows that while the mass of the body is the same on the moon as on the earth, its weight on the earth is about 6 times greater than that on the moon.

68. Apparent Weight of a man in a moving Lift.—When a man is ascending or descending in a lift with uniform velocity, his weight exerted on the floor of the lift is equal and opposite to the reaction of the floor. When, however, the lift is rising upwards, the reaction is greater than the man's weight; and, when it is going downwards, the reaction is less than the man's weight.

Let m be the mass of the man, R the thrust on the floor of the lift, which is equal and opposite to the reaction of the floor on the man, and which may be called the man's "apparent weight."

The forces acting on the man are (a) his weight mg acting downwards, and (b) R acting upwards. Suppose the lift is *descending* with an acceleration f: Remembering that, force = mass × acceleration,

we have, $mg - R = mf$; $\therefore$ $R = m(g-f) = mg\left(1 - \frac{f}{g}\right)$..............(1)

Hence the *man's apparent weight is less than his actual weight mg by f/g of the latter, i.e.*, the man will appear to be lighter. Similarly, when the lift is *ascending* with an upward acceleration f,

we have, $R = mg\left(1 + \frac{f}{g}\right)$..................(2)

Hence *the man will appear to be heavier by f/g of his actual weight.*

Example.—*If a man, weighing 16 stones stands on a lift which has an acceleration of 8 ft. per sec.², find the thrust on the floor due to his weight (i) when it is ascending, (ii) when it is descending.*

(i) We have, $R = mg\left(1 + \frac{f}{g}\right)$..............(see eq. 2, Art. 68)

$= 16$ stones wt. $\times (1 + \frac{8}{32}) = 20$ stones wt.

(ii) In this case, we have, $R = mg \times \left(1 - \frac{f}{g}\right)$...........(see eq. 1, Art. 68)

$= 16$ stones wt. $\times (1 - \frac{8}{32}) = 12$ stones wt.

69. Laws of Falling Bodies.—

(1) *In a vacuum all bodies fall with equal rapidity.*

Though the acceleration due to gravity is the same for all bodies at the same place, the resistance of air influences the general rate of fall. This will be evident by comparing the descent of a parachute with that of a lump of stone. The stone will fall very quickly and the observed difference in the rate of fall is due to the resistance offered by the air, the resistance increasing with the extent of surface of the falling body. Different objects will, however, fall at the same rate in a vacuum, where the resistance to motion is nil.

This was first stated and proved by Galileo in 1589, by dropping balls of different sizes and materials from the top of the leaning tower of Pisa. The balls reached the ground in practically the same time. Afterwards in 1650 it was proved in a conclusive manner by Newton in his *guinea-feather experiment.*

Guinea-Feather Experiment.—A wide glass tube (fig. 43) about a metre long, having a cap screwed at one end and a stop-cock at the other is taken. A piece of paper and a small coin are introduced into the tube. On suddenly inverting the tube, it is found that the coin reaches the other end earlier than the piece of paper. Next by opening the stop-cock to which end an air-pump may be connected, the air within the tube is exhausted. On now suddenly inverting the tube it is found that the coin and the paper fall together and reach the other end simultaneously.

The following simple experiment also proves the law. A piece of paper is laid on a metal disc of larger diameter and the combination is dropped down together. They are found to reach the ground simultaneously. Here the disc overcomes the resistance due to air and the paper easily accompanies it.

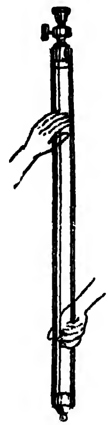


Fig. 43. Guinea-feather experiment.

(2) *The space traversed by a body falling freely from rest is proportional to the square of the time,*—*e.g.*, if the space traversed in one second is 4 feet, in two seconds it will be 16 feet (i.e. 4×2^2), in three seconds 36 feet (i.e. 4×3^2); and so on.

So, if s and t denote the space and time respectively, $s \propto t^2$.

This can be mathematically represented by the equation, $s = \frac{1}{2}gt^2$ (see Art. 28), where g is the acceleration due to gravity.

(3) *The velocity acquired by a body falling from rest is proportional to the time of its fall,—e.g.*, if the velocity at the end of one second is 8 feet per second, at the end of two seconds it is 16 (*i.e.* 2×8) feet per second, and at the end of three seconds it is 24 feet per second and so on. For this reason, a stone falling from a balloon at a great height will acquire so much velocity that it will strike the surface of the earth almost like a rifle bullet. So, if v denotes the velocity and t the time, $v \propto t$. This can be mathematically represented by the equation $v = gt$ (see Art. 28).

70. Falling of Rain-drops—A small rain-drop does not fall so quickly as a larger one, as the rate of fall of a smaller one is retarded more by the air.

The resistance of air $\propto$ the area of cross-section through the centre of the drop ; *i.e.*, $\propto \pi \times (\text{radius})^2$.

But the weight of the drop $\propto$ its volume $\propto \frac{4}{3}\pi \times (\text{radius})^3$.

Hence, when the radius increases—*i.e.* for a drop of larger size—the weight increases more rapidly than the resistance of the air. So *a larger drop falls more rapidly than a smaller one.*

It has been found that the maximum velocity of a very small rain-drop of diameter equal to $\frac{1}{5000}$ mm. is about 1·3 cm. per sec., and that of a larger drop of diameter equal to 0·46 cm. may be about 800 cm. per second.

71. Bodies projected vertically Downwards—If a body be projected vertically downwards with an initial velocity u, the equations of Art. 28 become

$v = u + gt$, where v is the velocity after a time t ;

$s = ut + \frac{1}{2}gt^2$, ,, s ,, distance fallen through ;

$v^2 = u^2 + 2gs$, ,, v ,, velocity at a given height s ;

g being the acceleration due to gravity.

72. Bodies projected vertically Upwards.—If a body is projected vertically upwards with an initial velocity u, we must substitute $-g$ for f, and the equations of Art. 71 now become

$$v = u - gt \;;\quad s = vt - \tfrac{1}{2}gt^2 \;;\quad v^2 = u^2 - 2gs.$$

Greatest Height attained.—At the highest point the velocity of the body is zero, so if x be the greatest height attained by the body, we have

$$0 = u^2 - 2gx.$$

Hence the greatest height attained $= u^2/2g$.

Again, the time t to reach the highest point is given by

$$0 = u - gt, \text{ whence } t = u/g.$$

Similarly u/g will be the time to reach the ground from the highest point. So the whole time of flight $= 2u/g$.

Example.—(1) *A body is thrown vertically upwards with a velocity of 100 ft. per sec. Find (a) how high it will go, (b) the time taken to reach the highest point, (c) the time of its returning to the ground.*

At the highest point the velocity of the body will be momentarily zero, and the body will then fall.

(a) Here, $u = 100$ ft. ; $v = 0$ at the highest point, $g = 32$ ft./sec^2 ; $s = ?$

We have $v^2 = u^2 - 2gs$.

$\therefore\ 0 = 100^2 - 2 \times 32 \times s$; $\qquad \therefore\ s = \dfrac{100 \times 100}{64} = 156{\cdot}25$ *ft.*

(b) Here $u = 100$ ft./sec. ; $v = 0$; $g = 32$ ft./sec.^{2} ; $t = ?$

We have, $v = 100 - 32\,t$. ; $\qquad \therefore\ t = \dfrac{100}{32} = 3\frac{1}{8}$ sec.

(c) Here $u = 100$ ft./sec. ; $g = 32$ ft./sec. ; $s = 0$; $t = ?$

We have $s = ut - \frac{1}{2}gt^2$; or, $0 = ut - \frac{1}{2}gt^2$;

or, $t(u - \frac{1}{2}gt) = 0$; whence either $t = 0$, which is rejected ;

or $u = \frac{1}{2}gt$; *i.e.* $t = \dfrac{2u}{g}$; $\quad \therefore\quad t = \dfrac{2 \times 100}{32} = 6\frac{1}{4}$ *sec.*

(2) *Two stones are projected vertically upwards at the same instant. One ascends 112 ft. higher than the other and returns to earth 2 seconds later. Find the velocities of projection of the stones ($g = 32$ ft. per sec. per sec.)* (*C. U. 1935.*)

At the highest point v will be momentarily zero, so we have, $0 = u^2 - 2gs$

or, $s = \dfrac{u^2}{2g}$ for one stone. For the other stone, $s + 112 = \dfrac{u_1^2}{2g}$.

$$\therefore\ 112 = \frac{u_1^2 - u^2}{2g} \ldots\ldots(1)$$

At the highest point $u = gt = 0$: or, $t = u/g$. $\therefore$ Total time of flight $t_1 = 2u/g$; and for the other, the total time of flight $(t_1 + 2) = 2u_1/g$.

$$2 = \frac{2(u_1 - u)}{g} = \frac{u_1 - u}{16} \ldots\ldots\ldots(2)$$

From (1), $\quad 112 = \dfrac{u_1^2 - u^2}{64} = \dfrac{u_1 - u}{16} \times \dfrac{(u_1 + u)}{4} = 2 \times \dfrac{u_1 + u}{4} = \dfrac{u_1 + u}{2}$;

$\therefore\ u_1 + u = 224$; and $u_1 - u = 32$......from (2)

or $\ u_1 = 128$ ft./sec. ; and $u = 96$ ft./sec.

(3) *A stone is dropped from a balloon at a height of 200 feet above the ground and it reaches the ground in 6 seconds. What was the velocity of the balloon with which it was rising?*

At the moment when the stone was dropped it was moving upwards with the same velocity as the balloon. Let this velocity be u ft. per sec. upwards. So, here u is negative, and g is positive as the stone is falling downwards.

Here $u = ?$; $s = 200$ *ft.* ; $t = 6$ *sec.* ; $g = 32$ *ft./sec*^{2}.

We have, $s = (-u)t + \frac{1}{2}gt^2$; or $200 = -u \times 6 + \frac{1}{2} \times 32 \times 6^2 = -u.6 + 576$;

$\therefore \quad 6u = 576 - 200 = 376$; or $u = 62\frac{2}{3}$ *ft. per sec.*

(4) *It is required to pierce a war balloon at an elevation of ¾ mile by means of a rifle bullet fired immediately under it. If to pierce the balloon, the bullet must have a velocity of 40 ft. per sec. on reaching the balloon, with what velocity must it leave the muzzle of the gun?* (*Pat. 1932*)

$$\tfrac{3}{4} \text{ mile} = \frac{1760 \times 3}{4} = 1320 \textit{ ft.}$$

Here $v = 40$ *ft./sec.* ; $u = ?$; $g = 32$ *ft./sec.*^{2} ; $s = 1320$ *ft.*

We have, $v^2 = u^2 + 2gs$; or $40^2 = u^2 + 2(-32) \times 1320$;

$\therefore \quad u^2 = 40^2 + 64 \times 1320 = 86080$; $u = 293{\cdot}4$ *ft. per sec.*

Pendulum

73 Some Terms.—

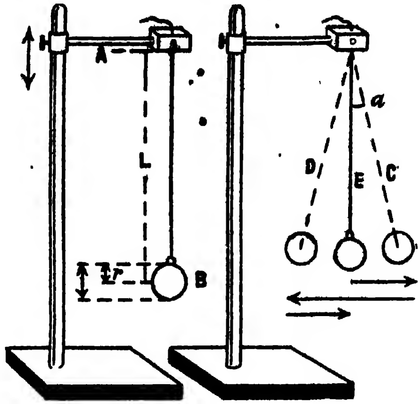
Simple Pendulum.—A simple pendulum is defined as a heavy particle suspended by a weightless, inextensible but perfectly flexible thread from a fixed point on a rigid support about which the pendulum oscillates without friction. In practice, however, a small metal bob suspended by a very fine thread is taken.

Compound Pendulum—A compound pendulum may be defined as a heavy body capable of oscillating to-and-fro about a fixed point, or a fixed line, as an axis. The metallic rod inside a clock, carrying at its lower end a heavy lens-shaped mass of metal, known as the bob, is an example of a compound pendulum.

Second's Pendulum.—It is a simple pendulum which takes one second in making half a complete oscillation. It has a period of two seconds.

Length of a simple Pendulum.—It is the distance from the point

of suspension upto the centre of gravity of the bob *i.e.*, the distance between A and B (fig. 44a). That is, it is the length of the suspension thread plus the vertical radius r of the bob. It is also called the *effective length* of the pendulum.



(a) Fig. 44 (b)

Amplitude.—The maximum angular displacement α (fig. 44b) of the bob from its undisturbed position (given by the vertical position E) on either side as at C or D is called its amplitude. It should not exceed 4°, for the motion to be *simple harmonic*. The amplitude gradually decreases as the bob swings, on account of air resistance mainly.

Time period or simply period—It is the time taken by a pendulum to make *one complete oscillation.* One complete oscillation means two swings—one forward, another backward. It is usually reckoned from the extreme position D (fig. 44, b) to the other extreme position C and back to D next time or from the undisturbed position E (pendulum vertical) when, say, moving to the right, until when it passes through E again moving in the same direction as shown by the arrows.

One *vibration* means the motion from one extreme position, say D, to the other extreme position C *i.e.*, it is half of an oscillation.

Frequency—It is the number of complete oscillations made by a pendulum per sec. at a place. Thus if n = frequency and T = time period, $nT = 1$ or $n = \frac{1}{T}$.

Phase.—The phase of a pendulum gives its state of motion at a particular instant and determines the position of the pendulum in the path as well as the direction of motion at that instant.

74 Laws of Pendulum.—The laws of oscillation of a simple pendulum are given by the following relation,—

$$t = 2\pi\sqrt{\frac{l}{g}}$$

where t = period of the pendulum ; l = effective length ; g = acceleration due to gravity.

Law (1). *The period of a pendulum is isochronous.* That is, a pendulum takes equal time to complete one oscillation whatever is the amplitude provided the latter is small (within 4°). So time period is independent of the amplitude of vibration. *This is known as the law of isochronism.*

Law (2). *The period of oscillation varies directly as the square root of the length.* Mathematically, $t \propto \sqrt{l}$ or $l/t^2 = a$ constant for the place of observation. Thus, if the length be increased four times, the period becomes double. *This is known as the law of length.*

[**Note.**—The length of a pendulum changes with temperature : so the period t of a pendulum changes with temperature.]

Law (3). *The period varies inversely as the square root of the acceleration due to gravity at the place of observation. This is known as the law of acceleration.* Mathematically $t \propto 1/\sqrt{g}$ or $t^2.g. =$ a constant for the same pendulum.

Thus, if g be greater at a place, t will be less, *i.e.*, the pendulum will vibrate more rapidly.

Law (4). *The period does not depend on the mass or material of the bob of the pendulum, provided the length remains constant. This is known as the law of mass.*

Motion of a simple pendulum is simple Harmonic.—Let the bob of mass m of a pendulum of length l [fig. 44(c)] be displaced through an angle θ from its undisturbed position B to the position C. If g be the acceleration due to gravity at the place the wt. mg of the bob can be resolved into the two components $mg \cos\theta$ acting along CF, the direction of the string which is kept tant thereby and $mg \sin\theta$ acting along CE at rt. angles to CF. The former is balanced by the tension T of the string while the latter tends to bring the bob back to its original position B with an acceleration $g \sin\theta$. If θ does not exceed 4°, $\sin\theta$ may be taken equal to θ and so the acceleration of the bob $g \sin\theta = g.\theta$. After crossing the mean position B, when the bob moves towards BD in virtue of its inertia and acquired velocity, the acceleration acts in the opposite direction *i.e.* towards B and so the motion decreases and vanishes at the other extreme position D when the direction of motion is reversed. So the acceleration is always directed towards the mean position B.

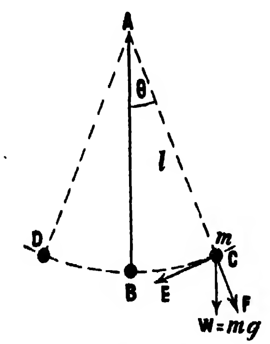


Fig. 44(c)

$$\text{Again } \theta = \frac{\text{Arc } BC}{\text{length } AB} = \frac{\text{displacement}}{\text{length of pendulum } (l)}$$

$$\therefore \text{ Acceleration} = g.\ \theta. = \frac{g}{l} \times \text{displacement.}$$

That is, acceleration is proportional to displacement since g and l are constants for the pendulum at the given place.

Thus acceleration being proportional to displacement and always directed to a fixed position B in the path of motion, the motion is simple hormonic, according to definition of simple harmonic motion.

Period of a simple pendulum.—Mathematically, motion of a pendulum which is simple harmonic, is given by,

$$\frac{\text{Acceleration}}{\text{Displacement}} = w^2, \text{ where } w = \text{angular velocity}$$

$$= \left(\frac{2\pi}{T}\right)^2 \text{ where } T = \text{time period.}$$

$$i.e., \quad T^2 = 4\pi^2 \times \frac{\text{displacement}}{\text{acceleration}} = 4\pi^2 \times \frac{l}{g}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}}.$$

75. Verification of the laws.

Law (1)—(*Law of Isochronism*).—To verify the first law, note with a stop-watch the total times of, say, 20 oscillations with different amplitudes, keeping the length constant. It will be found that the period t in each case remains constant.

It should be noted that the law is true only for a small angle of amplitude (about 4°) ; so when noting the times of oscillations with different amplitudes, care should be taken not to exceed the maximum limit of 4°.

Law (2)—*The Law of Length*—Find the *vertical* radius of the bob by means of a slide-calipers, and hence determine the length from the point of suspension up to the centre of gravity of the bob. Observe the time taken for 20 complete oscillations, and thus find t, the period

Change the length of the pendulum and again find the period. In this way get several values of the period for the corresponding lengths. It will be found that $t \propto \sqrt{l}$, or the value l/t^2 will be constant. The

variation between t^2 and l can be represented by a *straight line* (Fig. 45).

Law (3)—(*The Law of Acceleration*).—This law can be verified by taking a pendulum at different places having different values of g. It will be seen that at a place where g is greater, the vibrations will be quicker. $t^2.g$ will be found constant at the different places for the same length of the pendulum.

Law (4)—(*The Law of Mass*).—Keeping the length of the pendulum the same in every case, if the bob be replaced by another of different size of a different material, it will be found that t will remain unaltered.

By performing this experiment with bobs of different substances (such as wood, iron, brass, etc.) it can be shown that *the acceleration due to gravity at the same place is the same for all bodies.*

Graph.—Draw a graph with l (on the X-axis) and t^2 (on the Y-axis). The graph in Fig. 45, which is a *straight line*, represents the relation between l *and* t^2. The relation between l and t will be an arm of a *parabola*. From any of these graphs the length of the pendulum corresponding to a given time of oscillation can be determined, but it is better to take the help of l and t^2 graph (straight line) for this purpose.

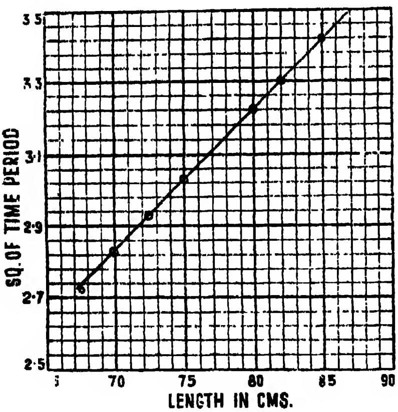


Fig. 45.

76. Second's Pendulum.—The period of a second's pendulum is 2 seconds. Hence, from the formula for the period of oscillation, we have,

$$1 = \pi\sqrt{\frac{l}{g}} \;\; ; \text{ or } l = \frac{g}{\pi^2} \ldots (1)$$

So the length of the second's pendulum changes at different places having different values of g.

Taking the mean value of g to be 981 cm. per sec. per sec., the length of the second's pendulum becomes [from eq. (1)]

$$l = \frac{981}{\pi^2} = \frac{981}{9{\cdot}87} = 99{\cdot}39 \text{ cms.}$$

Taking the value of g to be 32·2 ft. per sec. per sec.,

$$l=\frac{32·2}{\pi^2}=\frac{32·2}{9·87}=3·26 \text{ ft.}=39·19 \text{ inches.}$$

Graph.—To determine the length of the Second's pendulum from the graph, draw the $l-t^2$ graph and find the length corresponding to $t^2=4$ (Fig. 45).

77. The Value of 'g' by a Pendulum.—By carefully measuring the length and the corresponding period of a simple pendulum, the value of g at any place can be determined from the formula,

$$t=2\pi\sqrt{\frac{l}{g}}\text{; whence } g=\frac{4\pi^2 l}{t^2}.$$

Thus when the value of l/t^2 at a place is (say) 24·84, g is given by,

$$g=4\pi^2\times l/t^2=4\times 9·87\times 24·84=980·68 \text{ cms./sec}^2.$$

Loss or Gain of time by a clock on change of place.—The value of g varies with the latitude of a place (Art. 50). It is minimum at the Equator and increases gradually towards a Pole. But as the time-period t of a simple pendulum varies inversely as the square root of g, the t of a pendulum will increase as it is taken from the Equator to a Pole. So a pendulum clock will gradually gain time, *i.e.* will go *fast*, when taken from the Equator to a Pole.

Again, as the value of g diminishes with the distance above, and also below, the surface of the earth, the time-period t of a pendulum clock will increase, and so the clock will lose time, *i.e.* will go *slower* when taken to the top of a mountain or to the bottom of a mine.

78. Disadvantages of a Simple Pendulum.—(*i*) In obtaining the formula for the simple pendulum, the thread was assumed to be weightless and all the mass of the bob was assumed to be concentrated at its centre ; but, in practice, neither of these conditions is strictly true.

(*ii*) The formula for the simple pendulum is true only for very small amplitudes, and corrections should be made for large amplitudes.

(*iii*) Corrections should also be applied for the effect of resistance to motion, and the buoyancy of the air, which raises the centre of gravity of the pendulum.

(*iv*) Errors are also introduced due to the slackening of the thread when approaching the limit of swing, and *also due to the friction at the point of suspension* which may interfere with the free movement of the pendulum.

6

Examples.—(1) *Find the length of a second's pendulum at a place where $g=981$.* *(C. U. 1912, '19.)*

For a simple pendulum, we have, $t=2\pi\sqrt{l/g}$.

For a second's pendulum, $t=2$ secs ; $\therefore\ 2=2\times\frac{22}{7}\sqrt{\frac{l}{981}}$.

Hence $l=\frac{49}{22\times 22}\times 981=99{\cdot}39$ cms. nearly.

(2) *Two pendulums of lengths 1 metre and 1·1 metre respectively start swinging together with the same amplitude. Find the number of swings that will be executed by the longer pendulum before they are again swinging together* ($g=978$ *cms. per sec.*^{2}) *(C. U. 1909)*

Let t_1 and t_2 be the periods of oscillation of the pendulums of lengths 1 metre and 1·1 metre respectively. 1 metre = 100 cms., and 1·1 metre = 110 cms.

Then we have,

$$t_1=2\pi\sqrt{\frac{100}{978}}; \quad \text{and} \quad t_2=2\pi\sqrt{\frac{110}{978}}.$$

Suppose the pendulum of 1·1 metre length makes n_1 swings, and the other makes (n_1+n_2) swings before they again swing together,

then, $n_1t_2=(n_1+n_2)t_1$; or $n_1(t_2-t_1)=n_2t_1$..........(1)

But $(t_2-t_1)=\frac{2\pi}{\sqrt{978}}(\sqrt{110}-\sqrt{100})$

$\therefore$ from (1) $\frac{2\pi n_1}{\sqrt{978}}(\sqrt{110}-\sqrt{100})=\frac{2\pi n_2\sqrt{100}}{\sqrt{978}}$;

or $n_1=\frac{10}{\sqrt{110}-10}n_2=\frac{10(\sqrt{110}+10)}{10}n_2$;

$=\sqrt{110}+10)n_2=20{\cdot}5n_2$ (nearly) $=\frac{41}{2}n_2$ (nearly).

To get a whole number, the least value for n_2 is 2, and, therefore, $n_1=41$ nearly.

(3) *Supposing a pendulum to be so constructed that it beats seconds at a place where $g=980$, how would its length have to be changed so that it may beat seconds at a place where $g=340$?*

The period of a Second's pendulum is 2 seconds.

We have, $t=2\pi\sqrt{\frac{l}{g}}$. Hence $2=2\pi\sqrt{\frac{l}{980}}$.

Again, $2=2\pi\sqrt{\frac{l_1}{340}}$; $\therefore\ \sqrt{\frac{l}{980}}=\sqrt{\frac{l_1}{340}}$;

or, $\frac{l_1}{l}=\frac{340}{980}$; $\therefore\ l_1=\frac{17}{49}l$.

Hence the length has to be shortened by $\frac{17}{49}$ of its original length.

(4) *A pendulum which beats seconds at a place where $g = 32{\cdot}2$ is taken to a place where $g = 32{\cdot}197$. How many seconds does it lose or gain in a day?*

Let t_1 be the original period, and t_2 the new period of the pendulum. In this case t_1 is equal to 2 secs., but this fact is not required.

We have, $\quad t_1 = 2\pi\sqrt{\dfrac{l}{32{\cdot}2}}; \qquad t_2 = 2\pi\sqrt{\dfrac{l}{32{\cdot}197}}.$

$$\text{Hence} \qquad \frac{t_1}{t_2} = \sqrt{\frac{32{\cdot}197}{32{\cdot}2}} = \sqrt{\frac{32{\cdot}2 - 0{\cdot}003}{32{\cdot}2}} = \sqrt{1 - \frac{0{\cdot}003}{32{\cdot}2}}.$$

Because period $\propto \dfrac{1}{\sqrt{g}}$, we have $t_2 > t_1$, and so the pendulum will lose.

Let n = no. of secs. lost per day. The number of secs. in a day is $24 \times 60 \times 60$, or 86400. $\therefore\ (86400 - n)t_2 = 86400 \times t_1$;

$$or \quad (86400 - n) = 86400 \times \frac{t_1}{t_2} = 86400 \times \sqrt{1 - \frac{0{\cdot}003}{32{\cdot}2}} = 86400\left(1 - \frac{0{\cdot}003}{32{\cdot}2}\right)^{\frac{1}{2}}$$

$$= 86400\left(1 - \tfrac{1}{2} \times \frac{0{\cdot}003}{32{\cdot}2}\right)\text{approx.} = 86400 - 4\ ; \quad \therefore\ n = 4 \text{ sec.}$$

Hence the pendulum loses 4 secs. per day.

(5) *A pendulum which beats seconds at the Equator gains five minutes per day at the Poles. Compare the values of g at the two places*

Let g_1 and t_1 denote the value of g and period respectively at the Equator, and g_2 and t_2 those at the Poles.

Because the pendulum beats seconds at the Equator, $t_1 = 2$ seconds,

We have, $\quad t_1^2 = 4\pi^2\dfrac{l}{g_1}; \quad or, \quad 4 = 4\pi^2\dfrac{l}{g_1}; \quad or, \quad g_1 = \pi^2 l$ ....(1)

Now, at the Poles, the pendulum gains 5 minutes per day, that is (5×60) seconds in $(24 \times 60 \times 60)$ secs. $\therefore$ It gains $\dfrac{5 \times 60}{24 \times 60 \times 60}$ sec. per sec.

i.e. it gains $\dfrac{1}{288}$ sec. in one vibration; *or,* $\dfrac{2}{288}$ sec. in the complete oscillation.

Because it gains $\dfrac{2}{288}$ sec. in one oscillation, its period $t_2 = \left(2 - \dfrac{2}{288}\right) = \dfrac{574}{288}$ secs.

$$\therefore \quad \left(\frac{574}{288}\right)^2 = 4\pi^2\frac{l}{g_2}; \qquad or, \quad g_2 = 4\pi^2 l \times \frac{288^2}{574^2} \text{ ....(2)}$$

$$\text{From (1) and (2)} \quad \frac{g_1}{g_2} = \frac{\pi^2 l}{4\pi^2 l \times \dfrac{288^2}{574^2}} = \frac{\pi^2 l}{4\pi^2 l \times \dfrac{288^2}{4 \times 287^2}} = \frac{287^2}{288^2} = \frac{143}{144} \text{ approx.}$$

second's

(6) A pendulum of length l loses 5 secs. in a day. By how much must it be shortened to keep correct time? *(C. U. 1932)*

There are 86400 seconds in a day. A Second's pendulum, which loses 5 secs. a day, beats (86400 − 5) or 86395 times in one day, *i.e.* in 86400 seconds.

∴ Time of one vibration $t=\frac{86400}{86395}$ (and not 1 sec.)

∴ We have, $\pi\sqrt{\frac{l}{g}}=\frac{86400}{86395}$. ∴ $\pi^2\frac{l}{g}=\left(\frac{86400}{86395}\right)^2$ ....(1)

In order to keep correct time, let the length of the pendulum be shortened by x cm. In this case, it becomes a true Second's pendulum and its time of one vibration becomes 1 second.

Then we have, $\pi\sqrt{\frac{l-x}{g}}=1$; ∴ $\pi^2\frac{l-x}{g}=1$..............(2)

From (1) and (2), $\pi^2\frac{x}{g}=\left(\frac{86400}{86395}\right)^2-1=\left(1+\frac{5}{86395}\right)^2-1.$

$=\left(1+\frac{2\times5}{86395}+\text{etc...}\right)-1$ from Binomial theorem.

$=\frac{10}{86395}$ (neglecting other terms)

∴ $x=\frac{g}{\pi^2}\times\frac{10}{86395}=\frac{981\times10}{9{\cdot}87\times86395}=0{\cdot}0115$ cm.

Questions

Arts. 50 & 51.

1. A body is weighed at the surface of the earth, at the sea-level and at the top of a mountain. State, in general terms, how the position will affect the weight and mass of a body. Give reasons for your answer as far as possible. (C. U. 1920 ; *cf.* Pat. 1932)

2. State where a body weighs the more—at the Poles or at the Equator. Give reasons. How do you prove this difference in weight experimentally?

(See also Art. 66) (C. U. 1931 ; '40)

3. What is meant by "acceleration of gravity"? How do you prove that it varies from place to place on the earth's surface? How does it vary? (C. U. 1933)

4. Describe a method of measuring 'g'. How does it vary from place to place? (All. 1930).

[See also simple pendulum, Art. 77].

Art. 54.

5 (a). Explain what is meant by Friction and define the terms 'coefficient of friction' and 'angle of friction'.

Show that once a body is just ready to slide down an inclined plane, the tangent of the angle of inclination of the plane is equal to the coefficient of friction. (Pat. 1927)

5 (b). State and explain the laws of friction.

6. State what do you mean by 'Limiting Friction', and the 'angle of friction'. (Dac. 1928)

7. What are levers ? Give examples of different classes of levers. (Pat. 1921)

Art. 55.

8. (a) Define 'machine' and 'mechanical advantage'. (Pat. 1947)

(b) Justify the statement : 'what is gained in power is lost in speed' by considering two important machines. (Pat. 1947)

Arts. 62, 62(a) & 62(b).

9. Describe with a sketch the balance you have used in your laboratory. What are the requisites of a good balance ? (C. U. 1922, '41 ; Pat. 1928)

10. What are the requisites of a good balance ? You are given an inaccurate balance ; explain how it can be used to obtain accurate results.

The only fault in a balance being the inequality in weights of the scale pans, what is the real weight of a body, which balances 10 lbs. when placed in one scale pan and 12 lbs. when placed in the other ? (Dac. 1933)

[*Ans.* 11 lbs.]

11. What are the requisites of a good balance ? A balance with unequal arms is used for weighing. The apparent weights of the same body when placed in the two pans are 158·0 and 158·25 gms. respectively. Find the ratio of the balance arms. (Dac. 1934 ; *cf.* Pat. 1928 ; '44 ; *cf.* All. 1946)

Arts. 62(b) & 64.

12. (a) How would you determine whether the arms of a balance are of equal length, and how would you eliminate error due to such an irregularity ? (Pat. 1928)

(b) State and discuss the requisites of a good balance. (All. 1946)

13. Explain with a neat sketch the principle and construction of a physical balance. What is the method of double weighing adopted in the case of an inaccurate balance ? (C. U. 1930 ; All. 1946)

Art. 66.

14. Describe and explain the action of a spring balance.

"In a common balance we compare masses of two bodies while from a spring balance we can get the true weight of a body". Explain. (C. U. 1927, '40, '47 ; Dac. 1929)

Art. 67.

15. Distinguish between mass and weight. How are the mass and weight of a body affected by variations of latitude ? Is weight an essential

property of matter ? Explain why a very delicate spring balance would show slight difference in the weight of a body at different places on the earth, though a common balance would give no indications of any difference.

(Pat. 1920, '32 ; *cf.* C. U. 1920)

16. Define weight and discuss as fully as you can the factors on which it depends. Describe experiments to illustrate your answer. (Pat. 1931)

17. Describe experiments by which it can be shown that the mass of a body is proportional to its weight and explain carefully the reasoning by which this conclusion is drawn from the results of the experiments.

What is meant by the statement that weight is not the essential property of matter ? (Pat. 1932)

Art. 68.

18. A man weighing 10 stone is sitting in a lift which is moving vertically with an acceleration of 8 ft. per sec. Prove that the pressure on the base of the lift is greater when it is ascending than when it is descending and compare the pressures. (Pat. 1931)

$$\left[\textit{Ans.}: \quad \frac{R}{R^1} = \frac{25}{15} = \frac{5}{3} \right].$$

Art. 69.

19. State the laws of falling bodies, and illustrate them by suitable examples. Describe and explain the celebrated guinea and feather experiment.

(C. U. 1926, '37, '39, '41, '46)

19(a). Show that for a falling body the distance it falls down during a given number of secs. is equal to the distance travelled during the first second multiplied by the sq. of the number of secs. (C. U. '46)

20. A body of mass 50 gms. is allowed to fall freely under the action of gravity. What is the force acting upon it ? Calculate the momentum and the kinetic energy it possesses after 5 seconds. ($g=980$ cm. per sec^2.)

(C. U. 1937)

[*Ans.* : 49×10^3 dynes ; 245×10^3 F. P. S. units ; 60025×10^4 ergs.]

Arts. 73 & 74.

21. (a) What is the simple pendulum ?

(b) How is the period of swing of a pendulum related to the wt. of the bob, its length, and the amplitude of the swing ? Hence state the laws of oscillation of a simple pendulum and state how would you verify them experimentally.

What is meant by effective length ?

(C. U. 1913, '15, '17, '19, '21, '24, '32, '36, '40, '47 ; Pat. '46)

(c) Find the length of a Second's pendulum at a place where $g=981$. What is the exact meaning of the statement '$g=981$' ?

[*Ans.* : 99·39 cms. nearly]. (C. U. 1912, '17, '19, '36 ; *cf* Dac. 1931)

(d) Explain why a pendulum should oscillate if the bob is drawn aside and let go. (Pat. 1946)

Art. 76.

22. How will you proceed to determine the 'g' of a place with a pendulum? Give the practical directions necessary and state reasons.

What is the effect of the height above, or the depth below, the surface of the earth, on the periodic time of a pendulum?

(For the second part see Art. 51. As g decreases the periodic time of a pendulum increases and hence a clock will go slower).

Art. 77.

23. A pendulum which beats seconds at a certain place when 'g' is 981 cm./sec^2 is taken elsewhere when 'g' is 978·3 cm./sec^2. Calculate the number of seconds it loses or gains in a day? (Pat. 1939)

[*Ans.* : 1 minute 58·89 second.] (See ex. (4) p. 83).

24. Will a pendulum clock gain or lose when taken to the top of a mountain? (C. U. 1917, '19)

25. When a ball suspended by a string is made into a 'Second's pendulum' does the actual length of its string equal the length of the equivalent simple pendulum? If not why? (C. U. 1912)

[**Hints.**—As the ball has a certain dimension, the actual length of the string will not be equal to the length of the equivalent simple pendulum. The distance between the point of suspension and the centre of gravity of the ball will be the length of the equivalent simple pendulum].

26. Explain how you would deduce the value of g with a simple pendulum.

A clock which keeps correct time when its pendulum beats seconds was found to be losing 4 minutes a day. On altering the length of the pendulum, it gained $2\frac{1}{2}$ minutes a day. By how much was the length altered, if the length of the Second's pendulum is 99·177 cm.?

[*Ans.* : 8·97 mm.]

CHAPTER VI

Work, Energy, Power

79. Work.—Work is said to be done *by a force* when the point of application of it moves in the *direction in which the force acts*; and if the point of application moves in a *direction opposite to that of the force*, work is said to be done *against the force.*

When we raise a body from the ground we do work *against* its weight, which is a force acting downwards. If the body falls to the ground, it moves in the direction of the force, so its weight does work.

Work = force × distance through which the point of application moves in the direction of the force.

It should be noted that *no work* is done by or against a force at *right angles* to its own direction.

80. Units of Works.—*Unit work is done when unit force moves its point of application, in its own direction, through unit distance.* As the unit of force is measured in the two systems, the *absolute* and *gravitational*, so unit work may also be measured in the above two systems.

(*a*) *The absolute unit of work* in the C. G. S. system is one **Erg**: *it is the work done when a force of one dyne acts through a distance of one centimetre.*

The absolute unit of work in the F. P. S. system is the **Foot-poundal**: *it is the work done when a force of one poundal acts through a distance of one foot.*

(*b*) *The gravitational unit of work* in the C. G. S. system is the **Gram-centimetre**: *it is the work done in lifting a mass of one gram through a vertical distance of one centimetre.*

(For practical purposes the unit chosen by the engineers is the *Kilogram-metre*).

The gravitational unit of work in the F. P. S. system is the **Foot-pound** (ft.-lb.): *it is the work done in raising a mass of one pound through a vertical distance of one foot.*

Since the weight of a gram is nearly 981 dynes, 1 gram-centimetre = 981 ergs.

1 erg = 1 dyne-cm.; 1 foot-poundal = 421,390 ergs.

Note. The erg being very small, **three additional units of work** (or energy) are used by electrical engineers for practical purposes, *viz.*—

(i) **The Joule** $= 10^7$ ergs.

(ii) The Watt-hour $= 3,600$ joules $= (3,600 \times 10^7)$ ergs.

i.e. one joule per second for one hour.

(iii) The Kilowatt-hour $= 3,600,000$ joules $= (1000 \times 3600 \times 10^7)$ ergs. *i.e.* 1000 joules per second for one hour $= 36 \times 10^{12}$ ergs.

This is the legal supply unit fixed by the Board of Trade and is called the **Board of Trade (B. O. T.) unit.**

81. Conversion of Foot-Poundals into Ergs.—

$$1 \text{ poundal} = \frac{1}{32\cdot2} = \text{wt. of 1 lb.} = \left(\frac{1}{32\cdot2} \times 453\cdot6\right) \text{grams}$$

$$= \left(\frac{1}{32\cdot2} \times 453\cdot6 \times 981\right) \text{dynes ; and 1 foot} = 30\cdot48 \text{ cms.}$$

$$\text{Hence,} \quad 1 \text{ foot-poundal} = \frac{30\cdot48 \times 453\cdot6 \times 981}{32\cdot2} \text{ergs} = (4\cdot212 \times 10^5) \text{ ergs.}$$

82. Relation between the two units of work.—Since the gravitational unit of force is g times the absolute unit of force,

gravitational unit of work $= g \times$ absolute unit of work.

Since the weight of a pound is 32·2 poundals,

1 foot-pound $= 32\cdot2$ foot-poundals $= 32\cdot2 \times 30\cdot48 \times 13,125$

$= 1\cdot356 \times 10^7$ ergs $=$ **1·356 joules,**

(since, 1 foot $= 30\cdot48$ cms., and 1 poundal $= 13,825$ dynes).

83. Power.—The power of an agent, say a dynamo or an engine, is the rate at which it is doing work.

When we consider the time taken by an agent to perform any work, we consider what is called the **power** of the agent. The power used in any operation is the ratio of $\dfrac{\text{Total work done}}{\text{Time occupied}}$.

84. Units of Power.—(a) The **C. G. S. absolute unit** of power is **one erg per second.**

This being too small for practical purposes, **two additional units** are employed in electrical engineering, *viz.*—

(i) The Watt $= 1$ joule per sec. $= 10^7$ ergs per sec.

(ii) The Kilowatt $= 1,000$ watts.

(b) The **F. P. S. absolute unit** of power is **one foot-poundal per second.**

The **Gravitational unit** of power is **one foot-pound per second.**

Horse-Power.—It is the British practical unit of power and is used in Mechanical Engineering very largely.

One Horse-power (H. P.) = 33,000 ft.-lbs./min. = 550 ft.-lbs./sec.

In order to find out the working capacity of a horse, James Watt, the inventor of the Steam Engine carried out an experiment in which a weight of 150 lbs. was lifted upwards from a coal pit by a horse through a distance of 220 ft. in one minute. Thus the work done was $(150 \times 220) = 33{,}000$ ft.-lbs. in 1 minute = 550 ft.-lbs. in 1 second.

So James Watt adopted this as a unit of power, which he termed one **Horse-Power** (H. P.)

85. Conversion of Horse-Power into Watts :—

Since 1 foot-pound $= (1{\cdot}356 \times 10^7)$ ergs, 550 ft.-lbs. $= (746 \times 10^7)$ ergs.

Hence **1 H. P.** = 550 ft.-lbs. per sec. $= 746 \times 10^7$ ergs per sec.

$= \mathbf{746}$ **watts** ; ($\because$ 1 watt $= 10^7$ ergs per sec.)

$$\text{and 1 Kilowatt} = \frac{1000}{746} = 1{\cdot}34 \text{ H. P.}$$

86. Conversion of Kilowatt-hour into Foot-Pounds —

Since 1 Kilowatt = 1·34 H. P. $= (1{\cdot}34 \times 550)$ ft. lbs. per sec.

and Work = Power × Time in seconds,

we have, **1 Kilowatt-hour** $= (1{\cdot}34 \times 550) \times (60 \times 60)$ ft.-lbs.

$= \mathbf{2{,}653{,}200}$ **foot-pounds**

[**Remember.**—The amount of work done by an average horse is only $\frac{3}{4}$ H. P. The average amount of work done by an active man is $\frac{1}{7}$ H.P. The power of motor car engines vary from 6 to 30 H. P., that of a jeep from 20 to 80 H. P. ; those of gas-engines from $\frac{1}{2}$ to 270, while the power of a large battle cruiser may reach upto about 120,000 H. P.]

87. Distinction between Work and Power.—Remember that power involves a time-unit and is measured by the ratio of the work done to the time occupied in doing the work : *e.g.*

1 H. P. = 550 ft.-lbs. per sec. : 1 watt $= 10^7$ ergs per sec., etc. Thus

Work = Power × Time.

Hence 'watt-hour' or 'kilowatt-hour' which are products of 'power' and 'time intervals' are units of work.

Examples.—(*1*) *A man whose weight is 10 stone runs up a flight of stairs, carrying a load of 10 lbs. to a height of 20 ft. in 10 seconds. Find the work done in H. P. during this interval.*

10 stone $= 14 \times 10 = 140$ lbs.

Total work done in 10 secs. $= (140+10) \times 20 = 3000$ ft. lbs.

$\therefore$ The work done in 1 sec. $= \frac{3000}{10} = 300$ ft.-lb. So work done $= \frac{300}{550} = 0{\cdot}545$ H. P.

(2) *A man weighing 140 lbs. takes his seat in a lift which weighs 2 tons. He is taken to the 3rd floor, which is at a height of 75 ft from the ground floor, in 2 minutes. Calculate the work done and the power required in this process.*
1 ton $= 2240$ lbs. *(Pat. 1929)*

The total weight of the man and the lift $= 140 + 2240 \times 2 = 4620$ lbs.

The work done in raising 4620 lbs. through 75 ft.

$=$ force $\times$ distance $= 4620 \times 75 = 346{,}500$ ft.-lbs.

The unit of power in the F.P.S. system is one horse-power, which is 550 ft.-lbs. per second. $\therefore$ Power $=$ rate of doing work,

$$= \frac{346500}{2 \times 60} \text{ ft.-lbs. per second} = \frac{346500}{2 \times 60 \times 550} \text{ H.P.} = 5{\cdot}25 \text{ H. P.}$$

88. Energy.—A body is said to possess energy when by reason of its position or condition it can do work ; hence the capacity of a body for doing work is known as its *energy*.

Wind has got energy since work is done by it when it drives a boat.

The falling water at Niagra does work in driving the dynamos which generate electricity. Hence that elevated water has got energy.

The work a body can do is a measure of its energy. So the units in which energy is measured are identical with those of work. Therefore, *erg*, *foot-pound*, *joule*, etc., are also **units of energy.**

89. Forms of Energy.—Principally there are *two* forms of energy, *potential* and *kinetic*.

Potential Energy.—A body may have energy due to its position, as when a body is raised up above the ground. This form of energy which is obtained in *virtue of its position* is known as *potential energy*.

A lifted weight can do work in falling down, under the force of gravity, to its original position. Hence it possesses potential energy in virtue of its higher position. The coiled spring of a clock has potential energy, for it is capable of doing work by way of unwinding. A piece of coal possesses potential energy because due to its combination with oxygen, heat, light, and mechanical work are obtained therefrom. A body charged with electricity possesses potential energy, for a charged body can do work in attracting other bodies to it, or sparks may be obtained from it, in which case the potential energy of the body is converted partly into *heat*, partly into *light*, and a part of it gives rise to *sound* ; which are also known as different forms of energy.

Potential energy of a raised body.—

If m = mass of a body ; g = acceleration due to gravity ; h = vertical height through which the body is raised ;

The potential energy = work done in raising the body = mgh.
If m be taken in pounds and h in feet, then the potential energy,

P. E. = mgh **ft.-poundals**, (*where* $g = 32{\cdot}2$) = mh **ft.-pounds.**

If m be taken in grams and h in centimetres,

P. E. = mgh **ergs**, (*where* $g = 981$) = mh **gm.-cm.**

Kinetic Energy.—A body may have energy due to its motion, which is known as *kinetic energy*. The bullet fired from a rifle, or the rotating fly wheel of an engine has got kinetic energy.

If m = mass of a body, v = velocity with which the body is moving at any moment after the lapse of t secs.,

Kinetic energy of the body = force × distance moved through = (mass × acceleration) × (average vel. × time)

$$= \left(m \times \frac{v}{t}\right) \times \left(\frac{o+v}{2} \times t\right) = \tfrac{1}{2}mv^2.$$

If m be taken in pounds and v in feet per second, the kinetic energy, **K. E.** = $\frac{1}{2}mv^2$ (lb. × ft.^{2}/sec^2 = ft. × lb. × ft./sec.^{2}) = **ft.-poundals**

= $\frac{1}{2}mv^2/g$ ft.-pounds, (*where* $g = 32{\cdot}2$).

If m be taken in grams and v in cm. per second,

K. E. = $\frac{1}{2}mv^2$ **ergs** (gm. × cm.^{2}/sec.^{2} = cm. × gm. × cm./sec.^{2}

= cm. × dynes) = **ergs** = $\frac{1}{2}mv^2/g$ **gm.-cm.**, (*where* $g = 981$).

89(a). Potential Energy and State of Equilibrium.—*The state of stable equilibrium of a* body corresponds to a **minimum** *of potential energy ;* because the centre of gravity of a body, when in stable equilibrium, occupies the lowest possible position inside the body, and any displacement tends to raise the position of the centre of gravity and thus increases the potential energy of the body. When, however, *the potential energy of the body is* **maximum** any displacement will give rise to a couple tending to move the body further, and thus, in this position, *the equilibrium of the body is* **unstable.** Again, when the body is *in the state of* **neutral** *equilibrium*, its *potential energy will remain* **constant** for any small displacement.

90. Transformation and Conservation of Energy.—If a body is at some height above the ground, it has got some gravitational potential energy. If it is now allowed to fall freely through a distance, it loses an amount of potential energy equivalent to the work done by

the weight of the body, but gains an equal amount of kinetic energy. Just before the body strikes the ground, it has only kinetic energy equal to the amount of potential energy in the beginning, that is, the potential energy is wholly transformed into kinetic energy, which again gives rise to *heat* and *sound* energies when it strikes the ground. Again when a clock is wound up, its spring possesses potential energy exactly equal to the amount of work done in winding it up. This potential energy, which is stored up in the spring, is converted into kinetic energy continually as the spring becomes unwound in keeping the clock going. In the same way the potential energy of the compressed spring in a spring-gun is converted into the kinetic energy of the bullet. These are several cases of **transformation of energy**, according to which a body may lose energy in one form, but will gain an *equal amount* in another form. Consider also the various transformations of energy in the case of an ordinary steam engine connected to a dynamo for the generation of electricity. When the coal burns we get *heat energy*. The heat does work in changing water to steam, which then expands. The expanded steam exerts pressure and causes the piston to move, and thus runs the engine. Thus the heat energy is transformed into *mechanical energy*, and when the engine drives a dynamo, which generates electricity, the mechanical energy is converted into *electrical energy*. This energy can be transmitted by wires and made to do useful works, such as driving tram cars, where electrical energy is reconverted into *mechanical energy*; lighting lamps in houses, where electrical energy is reconverted into *light energy*; and in this way various other transformations may also take place.

Conservation of Energy.—*A pendulum bob* in motion affords *an example* of the transformation as well as conservation of energy. When the bob is at either of its extreme positions, it has got only gravitational potential energy, and, as it falls, it loses potential energy and gains an equal amount of kinetic energy until it reaches the lowest position when all the energy is transformed into kinetic. So, at any point of the swing, the total amount of energy (kinetic + potential) remains constant. The opposite process is repeated, until it reaches the other extreme position, when all the kinetic energy is transformed into potential again.

Total energy of a falling body is constant.—The potential energy of a body of mass m at a height h above the ground $= mgh$.

When it falls through a distance x, its potential energy at the time $= mg(h-x)$.

Its kinetic energy at that instant $= \frac{1}{2}mv^2$ (where v is the velocity acquired during this interval) $= \frac{1}{2}m \times 2gx$ ($\because$ $v^2 = 2gx$) $= mgx$.

∴ At that instant, potential energy + kinetic energy

$= mg(h - x) + mgx = mgh =$ potential energy in the beginning.

Hence, neglecting the effects of air resistance, it is seen that the total amount of energy (kinetic + potential) of the body remains constant as it falls. When the body strikes the ground, it is brought to rest and loses its kinetic energy. Then the potential energy has also disappeared. The energy, however, has not been destroyed. It has been converted mainly into heat, the body and the ground being warmer as the result of the impact.

Principle of Conservation of Energy.—From the illustrations given above it is clear that the total energy of a body always remains constant. It can only change its form. This is known as the **principle of conservation of energy**, which states—

The total amount of energy in the universe remains always constant. It can never be created nor destroyed, but can be transformed from one form to another, or to a number of other forms.

90(a). Perpetual Motion.—The above law of nature indicates the impossibity of the existence of a "perpetual motion" machine—*i.e.* a machine which, when once set in motion, will continue its motion perpetually without the supply of an equivalent amount of energy from outside. Even when no useful work is done by the machine the energy supplied in the beginning will be gradually used up in overcoming frictional and other resistances, and the machine will ultimately come to a stop.

91. The velocity of the bob of a Pendulum at its lowest point.—When the bob of the pendulum, of length l cm., is allowed to go from its extreme position C, it moves in an arc of a circle CBD, B being the lowest position (Fig. 46). From C draw a perpendicular CK on AB. At C the bob of the pendulum has potential energy $m.g.BK$, which represents the work done in raising the bob from B to C. When the bob is released from the position C, it gradually loses the potential energy and gains kinetic energy. At the lowest point B, it loses all its potential energy $m.g.BK$, and the kinetic energy $\frac{1}{2}mv^2$, which it gains, is equal to this.

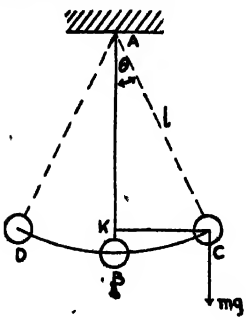


Fig. 46

∴ $m.g.BK = \frac{1}{2}mv^2$, where v is the velocity of the bob at B.

$$or \quad g(AB - AK) = \tfrac{1}{2}v^2 \; ; \quad or \quad g(l - l\cos\theta) = \tfrac{1}{2}v^2 \; ;$$

$$or \quad v^2 = 2gl(1 - \cos\theta) \; ; \quad \therefore \quad v = \sqrt{2gl(1 - \cos\theta)};$$

$$or \quad v = \sqrt{2gl \times 2\sin^2\frac{\theta}{2}} = 2\sqrt{gl} \times \sin\frac{\theta}{2}.$$

Example.—*The heavy bob of a simple pendulum is drawn aside so that the string makes an angle of 60° with the horizontal and then let go. Find the velocity with which the bob passes through its position of rest.* **(Pat. 1940)**

(Draw the diagram and proceed as explained above).

Here $\theta = (90 - 60°) = 30°$; $\therefore \; v = \sqrt{2gl\left(1 - \frac{\sqrt{3}}{2}\right)}$ *or* $v = \sqrt{0{\cdot}268 \times gl}$.

92 Other Forms of Energy.—From the foregoing statements it is evident that though the energy which a body possesses may be put into either of the two main forms, kinetic and potential, there are other forms also in which energy may appear ; such as heat, light, sound, electric current, magnetisation, and chemical action.

The Sun is the ultimate source of all energy.—The sun is generally considered to be the ultimate source of different forms of energy. We get considerable amount of energy from solar radiation in the form of heat, light, etc. For example, the energy of the steam engine is derived from coal. Coal again is nothing but wood subjected to great pressure of the earth for thousands of years. The energy in the wood is due to the sun's action on trees and plants. When the coal burns, the stored-up potential chemical energy due to solar radiation is given back as heat and light.

93. Examples of various Cases of Transformation of Energy.—

1. Muscular energy to potential energy :—the potential energy of a raised weight.
2. Muscular to kinetic :—the kinetic energy of a thrown stone.
3. Kinetic to potential :—the bob of a pendulum at the extreme positions of displacement.
4. Potential to kinetic :—a body dropped from a height, when fallen through a certain distance.
5. Kinetic to heat energy :—a moving wheel of an engine when stopped by applying brakes.
6. Gravitational to electrical energy :—water turbines.
7. Heat to light energy :—a red hot or white hot metal ball.

8. Light to chemical energy :—the chemical action of light on the photographic plate.
9. Heat to chemical and sound energy :—when a flame is applied to a mixture of oxygen and hydrogen, the two gases combine and a report is heard.
10. Heat to electrical energy :—electric current is produced by heating one of the junctions of two dissimilar metals (say iron and copper) (See Chapter VI, Part VII).
11. Heat to kinetic :—steam engine.
12. Chemical to heat :—burning coal.
13. Chemical to electrical energy :—voltaic cells, where the chemical energy reappears in the form of electric current.
14. Electrical to chemical :—electrolysis.
15. Electrical to heat and light :—electric heater and glow lamp.
16. Electrical to mechanical :—tram cars ; electric fans ; etc.
17. Mechanical to electrical :—dynamo.
18. Electrical to sound energy :—electric bells : sound is heard when discharge passes in frictional machines or induction coils.

93(a). Different Examples of Work done.—Work is measured by the product of the force and the distance through which the point of application of the force moves.

(*a*) *Work done in raising a load vertically upwards.*

If w represents the work done, $w = mgh$, where m is the mass of the load and h the vertical height through which the load is raised.

(*b*) *Work done in taking a load up along an inclined plane.*—In this case, $w = mg \sin \theta . l$, where l is the length of the inclined plane (see Art. 54*b*) $= mg \times l \sin \theta = mgh$, where h is the height of the inclined plane. Thus the work done in taking the load up the inclined plane is the same as that required to raise the load m vertically through a height h. Hence the work done in raising a body to a height h against gravity is independent of the path along which the body is taken and depends only on the vertical height.

(*c*) *Work required to generate a velocity v in a body originally at rest.*

$W = P \times S$, where P is a force which generates an acceleration f in a body of mass m ; and S is the distance traversed by the body in time t.

Here $P = mf$; and $S = \frac{1}{2}ft^2$. $\therefore W = P \times S$,
$= mf \times \frac{1}{2}ft^2 = \frac{1}{2}m(ft^2) = \frac{1}{2}mv^2$,
where v is the velocity acquired by the body after time t starting from rest.

94. Summary of Results.—

Quantity	Symbol	Quantity	Symbol
Displacement or distance	s	Relation between distance & speed	$v^2 = u^2 + 2fs$
Time	t	Mass	m
Velocity	$v = \frac{s}{t}$	Force	$P = mf$
Acceleration	$f = \frac{v_2 - v_1}{t}$	Momentum	$M = mv$
Distance	$s = vt$	Kinetic energy	K.E. $= \frac{1}{2}mv^2$
Relation between speed and time	$v = u + ft$	Potential energy	P.E. $= mgh$
Relation between distance & time	$s = ut + \frac{1}{2}ft^2$	Work	Force × distance moved.

Units of Force, Work, and Power.

C. G. S. SYSTEM

Quantity	Theoretical unit	Practical unit
Force	The *Dyne*	The *Gram Weight* = 981 dynes (gravitational unit)
Work	The *Erg*	(i) The *Joule* = 10^7 ergs (ii) The *Kilowatt-hour* = 36×10^{12} ergs (iii) The Gram-centimetre = 981 ergs
Power	One erg per sec. (No special name)	The *Watt* = 1 joule per sec. = 10^7 ergs per sec.

∴ F. P. S. SYSTEM

Quantity	Theoretical unit	Practical unit
Force	The *Poundal*	The *Pound Weight* = 32 poundals (gravitational unit)
Work	The *Foot-poundal*	The *Foot-pound* = 32 foot-poundals (gravitational unit)
Power	One foot-poundal per second (No special name)	The *Horse-Power* (H.P.) = 550 ft.-pounds per sec.

Examples

(1) A rifle bullet of mass 14 gms travelling at the rate of 36,000 cms. per sec. is just able to pierce a block of wood 21 cms. thick. Find the average force of the bullet while penetrating the wood.

Let F be the force in dynes ; then $F \times s = \frac{1}{2}mv^2$,

$$\text{or } F = \frac{mv^2}{2s} = \frac{14 \times (36000)^2}{2 \times 21} = 432 \times 10^6 \text{ dynes.}$$

(2) A train of mass 100 tons is travelling at 30 miles per hour. Calculate the force necessary to bring it to rest (a) in a distance of 120 ft. (b) in 10 seconds.

(*a*) Let F be the force applied which produces a negative acceleration f to bring the train to rest, then

$F = -mf$, (where m is the mass of the train) ; or $f = -F/m$.

We have, $v^2 = u^2 + 2fs$; $\therefore \quad 0 = u^2 - 2\frac{Fs}{m}$; or $Fs = \frac{1}{2}mu^2$.

Here $m = 100 \times 2240 = 224{,}000$ lbs. ; $u = 30$ miles per hour $= 44$ ft. per sec.

$$\therefore \quad F = \frac{mu^2}{2S} = \frac{224000 \times 44^2}{2 \times 120} = 1{,}806{,}933{\cdot}3 \text{ poundals}$$

$$= 56{,}466{\cdot}6 \text{ lbs. wt. (taking } g = 32)$$

(*b*) We have $v = u + ft$; or $0 = u - \frac{F}{m}t$; or $Ft = mu$;

$$\therefore \quad F = \frac{224{,}000 \times 44}{10} = 985{,}600 \text{ poundals} = \frac{985600}{32} = 30{,}800 \text{ lbs.wt.}, (\because g = 32).$$

(3) Find the energy stored in a train weighing 250 tons and travelling at the rate of 60 miles per hour. How much energy must be added to the train to increase its speed to 65 miles per hour. (*C. U. 1925*)

Mass $= 250$ tons $= 250 \times 20 \times 4 \times 28$ lbs.

Velocity $= 60$ miles per hour $= \dfrac{60 \times 1760 \times 3}{60 \times 60}$ ft. per sec. $= 88$ ft. per sec.

$\therefore$ The kinetic energy of the train $= \frac{1}{2} \times (250 \times 20 \times 4 \times 28) \times 88^2$ foot-poundals.
$= 2{,}168{,}320{,}000$ ft. poundals.

Again, 65 miles per hour $= \dfrac{65 \times 1760 \times 3}{60 \times 60} = \dfrac{286}{3}$ ft. per sec.

$\therefore$ The K. E. of the train, when the speed is 65 miles per hour,

$$= \tfrac{1}{2} \times (250 \times 20 \times 4 \times 28) \times \left(\frac{286}{3}\right)^2 = 2{,}544{,}765{,}000 \text{ ft.-poundals.}$$

$\therefore$ The energy to be added $= 2{,}544{,}765{,}000 - 2{,}168{,}320{,}000$
$= 376{,}445 \times 10^3$ ft.-poundals.

(4) *If clouds are 1 mile above the earth and rainfall sufficient to cover 1 square mile at sea level, ½ inch deep, how much work was done in raising the water to the clouds.* *(C. U. 1920)*

If w lbs. be the mass of rain water, and h ft. the height of the clouds above the surface of the earth, the work done in raising w lbs. of water through h ft. $= w \times h$ foot-pounds. Here $h = 1760 \times 3 = 5280$ ft.

The volume of rain water $= 1$ square mile $\times \frac{1}{2}$ in. $= (5280)^2 \times \dfrac{1}{2 \times 12}$ cu. ft.

The mass of 1 cubic foot of water $= 62{\cdot}5$ lbs.

$\therefore$ Mass of rain water $= (5280)^2 \times 1/24 \times 62{\cdot}5$ ft.-pounds.

$\therefore$ The work done $= (5280)^2 \times \dfrac{1}{24} \times \dfrac{125}{2} \times (5280) = (5280)^3 \times \dfrac{1}{24} \times \dfrac{125}{2}$

$= 383{,}328 \times 10^6$ foot-pounds.

Questions

Art. 79.

1. Define the terms work, force and pressure. Show that if a piston is moved along a cylinder against a constant pressure, the work done in a stroke is equal to the product of the pressure into the volume swept out by the piston. Explain clearly the units in which the work will be given by this calculation. (Pat. 1921 ; C. U. 1941)

[**Hints.**—Pressure = force on unit area.

Work done = force × distance = (pressure × area) × distance through which the piston moves = pressure × volume swept out. The work is expressed in ergs, if pressure is measured in dynes per sq. cm. and volume in c.c.]

2. What is the work done when a weight of 500 kilograms falls through a height of 50 metres and is then stopped ? Assume the normal value of gravity. (Dec. 1938)

[*Ans.* : 22725×10^9 ergs.]

[24525×10^8 ergs

2. (a) A man supports a mass of 10 lbs. in his hand. What is the work done by him when (a) he is stationery, (b) he moves over a distance of 20 yds. on a level horizontal road, and (c) he moves over a distance of 20 yds. up an incline of 1 in 20 ? [Neglect friction]

Arts. 86 & 87.

3. Define power, and state the C. G. S. and the F. P. S. practical units of power. (Pat. 1947)

Water is pumped up from a well through a height of 30 feet by means of a 5 horse-power motor. If the efficiency of the pump is 85%, find in gallons the quantity of water pumped up per minute. (1 gallon of water weighs 10 lbs.) (Mysore)

[*Ans.* : 467·5]

4. Define the terms work, power and horse-power, and explain how their units are related. (Pat. 1947)

An engine is employed to pump 6000 gallons of water per minute from a well, through an average height of 21 feet. Find the horse-power of the engine, if 45% of the power is wasted. (M.U.)

Ans. : 69·42.]

Arts. 89 & 90.

5. Distinguish between potential and kinetic energy with illustrations. (C. U. 1918, '32, '36, ; Pat. 1925, '35)

A railway train is going up-hill with a constant velocity. What is the source from which the energy of the train is supplied ?

Describe the various transformations of energy that go on in this case.

(See also Art. 92). (C.U. 1918)

[**Hints.**—The energy of the train is derived primarily from the burning coal. This is utilised in running the train against friction and air resistance, and also in raising the train up-hill against the force of gravity and thus doing work. The energy of the coal is derived from the sun. So the sun is the ultimate source of supply of energy.]

6. A solid mass of 100 gms. is allowed to drop from a height of 10 metres. Calculate the amount of kinetic energy gained by the body, g being 980 cms. per sec.^{2} (Dac. 1930)

[*Ans.* : 98×10^6 ergs.]

7. A shot travelling at the rate of 200 metres per second is just able to pierce a plank 2 inches thick. What velocity is required to pierce a plank 6 inches thick ? (Pat. 1941)

[*Ans.* : $200\sqrt{3}$ metres. (See examples (1) and (2) p. 98)]

8. Distinguish between pound, poundal, and pound-weight.

Prove that in the case of a body falling freely under gravity, the sum of the potential and kinetic energies is constant. (Pat. 1925, '36 ; C.U. '32, '41).

9. A pendulum consisting of a ten-gram bob at the end of a string thirty centimetres long oscillates through a semi-circle ; find its velocity and kinetic energy when it passes its lowest point. Specify the units in which your answer is given. (Pat. 1935)

[**Hints.**—At the starting point the bob has got only potential energy $= mgh$. At the lowest point the energy is only kinetic $(=\frac{1}{2}mv^2)$, which is equal to $mgh = (10 \times 981 \times 30)$ ergs. Hence find v.]

Ans. : Vel. = 242·61 cm. per sec. ; K. E. = 294,300 ergs.]

10. A body falls under gravity and strikes the ground. Explain how the phenomenon supplies an illustration of the transformation of energy. Does it also illustrate the principle of conservation of energy ?

(C.U. 1917, '36 ; Pat. 1931)

Arts. 90 & 93.

11. Define 'Work and Energy' Give some examples of transformation of energy. (C. U. 1916, '19, '23 ; Pat. '32, '47 ; Dac. '33)

State also the principle of Conservation of energy.

(C.U. 1911, '12, '20, '23 ; Pat. '41, ; *cf.* Dac. '31, '32)

12. Explain the terms 'Conservation and Transformation of Energy'. Illustrate your answer by reference to an electric light circuit, an oil engine used to drive the dynamo. (Pat. 1932 ; C.U. '36)

[See Arts. 90, 92, 93.]

13. What is meant by 'Energy' ? Describe suitable experiments to illustrate the following, and point out what they ultimately demonstrate :—(a) conversion of mechanical energy into electrical energy ; (b) conversion of electrical energy into heat energy ; (c) conversion of heat energy into luminous energy. (C.U. 1920)

CHAPTER VII

Properties of Matter

95. Constitution of Matter.—Every matter is composed of a very large number of tiny particles called *molecules*, separated from each other by minute distances which differ in the different states of matter. A *molecule* of any body is the smallest particle of it which can exist by itself and retain the properties of that body. Again, a molecule is composed of far smaller particles called *atoms*. The atoms are incapable

of free existence but by combining with each other form molecules which exist freely. When chemical reaction takes place between two substances, what happens is that the atoms of one substance combine with the atoms of the other. Formerly the atoms were supposed to be indivisible and were regarded as the ultimate particles with which all matter is built up. Recent researches have, however, established the existence of particles far smaller than the atom, such as *electrons*, *protons*, *Neutrons*, etc. (see Chapter I, Part VI).

The distances or spaces between consecutive molecules of a body are known as *intermoleculer spaces*, which can decrease or increase producing a change in volume of the body. The intermolecular spaces are not vacuous, but filled with a subtle imponderable fluid, called **the ether.**

The molecules of a body are held together in their positions, firmly in case of a solid and less so in a liquid, by their mutual force of attraction known as *intermolecular force of attraction* while in the case of a gas, the intermolecular force is supposed to be negligibly small.

The molecules of a solid execute vibration about their mean positions of rest which cannot be easily altered while in case of liquids, in addition, the mean positions of rest can be more easily altered. In the case of a gas, the molecules are at random motion perpetually. The state of motion in all the three states increases with the increase of temperature.

96. The Three States of Matter.—Matter exists in the three States—*solid*, *liquid* and *gas*.

Distinction between Solids, Liquids and Gases.—The molecular structure of matter is common to all the three states. *The difference lies only in the closeness with which the molecules composing them are packed.*

In a **solid**, the molecules are closely packed, and the force of attraction betweeen the molecules, spoken of as *cohesion* (Art. 97), is greater, which gives it a *definite shape* and *size*

In a **liquid**, the molecules are not so close together as in a solid and cohesion is much smaller. So it has got *no definite shape.* A liquid takes the shape of the vessel in which it is contained.

In a **gas**, the molecules are far apart from each other and cohesion is practically absent, and so the molecules spread themselves out as far as possible. A gas, therefore, has got *no definite shape or size.* Moreover, the molecules of a gas, unlike those of solids and liquids, are in perpetual motion in the most haphazard manner and have no mean position of rest.

Liquids and gases are classed together as *fluids* since they are both able to flow.

97. Properties of Matter.—

Cohesion and Adhesion—*Cohesion* is the force of attraction between molecules of the *same* nature; and *adhesion* is the force of attraction that exists between molecules of *different* nature. Cohesive force keeps the molecules together in a substance and adhesion is the cause of sticking together of two substances; for instance, wetting glass by water and other liquids, gluing wood to wood, 'tinning' metals with solder, etc. Cohesion holds together the particles of a crayon, but adhesion holds the chalk to the blackboard.

Impenetrability.—It is the property in virtue of which two bodies cannot occupay the same space at the same time. If a metal ball is immersed in a liquid, the liquid moves away to make room for the body.

Extension—It is the property in virtue of which every body occupies some definite space.

Divisibility.—It is the property in virtue of which a body can be divided into extremely minute parts.

Compressibility.—It is the property in virtue of which a body can be compressed in volume by applying external pressure.

Elasticity.—

It is the property in virtue of which a body offers resistance to any change of volume or shape, and resumes its original form when the deforming force is removed.

Solids resume their original form after the removal of the force provided it does not exceed a certain limit, called the *limit of elasticity*. If the force exceeds that limit (or maximum value), the body will not return to its original size when deforming force is removed. The force in this case is said to have exceeded the limit of elasticity.

Stress and Strain.

Strain.—When a force or system of forces acting upon a body produces a relative displacement between its parts, a change in volume or shape or in both may take place. The body is then said to be under strain. The strain produced in a body is measured by the change expressed as a fraction of the original. It is obviously a pure number.

Stress.—When a body is strained, internal forces of reaction are

automatically set up within the body in the opposite direction, due to which the body tends to return to its original size or shape on withdrawal of the deforming forces. This restoring force is called the stress. It is numerically equal to the deforming force, according to Newton's law of reaction, as long as the strain produced is within the elastic limit.

The stress acting normally to any section of a body is called the **normal stress** to that section. The stress which acts parallel to any section of the body is called the **tangential stress** on that section. The stress is measured by the force per unit area of the section and when uniform, is obtained by dividing the total force by the total area over which it acts.

Perfectly rigid body.—A perfectly rigid body is defined to be such that no relative displacement between its parts takes place whatever force is externally applied to it. No body is known to be perfectly rigid, though glass, steel etc. are nearly so.

Perfectly elastic body.—If a body perfectly recovers its original volume or shape when the deforming force acting on it is withdrawn, it is said to be *perfectly elastic.* No such body is known for all values of stress. A body, however, behaves as perfectly elastic, when the deforming force does not exceed a certain limiting (maximum) value depending on the nature of body, called the elastic limit of the body.

Elastic limit.—A body behaves as a perfectly elastic body only as long as the deforming force acting on it does not exceed a certain maximum value depending on the natue of the body and the nature of the strain. This limiting value of the stress is called the *elastic limit* of the body for the type of strain produced.

Different Kinds of Strain.—

(i) **Longitudinal (*or* tensile) strain.**—When a body is acted on by a stretching or compressive force, the *increase or decrease in the length* is referred to as the longitudinal or tensile strain. The corresponding stress is called longitudinal or tensile stress. Thus if due to stretching or compression, l = change in length of a body of length L,

$$\text{longitudinal (or tensile) strain} = \frac{l}{L}.$$

It is a pure number having no unit for its measurement. *Only solids can have such strains.*

When a body is acted on by a stretching force, the extension in the direction of the applied force is accompanied by a lateral contrac-

tion in all directions at right angles to the applied force. It is found that this lateral strain is always proportional to the direct strain *i.e.*

$$\frac{\text{lateral strain}}{\text{longitudinal strain}} = \delta = \text{a constant, called the } \textbf{Poisson's Ratio},$$

whose value depends only on the nature of the material in question and not at all on the stress applied. Poisson's Ratio for a stretching force is the same as that for a compressive force in which case there is lateral expansion.

(2). **Volume (*or* Bulk) Strain.**—In such strains there is a change in volume only without any change in shape. This takes place when a body is subjected to uniform pressure acting normally at every point on the surface. The corresponding stress (force per unit area) is called the volume stress. If V be the original volume of the body and v, the change produced in the volume,

$$\text{Volume strain} = \frac{v}{V}.$$

It is a pure number and has no unit for its measurement. Volume strain, even for very large deforming forces, is small for solids and liquids while in case of gases even a very small force produces a very large strain.

(3) **Shear Strain** or **Shear.**—When the strain produced in a body is such that there is only change in shape or form of it but no change in volume, it is said to be a shearing strain or simply a *shear*. It is a special property of solids only because they only have a definite shape of their own.

Let $ABCD$ (fig. 47) represent the section of a rect. block whose face CD is rigidly fixed. Let a force P act uniformly and tangentially over the face AB (area α) so that this face is displaced relatively to CD and assumes the rhombic form A_1B_1CD. The material of the block suffers a change in form only without any change in volume. The strain produced is a case of *shear* and is measured by the angle ADA_1 ($=\theta=$ the angle BCB_1) which is called **the angle of shear**. Let AA_1 be x and $AD = b$, then,

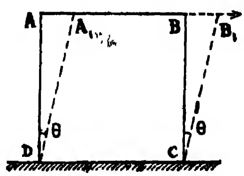


Fig. 47

$$\textit{Shearing strain} = \theta = \tan\theta \; (\because \; \theta \text{ is small}).$$

$$= \frac{x}{b}$$

$$= \frac{\text{relative displacement}}{\text{distance of separation}}.$$

= relative displacement for planes at unit distance apart

= *displacement gradient.*

The corresponding stress is called the shearing stress and is given by P/α.

98. **Hooke's law.**—This is the basic law of elasticity. It was established in 1678 by Robert Hooke, an Enlishman. It may be stated thus,

Within the elastic limit, stress is proportional to strain.

That is, within the elastic limit, $\frac{\text{stress}}{\text{strain}}$ = a constant = E, (say).

This constant is called the **Modulus** (*or* **Co-efficient) of elasticity** of the material of a body. As strain is a pure number, it is measured in the same unit as that of stress and is expressed in dynes/cm^2 or lbs. sq. inch.

The law holds good for all cases of strain, such as tensile, volume, twisting, bending, etc. But for different cases of strain, the modulus has a different value and is differently named. Thus,

$$\text{Young's modulus} = \frac{\text{tensile stress}}{\text{tensile strain}}.$$

$$\text{Bulk modulus} = \frac{\text{volume stress}}{\text{volume strain}}$$

$$\text{Rigidity modulus} = \frac{\text{shearing stress}}{\text{shearing strain}}.$$

98(a). **Verification of Hooke's law**—Hooke's Law can be easily verified by a steel spring-balance. By placing different weights on the pan and noting the corresponding elongations a *curve* can be plotted with *load* and *extension.* The law will be verified if the curve is a straight line (see p. 66). It should be noted that the elongation is proportional to the load within certain limits.

98(b). **Young's modulus**—In the case when an india-rubber cord or a wire fixed at one end is stretched in the direction of its length by a load attached at the other end, a change is produced both of volume and of shape, and the modulus of elasticity in this case is known as *Young's modulus* (Y) or *Co-efficient of tensile elasticity*, which is the ratio of the longitudinal stress to longitudinal strain.

Thus, if F be the force which acting along a length L of a wire of cross-section A stretches it by a small length l, then

the stress = force per unit area = $F/A = F/\pi r^2$, where r is the radius

of the wire; and the longitudinal strain = elongation per unit length = l/L.

$\therefore$ Y (*Young's modulus*) $= \dfrac{F/\pi r^2}{l/L} = \dfrac{FL}{\pi r^2 l}$ dynes per sq. cm. (or lbs per sq. inch).

Bulk modulus.—The bulk modulus applies to the case of a body subject to a uniform pressure distributed over the whole of its surface.

If V be the volume of the body which is diminished by an amount v when subjected to a pressure p,

the bulk or volume strain $= v/V$, the bulk stress $= p$

$$\textit{Bulk modulus} = p \div \frac{v}{V} = \frac{pV}{v} \text{ dynes/cm.}^2 \text{ or lbs/inch.}^2.$$

98(c). Modulus of elasticity and Young's modulus distinguished.

According to Hooke's law, $\dfrac{\text{stress}}{\text{strain}}$ = a constant, within the elastic limit. The general name for this constant for all cases of strain is the modulus (or Co-off.) of elasticity. When particularly the stress is such that the strain is a tensile one, the Co-officient is called the Young's modulus. Thus Young's modulus is the modulus of longitudinal (or tensile) elasticity.

99 Experiment.—Determination of Young's modulus for a Wire—Two exactly similar wires are fixed close together on the same support C. One A carrying a scale S has got a heavy load W attached to keep it stretched and free from kinks. The second wire (the experimental wire) carries a hanger on which any desired wt. F may be placed. A vernier V mounted on the second wire can slide along the scale S (Fig 48).

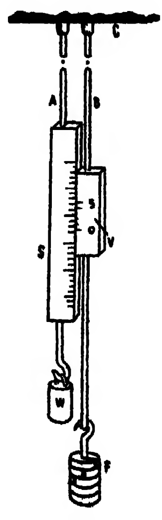


Fig. 48

First calculate theoretically the weight for which the wire will break, *i.e.* the *breaking weight* (see Art. 100) of the wire. This is obtained by multiplying the area of cross-section of the wire in square centimetres with the breaking stress for the metal of which the wire is made. In practice, care must be taken never to add on the hanger more than half the breaking weight which is the maximum permissible load and gives the elastic limit.

Now place almost half the breaking weight on the hanger and keep it there for a few minutes in order to allow the wire to remain stretched, and then remove the greater part

of the weight leaving only sufficient load on the hanger to keep the wire taut and free from kinks. Read the scale S and the vernier V. This is the initial reading.

Then increase the load by $\frac{1}{2}$ kgm. and again note the reading. Go on increasing the load by steps of $\frac{1}{2}$ kgm., and note the reading for each load up to the maximum safe load, *i.e.*, before reaching the elastic limit. This is the point beyond which Hooke's law does not hold. Now diminish the load by equal amounts (*i.e.*, by $\frac{1}{2}$ kgm.) till the original load of $\frac{1}{2}$ kgm. is reached, noting the reading in each case. Take the mean of the readings for each load as the actual reading corresponding to that load. The two sets of readings should closely agree, but if they differ appreciably, it is possible that the wire has been stretched beyond the elastic limit, in which case the experiment should be repeated with a new wire.

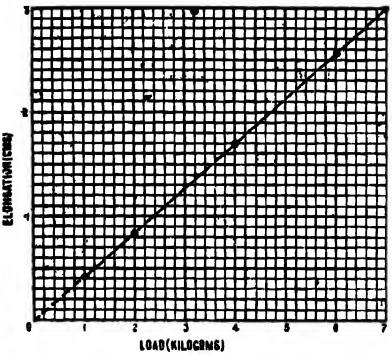


Fig. 48(a).

Measure the diameter of the wire very accurately with a micrometer screw-gauge at several places along the length of the wire from the point of support to the zero of the vernier twice in rt. angled directions at each place and find out the mean radius r therefrom.

Tabulate the readings of the vernier against the corresponding loads and find out the elongations for the various loads by subtracting each reading from the initial reading. Then plot a curve with loads as abscissa and the corresponding elongations as ordinates (Fig. 48(a). The graph should be a st. line passing through the origin, meaning thereby that elongation is zero for zero load. Hooke's law is verified if the graph is a st. line. From the graph find out the elongation l corresponding to any suitable load, say m grms. Measure the length L of the experimental wire from the point of support C up to the point where the vernier is attached. Then,

$$\text{Young's modulus} = \frac{F/\pi r^2}{l/L} = \frac{mg/\pi r^2}{l/L} \text{ dynes/cm}^2.$$

Note.—(i) As the wires are fixed on the same support, any yield of support will affect both the wires to the same extent and so there will be no relative motion of V over S.

(ii) As the two wires are made of the same material, any variation of temperature will affect both the wires by equal amounts and so the readings will not be affected.

Examples—(*1*) *A rubber cord 0·2 cm. radius is loaded with 13 kgm. weight. A length of 50 cms. is found to be extended to 51 cms. Calculate the Young's modulus of rubber.*

Here the pulling force $F = 13 \times 1000 \times 981 = 12{,}753{,}000$ dynes.

Stress $= 12{,}753{,}000 \div \pi \times (0{\cdot}2)^2$; Strain $= (51 - 50) \div 50 = 0{\cdot}02$;

$$\therefore \quad Y = \frac{12{,}753{,}000}{\pi \times (0{\cdot}2)^2 \times 0{\cdot}02} \text{ dynes per sq. cm.} = 0{\cdot}5 \times 10^{10} \text{ dynes per sq. cm.}$$

(*2*) *A mass of 20 kgm. is suspended from a vertical wire 600·5 cm. long and 1 sq. mm, in cross-section. When the load is removed, the wire is found to be shortend by 0 5 cm. Find Young's modulus for the material of the wire.* (*C. U. 1938*).

Pulling force $F = 20 \times 1000 \times 981$ dynes ;

$\therefore$ Stress $= F/(\text{area of cross-section}) = (20 \times 1000 \times 981) \div (0{\cdot}01)$.

$= 1{\cdot}962 \times 10^9$ dynes per sq. cm.

Strain $= l/L = \dfrac{0{\cdot}5}{600{\cdot}5} = \dfrac{1}{1201}$; $\therefore$ Young's modulus $(Y) = \dfrac{1{\cdot}962 \times 10^9}{\frac{1}{1201}}$

$= 1201 \times 1{\cdot}962 \times 10^9 = 2{\cdot}35 \times 10^{12}$ dynes per sq. cm.

100. Properties Peculiar to Solids.—

Ductility.—It is the property in virtue of which a body may change its form by the application of pressure. Due to this property a solid may be drawn into fine wires.

Malleability.—It is the property in virtue of which metals can be hammered into thin leaves. Gold, silver, led are malleable substances as they can be beaten into thin leaves. Lead is a malleable metal but not ductile, as it cannot be drawn into fine wires.

Tenacity.—It is the property of the solid, when in the form of wires, to support a weight without breaking. The weight required to break a wire is called its *breaking-weight*, and is the measure of the *tenacity* or *tensile strength* of the material of the wire.

Wrought-iron has more tenacity than cast-iron ; and the steel pianoforte wire is the most tenacious metal.

Hardness.—It is the relative property of the solid in virtue of which it offers resistance to being scratched by others. Diamond is the hardest known substance.

Rigidity.—It is the property of the solid due to which it does not readily alter its size or shape, and keeps its own volume and form unless subjected to a considerable force.

100(a). Properties peculiar to Liquids —

(*i*) **Surface tension.**—At any point inside a mass of a liquid a molecule is subject to attracting forces exerted by neighbouring molecules on all sides. But a molecule near to the surface is pulled more strongly inwards, than outwards, and this causes the surface to behave as though it were covered by a thin stretched skin. This tension along the surface is known as *surface tension.* The surface has a tendency to contract and reduce its area to a minimum.

The spherical shapes of soap bubbles, or rain-drops, are due to surface tension which pulls the drops into the shape of a sphere, as a sphere has the least surface area for a given volume. In large masses of liquids this effect is not so evident, as the attraction due to gravitation is in much excess over *cohesive forces.*

When the mutual attraction between the molecules of a liquid (*cohesion*) contained in a vessel is less than their attraction to the sides (*adhesion*), the liquid *wets* the side of the vessel, as is the case with water in a glass vessel ; but if the attraction of adhesion is less than that of co-hesion, as with mercury in a glass vessel, the liquid *does not wet* glass, and so mercurry sprinkled on glass surface separates out into spherical drops, whereas water or oil easily spreads over a a glass surface.

That the surface layer of a liquid acts as a stretched membrane may be understood by placing very carefully a slightly greased sewing-needle on the surface of water in a dish. The needle, which is eight times as dense as water, will be found to float in apparent contradiction to the law of Archimedes (Art. 111) according to which it should sink. The phenomenon of insects walking and running on the surface of water is also another illustration of the same effect.

(*ii*) **Capillarity.**—If a glass tube of small bore is dipped in a liquid, then, in a case where the liquid wets glass, as the case with water, the internal level of the liquid will be *higher than* the level outside [Fig. 49(a)], but with mercury, which does not wet glass, the interior surface is *below* the exterior surface [Fig. 49(b)]. The surface in the case of water in glass is *concave* upwards, but for mercury it is *convex* upwards.

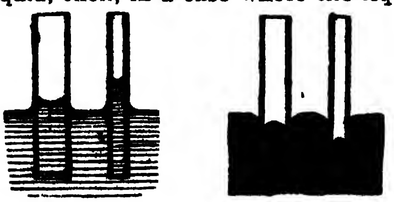


Fig. 49(a) Fig. 49(b)

These results are said to be due to what is known as *capillarity*, It is due to the fact that the molecular attraction of glass for water—

i.e. the force of adhesion between the solid and the liquid—is greater than the attraction (*i.e.* the force of cohesion) of water for water, and that the force of adhesion between glass and mercury is less than the force of cohesion between mercury and mercury. The elevation or depression of the liquid in the tube is inversely proportional to the diameter of the tube (Fig. 49) ; so the capillary effect can be clearly shown only in the case of very narrow tubes. For this reason, tubes of small internal diameter are called **capillary tubes**.

The rise of oil in wicks of lamps, the soaking up of ink by blotting paper, the retention of water in a piece of sponge, the rapid absorption of liquid by a lump of sugar, the wetting of a towel, when one end of it is allowed to stand in water, are all *instances of capillarity.*

(iii) **Viscosity**.—Water poured into a funnel runs out rapidly, but glycerine or thick oil runs through more slowly, and treacle still more slowly. They differ from one another in, what is termed, *viscosity*, which is due to friction in the interior of the liquids. Liquids like water which flow readily are termed *mobile*, while those of the treacle type are said to be *viscous*. The viscosity of a liquid decreases with the rise in temperature as seen in the case of treacle, when it is hot. Viscosity varies with the nature of the fluid. Gases are very mobile, yet the viscosity of air is seen in the effect of wind in causing waves.

Questions

Arts. 97 & 99.

1. State and explain Hooke's law and describe an experiment to illustrate it. (C. U. 1938 ; Pat. 1930).

2. State 'Hooke's Law' and define 'Young's modulus of Elasticity'. How would you determine 'Young's modulus' for a steel wire ?
(See also Art. 98). (C. U. 1932, '36, '38 ; All. '28, '30, '46)

3. Tell how you may, by use of Hooke's law and a 20 lb. weight, make the scale for a 32-lb. spring balance. (C. U. 1936.)

4. A wire of 0·4 cm. diameter is loaded with 25 kgm. wt. A length of 100 cm. is found to be extended to 102 cm. Calculate the Youngs' modulus of the wire. (All. '46).

HYDROSTATICS

CHAPTER VIII

Pressure in Liquids

101. Hydrostatics.—The study of *Hydrostatics* deals with the action of forces on fluids and with their conditions of equilibrium.

102. Pressure at a point in a Liquid,—*Pressure* at a point due to a liquid is the *thrust exerted by the liquid per unit area* surrounding the point. That is, pressure $P = \frac{Total\ force}{Total\ area}$; which is the same as *the force per unit area.*

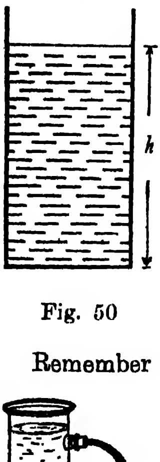


Fig. 50

Consider a cylindrical column of liquid of height h, the area of cross-section of the cylinder being A (Fig. 50). The weight of this column of liquid is the total thrust upon the base. Therefore the total thrust upon the base $= A\ h\ \rho\ g$.

[ρ (pronounced '*rho*') = density of the liquid]

$\therefore$ Pressure exerted by the liquid column

$$= \frac{A\ h\ \rho\ g}{A} = h\ \rho\ g.$$ That is,

the pressure at a point in a liquid is proportional to its depth.

Remember that *pressure is the force per unit area.*

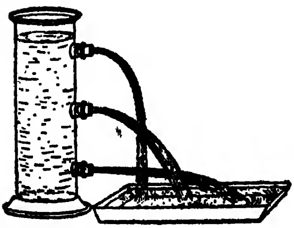


Fig. 51.

Experiments.—(i) Fig. 51 is a graphic method of showing that in a liquid the pressure is proportional to depth. It is a tall jar having three outlets at different depths. When the jar is filled with water, water-jets are seen to flow out through the outlets with different forces indicating difference of pressures at those levels.

(ii) A thistle funnel is bent as shown in Fig. 52, and across its mouth a thin india-rubber membrane is tightly tied with thread. Coloured water is poured into the bend to serve as a manometer (pressure measurer). Lower the funnel into a deep glass vessel

filled with water. The increase in pressure of the water on the membrane, transmitted by the air in the funnel, will be indicated by the drop of the liquid in the short tube and rise in the longer one. By lowering the funnel at different depths it will be shown that the *pressure is proportional to depth.* Move the funnel to and fro at the same level. It will be seen that the levels of the coloured water do not change, which shows that the *pressure at the same level is the same.*

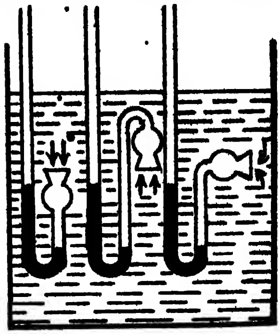


Fig. 52

Using thistle funnels bent at different angles (Fig. 52) it can be shown that a *pressure is exerted in a liquid in all directions.*

103. The Free Surface of a liquid at rest is always Horizontal —If possible let the surface be not horizontal (Fig. 53). Consider two points A and B in the liquid in the same horizontal plane.

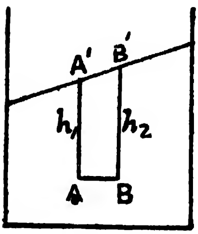


Fig. 53

The pressure at A due to the liquid is $g\rho h_1$, and that at B is $g\rho h_2$, where ρ is the density of the liquid, and h_1 and h_2 are the heights of the liquid at A and B respectively. Since h_2 is greater than h_1, the pressure at B is greater than that at A. Hence the liquid particles will move from places of higher pressure to those of lower ones until the state of equilibrium is reached, *i.e.* until the surface of the liquid is horizontal.

From this fact it follows also that if a liquid be poured into a series of connected vessels of varied shapes, the liquid, when at rest, will stand at the same level in all the vessels. Thus, in a tea-pot the water stands at the same level in the spout as in the vessel itself. This is commonly expressed by saying that **liquids find their own level.**

If several liquids which do not mix with one another are placed in the same vessel, they will arrange themselves one above another in the order of their densities, the heaviest of them being at the bottom and the lightest at the top. It will be found that the surface of separation in each case is horizontal.

103 (a). Some illustrations.—

(1) Spirit level.—This instrument is based on the principle explained above, and is used to test whether a surface is perfectly hori-

zontal or not. This consists of a slightly curved glass tube filled with alcohol, except for a small bubble of air, which naturally occupies the highest part of the tube (Fig. 54). The glass tube is fixed in a brass mount. The air-bubble occupies exactly the middle position of the tube if the instrument is placed on a perfectly horizontal surface, and the bubble will move in a different position if the surface is not horizontal.

Fig. 54—Spirit level.

(2) **City Water Supply**—The principle that water, or any other liquid, finds its own level is applied in supplying water to different houses of a city. In order that every house may have an adequate supply of water at a considerable pressure, water obtained from the source of supply, say a river or a well, is pumped up by steam-driven pumps into a large reservoir placed at the highest place in the neighbourhood, or on lofty water towers, specially erected for the purpose. The water from the reservoir is carried to different buildings by means of water-mains and branch pipes. The pressure of the water supply depends upon the vertical height—called the "**head of water**"—of the water surface in the reservoir. In practice, however, the water does not rise as high as the height of the water surface in the reservoir. This is due to the friction both of the air and of the pipes.

(3) **Tube-wells.**—There are water beds at different depths below the surface of the earth. These beds are fed by outlying rivers and lakes, and the currents of water in those beds are like water in a U-tube, the water in which always tries to find its own level in the two limbs. So, as soon as a boring is made anywhere below the surface of the earth reaching any of these water beds, water gushes forth upwards with a tendency to find its own level, which is the level of rivers, etc. A pump is generally required to raise the water up to surface of the earth.

Example.—*Neglecting the loss of pressure in the transit, calculate what head of water is necessary to produce a pressure of 200 lbs. per sq. inch in the street mains.*

1 cu. ft. of water weighs 62·5 lbs. ∴ For the head of water 1 ft. high, pressure per sq. foot = 62·5 lbs. ∴ Pressure per sq. inch $= \frac{62·5}{144} = 0·43$ lb.

∴ To maintain a pressure of 0·43 lb. per sq. inch, a column of water 1 foot high is necessary.

Hence to maintain a pressure of 200 lbs. per sq. inch, the height (head) of water necessary $= \frac{200}{0·43} = 464$ ft. (approx.)

That is, the water in the reservoir should stand 464 ft. above the point in question.

104. The pressure of a fluid at any point in the wall of a vessel acts in a direction perpendicular to the wall.—Fill the vessel (Fig. 55) with water and push the piston in. It will be found that the water is at once forced out of every hole, as shown in the figure, along a radius of the spherical vessel—*i.e.* in a direction perpendicular to the wall of the vessel.

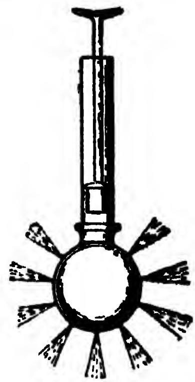


Fig. 55

This fact is also shown when a pipe containing a liquid at rest is pierced by a small hole. A thin jet squirts out at right-angles to the surface of the pipe.

Examples.—*1. A plate 10 metres square is placed horizontally 1 metre below the surface of water, when the height of the mercury barometer is 760 mm. What will be the total pressure on the plate? (The density of mercury = 13·6). (C. U. 1911.)*

1 metre = 100 cms.; 10 metres square = 10 metres × 10 metres = 1000 cms. × 1000 cms. = 10^6 sq. cms.

The pressure at a point 100 cms. below the surface of water = atmospheric pressure + the pressure due to a column of water of height 100 cms.

= Pressure due to (76 × 13·6 + 100) cms. height of water = that due to 1133·6 cms. height of water = that due to 1133·6 gms. weight = 1133·6 × 981 dynes (∵ 1 gm. wt. = 981 dynes)

This is the force exerted on unit area of the plate.

∴ The total pressure on the plate = 1133·6 × 981 × 10^6 dynes.
= 1·013 × 10^{12} dynes.

2 A U-tube open at one end and closed at the other is partially filled with mercury (density 13·6). The closed end of the tube contains some air and the mercury in the open limb stands 30 cms. higher than it does in the closed limb. Find in c. g. s. units intensity of pressure on the air in the closed end of the tube. (*C. U. 1910*)

The pressure of the enclosed air = Pressure due to (76 + 30) cms. of mercury
= (106 × 13·6 × 981) dynes = 1·41 × 10^6 dynes.

3. At what depth below the surface of water will the pressure be equal to two atmospheres if the atmospheric pressure be 1 megadyne (10^6 dynes) per sq. cm.? (g = 981 cm./sec.^{2}). (*C. U. 1931*).

Let h cms. be the required depth at which the pressure is equal to 2 megadynes.

∴ The pressure due to h cms. height of water = 1 megadyne = 10^6 dynes.

The pressure due to 1 cm. height of water (*i.e.* the wt. of 1 c.c. of water) = 1 gm.-wt. = 981 dynes.

∴ The pressure due to h cms. height of water = h × 981 dynes = 10^6 dynes.

$$\therefore \quad h = \frac{10^6}{981} = 101{,}936 \text{ cms.}$$

105. The Lateral Pressure—When a liquid is at rest, it exerts pressure on the sides of the containing vessel, which is known as *lateral pressure.*

Fig. 56 shows a vessel floating on water having a stop-cock in one side. Fill the vessel with water and open the stop cock. Water flows out of the cock, and the vessel is seen to move backwards in a direction *opposite* to that of the water jet.

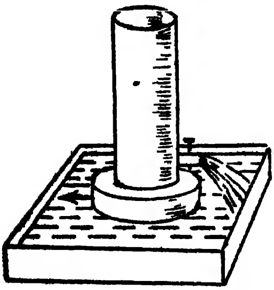


Fig. 56

When the liquid is at rest, the horizontal pressure on the two sides are equal and destroy each other . but when the pressure on one side is relieved by opening the stop-cock, the vessel moves in the opposite direction (in the direction of the arrow) due to the lateral pressure on that side.

106. The pressure at any particular Depth does not depend on the Shape of the Vessel.

Experiment.—The area of cross-section of all the four vessels A,B,C,D, (Fig. 57) known as *Pascal's vases*, are equal. They can be screwed on the platform and a plate E attached to one end of a lever is pressed against the bottom by adding weights W on the scale pan. Placing a suitable weight on the scale pan, water is poured into the vessel until the supporting plate just yields, and water escapes. Noting the depth h of water, the experiment is repeated with other vessels. It will be found that water begins to escape when it attains the same depth in every case, proving that *pressure depends only on the depth, and not on the size or shape of the vessel.*

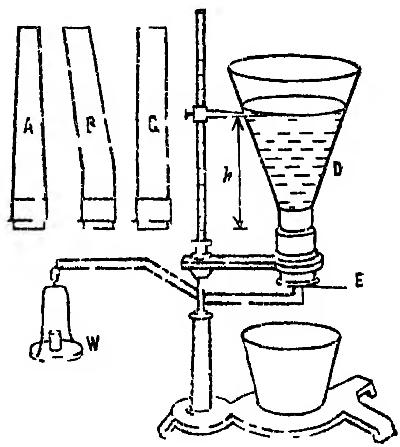


Fig. 57

This is also known as **the Hydrostatic Paradox.**

The result is at first puzzling but a moment's consideration will show that there is no real inconsistency. There are two vessels in Fig. 58 of different shape having bases of the same area. They

are filled with water to the same level. Though the amount of water in the two vessels is different, the pressure exerted on the base of the vessel is the same in the two cases. This is because the sides of the vessel exert pressure on the liquid at right angles to its surface. This pressure is represented by P in Fig 58(a), which can be resolved into two components, V acting upwards and H acting horizontally.

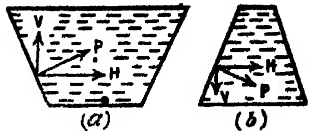


Fig. 58

All the vertical components like V serve to support the water on the sloping side. In Fig. 58(b), the slope of the side being opposite, the vertical component V is acting downwards, which is transmitted to the base. Due to this, the total pressure on the base is the same as if the vessel had vertical sides. This explains that the pressure depends only on the depth and not on the shape or size of the vessel.

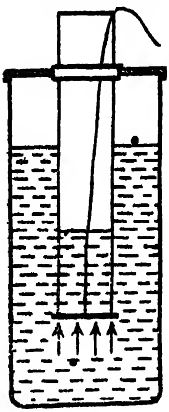


Fig. 59

107 The Upward Pressure at any Depth in a Liquid is Equal to the Downward Pressure —

Experiment —Take a glass cylinder with both ends open. A thin disc of tin is held tightly against the lower end by a string passing through its centre (Fig. 59). On lowering the whole into water and loosening the string, it will be found that the tin disc does not fall. This is due to the vertical upward thrust exerted by water underneath the disc.

Now carefully pour water inside the cylinder and note that the disc remains in its place so long as the level of water inside is less than the outside level, but the disc falls down by its own weight when the level of water inside and outside the cylinder is the same.

This proves that *the upward pressure, or the buoyancy, at any depth is equal to the downward pressure.*

108. Pascal's Law.—*The pressure exerted anywhere on a mass of a confined liquid is transmitted undiminished in all directions so as to act with equal force on every unit area of the containing vessel in a direction at right angles to the surface of the vessel exposed to the liquid.*

Experiment.—Take a stout glass flask fitted with a closely fitted piston at the neck. There are four tubes, bent upwards and attached to the flask, as shown in Fig 60. Put a little mercury into the bend of each of these tubes. Then each of these tubes serves as a manometer (or pressure measurer).

Remove the piston and fill the flask with water, and then apply a pressure by re-inserting the piston. The pressure is transmitted in all directions.

On pushing the piston, the mercury will be seen to rise to the same height in all the tubes, showing that the pressure exerted is the same in every case.

If each of the openings has got the same area, then the total force exerted (*i.e.* pressure × area) will also be equal in every case. If the area of one of the openings be twice that of another, the total force (here total force = 2 area × press.) exerted there will also be twice, but the pressure, *i.e.* the force on unit area, will be the same, and so the manometer will indicate the same difference of level.

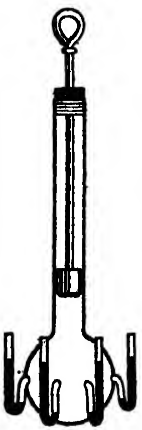


Fig. 60

This principle is applied in constructing *hydraulic presses* and *lifts*.

Historical.—The above law is known as Pascal's law because **Blaise Pascal** (1623—1662), a French Mathematician and Philosopher, was the first to state the law of the equal transmission of fluid pressure. Pascal was one of the founders of the science of **Hydrostatics.** He was the first to make a thrilling demonstration of the fact that a narrow vertical column of water contained in a long tube fixed to the top of a wooden barrel can exert so much pressure on its walls that the barrel may burst. At that time it was called a paradox. The *hydrostatic bellows* (Art. 110) is based on this principle.

109 Hydraulic Press (The Bramah Press).—

Principle of Action.—Consider two cylinders A and B (Fig. 61) of different areas fitted with pistons and communicating with each other through a pipe. Now, if a pressure be applied on the piston in A, an equal pressure will be transmitted to the piston in B. Remember that *it is the pressure which is transmitted, not the total force.* The pressure is the force per unit area. Hence the areas of the pistons must be taken into account in considering the transmitted force. So every unit area of the piston in B will be pressed upwards with the force exerted by every unit area of the piston in A.

Thus, if the diameter of B is four times the diameter of A, the area of cross-section of B will be sixteen times that of A. The pressure on the piston in B will be the same as that applied by the piston in A,

but since the total force is the product of pressure and area, the upward force on the platform W is sixteen times the force on the piston in A, or in other words, if α and β be the areas of the small and large pistons respectively, and f the force applied by the piston in A, then the force F on the piston in B will be given by, $F = \frac{\beta}{\alpha} \times f$.

This is the principle of Bramah's Press, where, by applying a small force to the piston in the narrow cylinder, a much heavier load may be raised or pressed against the upper platform, as shown in Fig. 61.

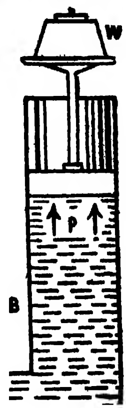


Fig. 61

Description.—The principle of the hydraulic press is applied for a variety of purposes, *e.g.* for compressing bales of cotton, lifting heavy weights, bending tram lines, extracting oil from seeds, etc.

Fig. 62 shows a section of a working type of hydraulic press. On the right-hand side of it, is a pump, the piston A of which works in a small cylinder C_1. When the piston is raised by the lever L, water from a cistern enters the cylinder C_1 through a valve V_1 at the bottom. When the piston descends the valve at the bottom closes, and the water is forced into the larger cylinder C_2 along the narrow tube through valve V_2. In this cylinder there is the large solid piston B (*the ram*), which carries a platform P at its top on which bales are placed to be compressed against a stout fixed plate R. The sectional area of the piston in cylinder C_2 is much larger than the area of the other piston, so the force exerted on the larger piston is multiplied many times according to the principle stated above.

"Paradox".—In the above case the force is magnified, but there is no gain in work, and though it appears to be paradoxical at first sight, there is nothing really peculiar in it. It is the case with all the simple machines, and the *Principle of Conservation of Energy* is satisfied in all cases.

In the hydraulic press, when a small force applied to the small piston in the narrow cylinder overcomes a large resistance applied to

the larger piston of the other cylinder, the small piston has to be moved through a considerable distance in order to move the larger one through a small distance, and the work done in the two cases is the same For example, let the area of cross section of the small piston be α and that of the other be β When the small piston moves down through x_1, $\alpha \times x_1$ vol of water must be driven from the small cylinder into the larger one, and for this the larger piston will be raised through $\alpha x_1/\beta$ as the area of cross section of this cylinder is β Now, if a force F_1 is produced on the small piston A, then the pressure is F_1/α and this pressure is transmitted undiminished to the larger piston, so the thrust on the larger piston = press × area $= \frac{F_1}{\alpha} \times \beta$, and it moves through $\frac{\alpha x_1}{\beta}$ Hence the work done (i e. force × distance) by the smaller piston $= F_1 x_1$, and the work done on the larger piston is $\frac{F_1}{\alpha} \beta \times \frac{\alpha x_1}{\beta}$ and they are equal

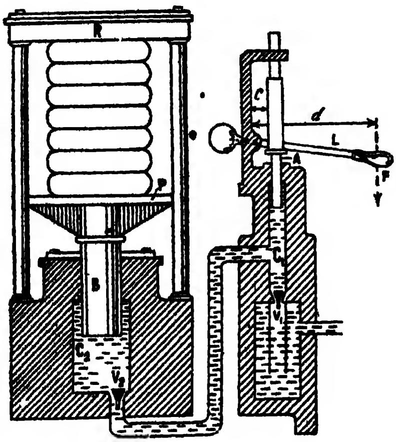


Fig 62—Hydraulic Press

Machanical advantage of the machine —Let a force F be applied to the end of the lever L If the thrust generated on the smaller piston A be F_1, then the *mechanical advantage* (m) of the lever is given by,

$$m = \frac{F_1}{F} = \frac{\text{Power arm}}{\text{Resistance arm}} = \frac{d}{c}$$

$$i\,e \quad F_1 = m\ F = \frac{d}{c}\ F$$

Let the total force produced on the larger piston B be F_2 Then,

$$F_2 = \frac{\beta}{\alpha}\ F_1 = \frac{\beta}{\alpha} \frac{d}{c} \cdot F$$

∴ **Mechanical advantage of the machine.**

$$= \frac{F_2}{F} = \frac{\beta}{\alpha} \cdot \frac{d}{c}$$

Example.—*A Bramah press has a piston whose cross-section is 144 sq. in. The cross-section of the pump is 2 sq. in. The shorter arm of the lever working the pump is 1 foot and the longer one is 4 feet in length. Calculate the total force obtained when an effort of 175 lbs. is applied to the end of the longer arm.*

By the principle of the lever, we have $175 \times 4 = W \times 1$, where W is the weight or load : $\therefore \quad W = \frac{175 \times 4}{1} = 700$ lbs.

That is, the pressure has been increased from 175 to 700 lbs.

Now, according to the principle of the hydraulic press, we have, $\frac{F}{700} = \frac{144}{2}$, where F is the total force.

$$\therefore F = \frac{700 \times 144}{2} = 50{,}400 \text{ lbs. wt.}$$

110. Hydrostatic Bellows.—Fig. 63 represents an instrument known as the *hydrostatic bellows*. This is another example of multiplication of force by the transmission of fluid pressure. The instrument consists of a stout leather bellows attached to a long vertical tube. The bladder and a part of the tube are filled with water. A heavy weight placed on the platform of the bladder will be supported simply by the weight of a column of water in the attached tube.

Fig. 63—Hydrostatic Bellows

Questions

Art. 102

1. How would you prove experimentally that a liquid exerts pressure in all directions ? (C. U. 1911 ; '14 ; '27).

2. The density of sea-water is 1·025. Find the pressure at the depth of 10 ft. below the surface in pounds per square foot, given that one cubic foot of water weighs 62·5 lbs. (C. U. 1927).

[*Ans :* 640·625 lbs. per sq. ft.]

3. Define intensity of pressure at a point in a liquid. Prove that the difference of pressure between the surface of a liquid and a point in the liquid z cms. below the surface is given by $P = gdz$, where d is the density of liquid and g is the acceleration due to gravity. (C. U. 1910 ; Pat. 1938).

[**Hints.**—Intensity of pressure at a point is the pressure per unit area surrounding that point. If p be the atmospheric pressure, *i.e.* pressure of air exerted on unit area, then the pressure due to atmosphere on an area A of the liquid surface $= p \times A$. The pressure due to the liquid column of the same area

A and height z cms. $= Agdz$. $\therefore$ Total pressure at a point in the liquid z cms. below the surface $= pA + Agdz$.

$\therefore$ The press. on unit area $= p + gdz$, and the press. on the surface $= p$.

$\therefore$ The difference of pressure $= (p + gdz) - p = gdz$.]

4. A rectangular tank 6 ft. deep, 8 ft. broad and 10 ft. long is filled with water. Calculate the thrust on each of the sides and on the base. (1 cu. ft. of water weighs 62·5 lbs) (Pat. 1929).

(See Art. 104.) [*Ans.* : On the base = 960,000 poundals ; on each of the shorter sides = 288,000 poundals ; on each of the longer sides = 360,000 poundals.]

Art. 105.

5. A tall vessel provided with a tap at the side near the bottom is filled with water and made to float upright on a thick plate of cork. Explain what will happen when the tap is opened. (C. U. 1914).

Art. 108.

6. State Pascal's Law as to the transmission of pressure in a liquid and describe a suitable experiment to verify it in the laboratory.

(C. U. 1922 ; Cf. Pat. 1929 ; '31 ; '41 ; '44).

7. The neck and bottom of a bottle are $\frac{1}{2}$ in. and 4 in. in diameter respectively. If when the bottle is full of oil, the cork in the neck is pressed in with a force of 1 lb. wt., what force is exerted on the bottom of the bottle ?

(Pat. 1944).

[*Ans.* : 64 lbs. wt.]

Art. 109.

8. Draw a neat diagram of the hydraulic press, and give a brief description of it with an explanation of its action. (C. U. 1922 ; '34 ; '37 ; Pat. '19)

9. Make a diagram showing the construction of a hydraulic press.

A force of 50 kgms. is applied to the smaller piston of such a machine. Neglecting friction, find the force exerted on the large piston, the diameters of the pistons being 2 and 10 cms. respectively. (Pat. 1922).

[*Ans.* : 1250 kgm. wt.]

10. Explain the action of the hydraulic press.

The area of the small piston of a hydraulic press is one sq. ft. and that of the large piston twenty sq. ft. How much wt. can be raised on the large piston by a force of 200 lbs. acting on the small piston ? (C. U. 1946)

[*Ans.* : 4,000 lbs.].

CHAPTER IX

Archimedes' Principle, Density and Specific Gravity

111. Archimedes' Principle.—*A body immersed partly or wholly in a fluid at rest appears to lose a part of its weight equal to the weight of the fluid displaced.*

Verification of Archimedes' Principle :—

The Archimedes' principle can be verified by a *Hydrostatic balance*, which is simply an ordinary balance by which the weight of a body immersed in a liquid can conveniently be obtained. In a special form of this balance, one of the pans having a hook at its bottom is suspended by a shorter suspending frame than the other. The body to be weighed is hung from the hook.

Expt—A solid metal cylinder *A* (fig. 64) is suspended from a hook fixed at the bottom of a hollow cylinder *B* into which the solid cylinder *A* exactly fits, so that the internal volume of the hollow cylinder is the same as the volume of the solid one. The whole thing is suspended from one arm of a balance and counterpoised. The solid cylinder is then totally immersed in water placed in a beaker *D* which rests on a small wooden bridge *C*. The balance is now disturbed as the lower cylinder has lost a part of its weight due to the upward thrust, *i.e.* **buoyancy of water.**

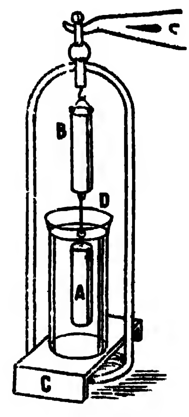


Fig. 64

Now fill the hollow cylinder *B* completely with water, and the balance will be restored again, showing that the solid cylinder lost a part of its weight equal to the weight of its own volume of water, which is the same as the weight of the displaced water ; or, in other words, *the upward thrust on the cylinder is equal to the weight of the liquid displaced by it.*

This verifies the principle of Archimedes.

Note.—It should be noted that the loss in weight of the cylinder is only an apparent one, for really the vessel of water together with the cylinder placed on the scale pan would weigh the same whether the

cylinder is placed outside or inside the vessel of water, as explained in the case (2) on downward thrust (See p. 126.)

111 (a) The **buoyancy** of a fluid may be defined as the resultant upward thrust experienced by a body when immersed in the fluid. When standing or lying in water you must have noticed that water tends to raise you or buoy you up. The result of this buoyancy of water can be observed by pushing a lead pencil, (or any other thing which floats) into water and then letting go, when the solid will be seen to float up through the water.

Theoretical Proof of the value of buoyancy.—Consider a solid rectangular block $ABCD$ inside a liquid (fig. 65). The liquid presses on the block all over The horizontal pressures on the two pairs of opposite vertical surfaces counteract each other as they are of equal magnitude and act in the same horizontal line. The top surface AB is pressed downwards by the weight of the column of liquid $AEFB$. The bottom surface CD, which is at a depth CF below the surface, is pressed upwards by the weight of the column of liquid $EDCF$ (see art. 107). It is clear that the upward force exceeds the downward force by the weight of the column of liquid $ADCB$, which is the quantity of liquid displaced by the block, *i.e.* the upward thrust exerted by the liquid is equal to the weight of the displaced liquid.

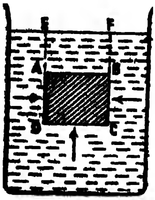


Fig. 65

Mathematical Proof.—Let EA and ED, *i.e.* the depths of AB and $CD = h$ and h' respectively ; area of the faces AB and $CD = A$; density of the liquid $= d$; acceleration due to gravity $= g$.

$\therefore$ The total *downward* pressure on the face $AB = Ahdg$,
And the total pressure on CD acting *vertically upwards* $= Ah'dg$.
$\therefore$ The resultant thrust on the block exerted by the liquid acting *vertically upwards* $= A(h' - h)dg$. But $A(h' - h)$ is the volume of the block, so the resultant upward thrust is equal to the weight of the volume of the liquid displaced by the block. This upward thrust is called the **buoyancy** of the liquid.

Besides the buoyancy there is another force acting on the body, which is the weight of the body acting vertically downwards. If W be this weight, the resultant force acting on the body is $\{W - A(h' - h)dg\}$; that is, *as the effect of immersion the body loses a part of its weight equal to the weight of the liquid displaced by it.*

The principle of Archimedes is also known as the **law of buoyancy.** It was discovered by **Archimedes** (212—287 B. C.), a celebrated mathematician and philosopher. The story of Hiero's crown has been

too well known. Hiero, the king of Italy, wished to be certain that the crown made for him was of pure gold, and he asked Archimedes to settle this. This puzzled Archimedes, but he made many experiments as a result of which he discovered the above principle by which he could prove that the crown was not made of pure gold.

111. (b) The principle of Archimedes is also true for gases.

A body will apparently weigh less in air than it would in vacuo, for the air exerts an upward thrust equal to the weight of the displaced air ; but the weight of the displaced air is so small that ordinarily it is not taken into account.

Expt.—It can be demonstrated by the **baroscope** (fig. 66). A large sphere of cork is suspended from one arm of a small balance and is equipoised by brass wts. W placed on the other arm. The whole system is then placed under the receiver R of an air pump. On drawing out air, the arm carrying the cork sphere is seen to sink down. The cork sphere owing to its larger volume displaces greater volume of air than the brass pieces and so the up thrust, or the buoyancy of air, is also greater on the cork sphere. As the apparent wts. of both in air are the same while the buoyancy on the cork sphere is greater, the true wt. of the cork sphere must be greater. That is why, in the absence of air, the cork sphere sinks down. If, however, the two are equipoised first in vacuo and then air is introduced, the cork will go up and the wts. sink down.

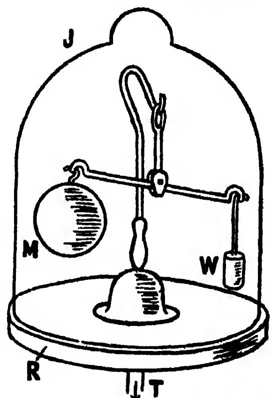


Fig. 66

111 (c) True weight of a body.—In very accurate weighings it is necessary to take account of the air displaced by the body in order to reduce the weight to vacuum.

Let W = true wt. of the body, *i.e.* its weight in vacuum,
W_1 = true wt. of the counterpoising weights.
d = density of the body ; d_1 = density of the wts. ;
ρ = density of air.

Then the volume of the body $= W/d$ and the volume of the counterpoising wts. $= W_1/d_1$: So the wt. of the air displaced by the body $= \rho . W/d$ and that by the wts. $= \rho . W_1/d_1$.

Hence, for equilibrium, we have,

$$W - \rho.W/d = W_1 - \rho.W_1/d_1$$

whence $W = W_1 \dfrac{(1-\rho/d_1)}{1-\rho/d} = W_1 + W_1.\rho.\left(\dfrac{1}{d} - \dfrac{1}{d_1}\right)$,

since ρ is small in comparison with d or d_1.

Example.—The wt. of a body in air is 30·5 gms. The density of the body is 0·76 gm./c.c., that of brass wts. is 8·4 gms./c.c., of air is 0·001293 gm./c.c. Calculate the true wt. of the body.

True wt. $W = W_1 + W_1.\rho.\left(\dfrac{1}{d} - \dfrac{1}{d_1}\right)$

$= 30·5 + 30·5 \times 0·001293\left(\dfrac{1}{0·76} - \dfrac{1}{8·4}\right)$

$= 30·5471$ gm.

Hence the true wt. is greater than the apparent wt. by 0·0471 gm.

112. Two Interesting Cases on Downward Thrust.—The following interesting cases should be noted carefully regarding the downward thrust on a liquid by an immersed body.—

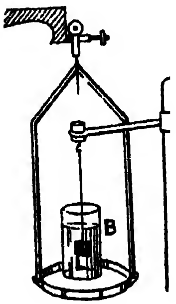


Fig. 66 (a)

(1) A beaker containing water is placed on one pan of a balance and counterpoised [Fig. 66 (a)]. Now a body L of known volume, say v *c.c.*, suspended by a thread from an external support (not from the balance beam), is allowed to sink into the water. What effect has this on the balance ?

It will be found that the equilibrium of the balance will be destroyed, to restore which weight on the other pan should be increased by v gm. This is because the liquid in the beaker displaced by the body raises the level by which the downward pressure on the bottom, which depends upon the depth of the liquid, is increased and so the total thrust on the bottom is increased.

(2) A beaker containing water is placed on the left pan of a balance and a body is also placed on the same pan outside the beaker and the two are counterpoised. Now the body is suspended from the left hook of the balance and is allowed to sink into the water. It will be found that equilibrium will not be disturbed in this case. The upward force of buoyancy, when the body is immersed, is equal to the weight of its own volume of the liquid. But, since the weight of the whole system on the left side remains unchanged, the beaker experiences an equal force downwards and so the equilibrium is not disturbed.

113. Density and Specific Gravity.—*The* **density** *of a substance is the mass per unit volume of the substance.* So,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}.$$

The **specific gravity** *of a substance is the ratio of the mass of any volume of the substance to the mass of an equal volume of water at 4°C.* Thus,

$$\text{Sp. gr. of a substance} = \frac{\text{Mass of V c.c. of the substance}}{\text{Mass of V c.c. of water at } 4^\circ\text{C.}}$$

$$= \frac{\text{Mass of unit volume of the substance}}{\text{Mass of unit volume of water at } 4^\circ\text{C.}}$$

$$= \frac{\text{Density of the substance}}{\text{Density of water at } 4^\circ\text{C}}.$$

So, specific gravity is called *Relative density.*

Note. (i) *Specific gravity is expressed as a ratio ;* it expresses the number of times a substance is heavier than an equal volume of water. So it is a pure **number** only, while density, which is the mass per unit volume, is not a mere number. *Density must be expressed in some unit, i.e. in grams per cubic centimetre, or in pounds or ounces per cubic foot.*

(ii) We may speak of *weight*, instead of *mass*, in defining specific gravity, because the ratio of two weights is equal to the ratio of their masses.

114. Relation between Density and Specific Gravity in the two systems of units.

(*a*) In C. G. S. units, the density of water at 4°C. = the mass of 1 c.c. of water at 4°C. = 1 gram.

Since 1 c.c. of water weighs 1 gm., the volume of a substance in c.c. is *numerically* equal to the mass in gm. for an equal volume of water. *So density of a substance can be written, for C. G. S. units*, as follows.

$$\text{Density} = \frac{\text{Mass of the substance}}{\text{Volume of the substance}}$$

$$= \frac{\text{Mass of the substance}}{\text{Mass of an equal volume of water}}.$$

But the ratio of the masses of two bodies is the same as the ratio of their weights. Hence, when measured in C. G. S. units,

$$\text{Density} = \frac{\text{Weight of a body}}{\text{Weight of an equal volume of water}}$$

$$= \text{Specific gravity of the body.}$$

Therefore, **the density of a substance in C. G. S. units is numerically equal to its specific gravity.** For example, the density of lead in C. G. S. units is 11·3 grams per c.c., and the sp. gr.

of lead $= \dfrac{\text{density of lead}}{\text{density of water at } 4^\circ\text{C.}} = \dfrac{11{\cdot}3}{1} = 11{\cdot}3.$

(b) In F. P. S. units, the density of water at 4°C. = the mass of 1 cu. ft. of water at 4°C. = 62·5 lbs. Since sp. gr. of a substance

$= \dfrac{\text{density of the substance}}{\text{density of water at } 4^\circ\text{C.}}$, *i. e.* density of a substance = density of

water at 4°C. × sp. gr. of the substance. So, **the density of a substance in F.P.S. units (lbs. per ft.^{3}) is numerically equal to 62·5 × sp. gr. of the substance** For example, (i) the density of lead in F. P. S. units is

709 pounds per cu. ft., and the sp. gr. of lead $= \dfrac{709}{62{\cdot}5} = 11{\cdot}3.$

Again, (ii) the density of iron in C. G. S. units is 7·8 grms per c.c.,

and the Sp. gr. of iron $= \dfrac{\text{Mass of 1 c.c. of iron}}{\text{Mass of 1 c.c. of water}} = \dfrac{7{\cdot}8}{1} = 7{\cdot}8.$

And the density of iron in F. P. S. units is 487·5 lbs. per cu. ft.

Then, the Sp. gr. of iron $= \dfrac{\text{Mass of 1 cu. ft. of iron.}}{\text{Mass of 1 cu. ft. of water}} = \dfrac{487{\cdot}5}{62{\cdot}5} = 7{\cdot}8.$

(c) In C. G. S. units,

$$\text{Sp. gr.} = \frac{\text{density of the substance}}{\text{density of water at } 4^\circ\text{C.}}$$

$$= \frac{\rho.\ \text{gms./c.c.}}{1.\ \text{gm./c.c.}} = \rho.$$

In F. P. S. units,

$$\text{Sp. gr.} = \frac{\text{density of the substance}}{\text{density of water at } 4^\circ\text{C.}}$$

$$= \frac{62{\cdot}5 \times \rho\ \text{lbs./cu. ft.}}{62{\cdot}5\ \text{lbs./cu. ft.}}$$

$$= \rho.$$

Thus Sp. gr. is the same in both the units.

The relations between the two systems will be clear by the following table :—

System	Density	Specific gravity
Metric (or C. G. S.) ...	x gm. per c. c.	x
British (or F. P. S.) ...	$62{\cdot}5 \times x$ lb. per cu. ft.	x

Examples.—*1. The crown of Hiero weighed 20 pounds. Archimedes found that immersed in water it lost 1·25 pounds. The crown was made of gold and silver. Find the weights of these metals. (Sp. gr. of gold = 19·3 ; sp. gr. of silver = 10·5).*

Let w_1 be the wt. of gold, and w_2 that of silver, then $w_1 + w_2 = 20$ lbs.

The specific gravity of gold is 19·3, hence the density of gold = $(19{\cdot}3 \times 62{\cdot}5)$ lb. per cu. ft. (See Art. 114.). Similarly, the density of silver = $(10{\cdot}5 \times 62{\cdot}5)$ lb. per cu. ft.

The volume of gold $= \dfrac{w_1}{19{\cdot}3 \times 62{\cdot}5}$ cu. ft. ; The volume of silver

$$= \frac{w_2}{10{\cdot}5 \times 62{\cdot}5} \text{ cu. ft.}$$

$\therefore$ The total volume of the crown $= \left(\dfrac{w_1}{19{\cdot}3} + \dfrac{w_2}{10{\cdot}5}\right) \times \dfrac{1}{62{\cdot}5}$ cu. ft.

Now, the weight of the displaced water = 1·25 lbs. The volume of this water $= \left(1{\cdot}25 \times \dfrac{1}{62{\cdot}5}\right)$ cu. ft., and this must be equal to the volume of crown.

$$\text{Hence,} \quad \left(\frac{w_1}{19{\cdot}3} + \frac{w_2}{10{\cdot}5}\right) \times \frac{1}{62{\cdot}5} = 1{\cdot}25 \times \frac{1}{62{\cdot}5} \; ; \text{ or } \frac{w_1}{19{\cdot}3} + \frac{w_2}{10{\cdot}5} = 1{\cdot}25.$$

Also we have $w_1 + w_2 = 20$. From these two equations we get, $w_1 = 15{\cdot}078$ lbs., and $w_2 = (20 - 15{\cdot}078) = 4{\cdot}922$ lbs.

2. The mass of an alloy of copper and lead is 320 gm. ; the total volume is 30 c. c Find the volume of each component. (Sp. gr. of copper = 8·8 : sp. gr. of lead = 11·3)

Let x c. c. = volume of copper ; y c.c. = volume of lead.

$\therefore$ Mass of copper $= x \times 8{\cdot}8$ gm. ; mass of lead $= y \times 11{\cdot}3$ gm.

Hence, $x \times 8{\cdot}8 + y \times 11{\cdot}3 = 320$; and $x + y = 30$.

Solving these equations, we get, $x = 7{\cdot}6$: $y = 22{\cdot}4$;

$\therefore$ Volume of copper is 7·6 c.c., and volume of lead 22·4 c.c.

3. Find a mathematical expression for the density of a mixture, when the densities and the masses of the components are known.

Calculate the quantity of pure gold in 100 gms. of an alloy of gold and copper of density 16. (Density of gold = 19, and that of copper = 9). (*Dac. 1930.*)

Let m_1 and m_2 be the masses of the components and d_1 and d_2 their respective densities, and let d be the density of the mixture.

Then, volume of the mixture $= \dfrac{m_1 + m_2}{d} = \left(\dfrac{m_1}{d_1} + \dfrac{m_2}{d_2}\right) =$ Volume of the components, whence d can be calculated.

If m gms. be the mass of pure gold in the alloy, the mass of copper $=$ $(100-m)$ gms. Now, volume of the alloy $= \dfrac{100}{16}$; vol. of gold $= \dfrac{m}{19}$; and vol. of copper $= \dfrac{100-m}{9}$.

$$\therefore \frac{100}{16} = \frac{m}{19} + \frac{100-m}{9}, \text{ whence } m = 83·12 \text{ gms.}$$

4. A cylindrical tube one metre long and one centimetre in internal diameter weighs 100 gms. when empty and 150 gms. when filled up with a liquid. Find the specific gravity of the liquid. (*Pat. 1928*).

The wt. of the liquid $= 150 - 100 = 50$ gms.

The volume of the liquid = internal volume of the cylinder

$$= \tfrac{22}{7} \times (0·5)^2 \times 100 = 78·57 \text{ c.c.}$$

$\therefore$ The density of the liquid $= \dfrac{50}{78·57}$ gm. per c.c. $= 0·636$ gm. per c.c.

But the density of water is 1 gm. per c.c. ;

$$\text{Hence sp. gr. of the liquid} = \frac{0·636}{1} = 0·636.$$

115. Immersed and Floating Bodies.—

Let W represent the weight of a body immersed in a liquid. It will displace its own volume of the liquid of weight, say W'.

Then W' is the upward thrust, or buoyancy, which will act in opposite direction to W which is acting downwards.

(1) If $W > W'$, the body will sink.

(2) If $W = W'$, the body will float being wholly immersed anywhere in the liquid.

(3) If $W < W$, the body will float being partly immersed in the

liquid ; the volume of the body inside the liquid displaces a weight of the liquid equal to the weight of the whole body, that is,

a body floats when the weight of the displaced liquid
= the weight of the body.

115(a). Conditions of Equilibrium of a Floating body.-

1. The wt. of the floating body must be equal to the wt. of the liquid displaced.

2. The C. G. of the body and the C. G. of the displaced liquid (centre of buoyancy) must lie in the same vertical line which is called *the centre line* of the body. In general, the former is above the latter. For a completely immersed body, the former should be below the latter.

[**Note.**—The equilibrium of a floating body is stable or unstable, according as the *meta-centre* is above or below the C. G. of the body. In the case of a floating spherical body the two coincide and the body is said to be in neutral equilibrium. The C. G. of a ship is kept below the meta-centre by weighting the bottom of the ship with ballast and thereby stability is increased.

Meta-centre.—If a body floating in equilibrium in a liquid leans on one side, the C. G. of the body and the centre of buoyancy of the liquid are both displaced in the direction in which the body leans. The point where the vertical line through the new position of the centre of buoyancy intersects the centre line of the body, is called the meta-centre of the body].

115(b). Densities of Immersed and Floating bodies.—Let the density of a liquid be d_1, in which a body of density d_2 and volume V is placed. Then, when the body is totally immersed, the mass of liquid displaced $= d_1 V$. The mass of the body $= d_2 V$. Hence, (1) if $d_2 V > d_1 V$, *i.e.* if $d_2 > d_1$, the body will sink.

(2) if $d_2 = d_1$, the body will float being wholly immersed anywhere in the liquid.

(3) if $d_2 < d_1$, the body will float partially immersed. In this case, if v be the volume of the liquid displaced by the immersed part of the body, $d_1 v = d_2 V$; or $\dfrac{v}{V} = \dfrac{d_2}{d_1}$.

i.e. $$\frac{\text{Volume of the immersed part}}{\text{Total volume}} = \frac{\text{density of the body}}{\text{density of the liquid}}.$$

116. Illustrations of the Principle of Buoyancy of Liquids.—

(1) **Why Ice floats on water ?**—It is known that 1 gm. of ice at 0°C. occupies 1/0·92 or 1·09 c.c., the density of ice being 0·92 gm.

per c.c., but 1 gm. of water at 0°C. occupies very nearly 1 c.c. Hence 1 c.c. of water at 0°C. becomes 1·09 c.c. when turned into ice at the same temperature, that is, when water freezes into ice, it increases in volume by about 9 per cent., *i.e.* 11 volumes of water at 0°C. become about 12 volumes of ice at the same temperature.

Hence the density of ice will be diminished in the same proportion. So, from the above relation, we get

$$\frac{\text{Volume of ice under water}}{\text{Total volume}} = \frac{11}{12},$$ *i.e.* ice will float in water with $\frac{11}{12}$ of its volume below the surface and $\frac{1}{12}$ above it.

Note. A body which floats in one liquid may sink in another which is lighter. Thus iron floats on mercury but sinks in water, oil floats on water but sinks in alcohol, wax floats on water but sinks in ether.

(2) **Why an Iron Ship floats on water?**—It is a well-known fact that a solid block of iron readily sinks in water, because the density of iron is greater than that of water ; but the mystery why an iron ship floats lies in its construction, that is, in its hollow shape, and due to this concave shape a porcelain saucer can be made to float on water. When the ship enters water, the volume of water displaced is much greater than the volume of iron immersed, and, as a solid cannot displace more than its own weight of a liquid, the ship sinks in water until the weight of the displaced water is equal to the weight of the ship, that is, the ship is immersed to such a depth that the weight of the ship with its contents, (*i.e.* the engines, cargo and passengers, etc.), is balanced by the upward thrust, or the force of buoyancy, of the displaced water.

[The carrying capacity of a ship is determined by the tonnage and is found by taking the difference of the weights of the water displaced by the empty ship and the fully loaded ship. The weight of a big ship with its contents may come up to 65,000 tons, *i.e.* 65,000 tons of water will be displaced by the vessel when afloat. It should be remembered that the depth of immersion of a ship is less in sea-water than in fresh water because the density of sea-water is a little greater than that of a fresh water, and so, in order to obtain the same upthrust, a smaller volume of sea-water must be displaced. Thus a ship can carry much larger cargo on sea-water than on fresh water. Now-a-days, according to law, every ship must bear a mark, called the *Plimsoll line*, showing the limit up to which it is permitted to immerse in sea-water of normal density].

It is known that a piece of marble can be made to float when tied to a suitable piece of cork. Thus bodies heavier than water can

be made to float by being tied up to lighter bodies of suitable size. This explains the principle of *life-belts*, which are found in steamers and ships.

(3) **Swimming.**—It is an art of moving in water keeping the head out of the surface of water. Though volume per volume the human body is lighter than water and it will float, the head is heavier and so it tends to sink in water. The secret of swimming, therefore, lies in keeping the head out of water by the movement of limbs. It is much easier to swim in salt water than in fresh water, because the density of salt water being greater, less force is required to prevent the body from sinking.

Example.—*The weight of a big liner is given as 64,000 tons. What must be the volume of a floating dock which will be able to support it ? (Sp. gr. of sea-water = 1·025).*

The volume of the dock must be equal to the volume of sea-water weighing 64,000 tons, *i.e.* (64,000 × 22,40) lbs.

1 cu. ft. of pure water weighs 62·5 lbs.

$\therefore$ The wt. of 1 cu. ft. of sea-water = 62·5 × 1·025 lbs.

$$\therefore \text{ Volume required} = \frac{64000 \times 2240}{62{\cdot}5 \times 1{\cdot}025} = 2{,}237{,}846 \text{ cu. ft. (approx.)}$$

117. The Cartesian Diver.—This is a hydrostatic toy invented by the great French philosopher Descartes (1596—1605) to illustrate the principle of Archimedes. The principles of equilibrium of a body floating in a liquid, transmission of fluid pressure, and compressibility of gases are demonstrated by it.

The diver is usually a small hollow doll having a tubular tail open at the end communicating with the inside (Fig. 67). In some cases the doll is solid and is attached to a hollow ball having a small opening at the bottom (shewn on the right of the jar), so that the two together can float in equilibrium.

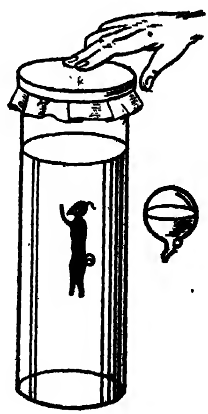


Fig. 67.

The diver is kept in a tall jar which is nearly filled with water. The top of the jar is closed air-tight by means of a sheet of rubber. The diver is partly filled with air and partly with water, the total mass being slightly less than the mass of water displaced, the diver floats partly immersed.

On pressing the rubber sheet by means of the fingers, the diver is seen to sink down and rise up again on releasing the pressure. By regulating the pressure, the diver may be kept stationary at any depth.

Explanation.—When the rubber sheet is pressed, vol. of the air enclosed on the water surface is diminished whereby the pressure of the air is increased. This pressure gets transmitted through water to the air inside the diver. As a result, the volume of the enclosed air is reduced and so an additional quantity of water enters into the diver through the opening whereby the diver is rendered heavier than the displaced water and so it sinks. When the pressure on the rubber sheet is released, the air inside the diver expands driving out the additional water and the diver is rendered lighter and so it rises up again.

If it were possible to make the diver sink to such a depth that the liquid pressure at that depth is too great for the inside air to expand adequately on the release of pressure, the diver will not rise up again.

Most *fishes* have an *air-bladder* below the spine, which they can compress or dilate at pleasure and thus can either rise up or sink in water.

118. The Submarine.—The principle of the 'Cartesian Diver' is applied to the submarine (Fig. 68). It can float on the surface of the sea like an ordinary ship or sink when necessary. It is supplied with large ballast tanks *T* both in the stern and bow, which can be filled with water or with air from compressed air reservoirs within the ship. When water is taken into the tanks (which are provided with trap-door), the weight of the boat is so increased as to make the vessel sink, and when the water is pumped out of the tanks by pumps worked by compressed air, the ship is made so light that it rises to the surface. Thus, by emptying or filling the tanks, the mass of the ship is so varied and controlled that the ship is made to rise or sink as desired.

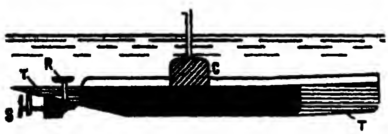


Fig. 68.—Submarine.

The act of filling or emptying the tanks can be done very quickly. Moreover, the ship can be kept steady at any depth by the help of the vertical rudder R and other horizontal rudders not shown in the figure. The *Conning tower* C in which a periscope is fitted always projects above the surface of water so that objects plying on the surface of the water may be viewed from within the boat.

119. (*a*) **Density of Ice.**—The density of ice can be determined by preparing a mixture of water and alcohol in such a proportion that when a piece of ice is placed in it, the ice will neither sink nor float but will remain anywhere within the liquid being completely

immersed. The density of ice is then equal to that of the liquid mixture, and it can be found by means of a hydrometer (see Art. 120). Its value is about 0·92 gm. per c.c.

(*b*) **Density of Wood, Wax, etc. by Flotation.**—Take a rectangular block of wood, whose length is l cm. and whose area of cross-section is α sq. cm. (Fig. 69).

∴ The volume of the block $= l\alpha$ c.c., and the weight of the block $= l\alpha \times d$ grams, where d is the density of wood ... ... (1)

Float the block vertically in water, and measure the depth (h cm.) to which it sinks. Then the volume of water displaced $= h\alpha$ c.c.

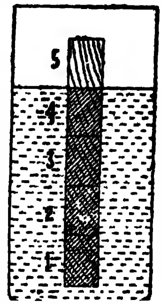


Fig. 69

∴ The weight of the displaced water $= h\alpha$ grams (∵ the density of water is 1 gram per c.c.) and this is equal to the weight of the block, according to the law of flotation, $l\alpha d = h\alpha$

$$\text{or} \qquad d = \frac{h}{l} = \frac{\text{length immersed in water.}}{\text{total length}}$$

N.B. *The method applies to other materials also, such as wax etc., which can be cut in the form of rectangular-blocks. Only a metre scale is sufficient and a balance is unnecessary in this method.*

Example—1. *A hollow spherical ball has an internal diameter of 10 cm. and an external diameter of 12 cm. It is found just to float in water. Find the density of the material of the ball. (The volume of a sphere varies as the cube of the diameter).* *(C. U. 1928 ; Dac. 1933.)*

Let V = volume of the sphere, and d = diameter of the sphere.

Then $V \propto d^3$; ∴ $V = kd^3$, where K is a constant.

The internal volume of the hollow ball $= K\,(10)^3 = 1000\,K$ c.c., and the external volume $= K(12)^3 = 1728\,K$ c.c.

∴ The volume occupied by the actual material of the ball $= 1728\,K - 1000K = 728\,K$ c.c.

As the ball is found just to float in water, the mass of the ball = the mass of the displaced water = volume of the displaced water × density of water = $1728K \times 1 = 1728\,K$ gm.

∴ The density of the material of the ball

$$= \frac{\text{mass of the ball}}{\text{volume occupied by the actual material of the ball}}$$

$$= \frac{1728\,K}{728\,K} = 2{\cdot}37 \text{ gm. per c.c.}$$

2. *A cylinder of iron of specific gravity 7·86 and volume 200 c.c. floats on mercury. Calculate the volume of mercury displaced. Calculate also the volume of mercury displaced by the iron, when water is poured on the top of mercury to cover the iron completely.* (*Sp. gr. of mercury* = 13·6).

If V be the volume of mercury displaced in the first case, we have, mass of mercury displaced = mass of iron ; or $V \times 13·6 = 200 \times 7·86$; $\therefore$ $V = 115·59$ c.c.

If V' be the volume of mercury displaced in the second case,

the volume of water displaced = $(200 - V')$c.c.

So the mass of water displaced = $(200 - V') \times 1$ gm. ; and

mass of mercury displaced = $(V' \times 13·6)$ gm. ; mass of iron = $200 \times 7·86$ gm.

We have, mass of mercury displaced + mass of water displaced = mass of iron

or $$(V' \times 13·6) + (200 - V') \times 1 = 200 \times 7·86 \;;$$

$$\therefore \quad 126V' = 200(7·86 - 1) = 200 \times 6·86 \;; \;\therefore\; V' = 108·9 \text{ c.c.}$$

3. *A block of wood of specific gravity 0·85 floats in water. Some kerosene of specific gravity 0·82 is poured on the surface of water until the wooden block is completely immersed. Calculate the fraction of the block lying below the surface of water.*

Let V be the volume of the block in the kerosene and V' the volume below the water surface.

So the total volume of the block = $V + V'$. $\therefore$ Wt. of block = $(V + V') \times 0·85$

The upthrust in kerosene = wt. of V c.c. of kerosene = $0·82\ V$ gm.

The upthrust in water = wt. of V' c.c. of water = V' gm.

$\therefore$ Total upthrust = $(0·82V + V')$ = wt. of the block = $(V + V') \times 0·85$

$$\therefore \quad 0·82V + V' = 0·85V + 0·85V'$$

$$\text{or} \quad \frac{V'}{V} = \frac{1}{5} \;;\text{ or } \frac{V'}{V'+V} = \frac{1}{1+5} = \frac{1}{6}.$$

Hence 1/6 of the block is below the water surface.

4. *A body of density δ is dropped gently on the surface of a layer of liquid of depth d and density δ' (δ' being less than δ). Show that it will reach the bottom of the liquid after a time* $\sqrt{\dfrac{2d\delta}{g(\delta - \delta')}}$, *g being the acceleration due to gravity.* (*Pat.* 1931)

If m be the mass of the body, the volume of the body = m/δ, which is also the volume of the displaced liquid. So the weight of the displaced liquid

$$= \left(\frac{m}{\delta} \times \delta'\right)g, \text{ which is the upthrust acting on the body.}$$

The force tending to bring the body down in the liquid = mg, the weight of the body, and the upthrust is $mg\ (\delta'/\delta)$. Hence the resultant downward force = $mg(1 - \delta'/\delta)$ = mass $m \times$ acceleration f with which it is going down the liquid,

$$\therefore \quad f = g\left(\frac{\delta - \delta'}{\delta}\right).$$

But we know that if d be the distance travelled in time t, $d=\frac{1}{2}ft^2=\frac{1}{2}g\left(\frac{\delta-\delta'}{\delta}\right)t^2$;

$$\text{or} \qquad t^2=\frac{2d\delta}{g(\delta-\delta')}; \qquad \therefore \ t=\sqrt{\frac{2d\delta}{g(\delta-\delta')}}.$$

5. *A body of of specific gravity 2·505 is dropped gently on the surface of a salty lake (sp. gr. 1·025) If the depth of the lake be a quarter of a mile, find the time the body takes to reach the bottom.* **(Pat. 1941).**

If m be the mass of the body, the volume of the body $=m/2\cdot505=$ the volume of the displaced liquid.

So, the weight of the displaced liquid $=\left(\frac{m}{2\cdot505}\times1\cdot025\right)g$.

$=$ the buoyancy, or the upthrust, acting on the body.

The force tending to bring the body down in the liquid $=mg$, the wt. of the body.

Hence the resultant downward force

$$=mg-\left(\frac{m}{2\cdot505}\times1\cdot025\right)g=mg\left(1-\frac{1\cdot025}{2\cdot505}\right)=mg\times\frac{1\cdot48}{2\cdot505}$$

$=mg\times$ acceleration f with which the body is going down the liquid.

$\therefore f=g\times\frac{1\cdot48}{2\cdot505}$. But if d be the distance travelled in time t, $d=\frac{1}{2}ft^2$ (the initial velocity u being zero.) ;

$$\text{Or} \quad t^2=\frac{2d}{f}=\frac{2\times(440\times3)\times2\cdot505}{1\cdot48\times g}=\frac{4468\cdot59}{g}; \text{ or } t=\frac{66\cdot8}{\sqrt{g}}\text{sec.}$$

6. *A toy man weighs 150 gms The density of man is 1·12, that of cork 0·24, and that of water 1. What weight of cork must be added to the man that he may just float in water.*

Let W be the wt. of cork required, then volume of cork $=\frac{W}{0\cdot24}$c.c.

Volume of man $=\frac{150}{1\cdot12}$ c.c. $\therefore$ Their total volume $=\left(\frac{150}{1\cdot12}+\frac{W}{0\cdot24}\right)$c.c.

The man and cork will just float in water when their total weight is equal to the weight of water displaced by them.

Hence the volume of displaced water $=\frac{150+W}{1}$ c.c.,

and this is equal to the total volume of man and cork.

$$\therefore \quad \frac{150}{1\cdot12}+\frac{W}{0\cdot21}=150+W; \qquad or \quad W=5\cdot075 \text{ gms.}$$

7. A cylider is 2 ft. high and the radius of the base is 3 ft.; its specific gravity is 0·7. It floats with its axis vertical. Find (a) how much of its axis will be under water, (b) the force required to raise it 1 inch? (*Pat. 1930*)

(*a*) Volume of the cylinder $= \frac{22}{7} \times (3)^2 \times 2 = \frac{396}{7} = 56·571$ cu. ft.

Sp. gr. of the substance of the cylinder $= \frac{\text{wt. of 1 cu. ft. of the substance}}{62·5} = 0·7$;

$\therefore$ Mass of 1 cu. ft. of the cylinder $= 62·5 \times 0·7$;

$\therefore$ Mass of the cylinder $= 62·5 \times 0·7 \times 56·571$ lbs.

This is equal to the mass of water displaced by the cylinder.

$\therefore$ Volume of water displaced $= \frac{62·5 \times 0·7 \times 56·571}{62·5}$

$= 0·7 \times 56·571$ cu. ft. = volume of the cylinder under water.

Area of the base of the cylinder $= \pi \times 3^2 = \frac{22}{7} \times 9 = \frac{198}{7} = \frac{56·571}{2}$ sq. ft.

$\therefore$ Length of the axis immersed $= (07· \times 56·571) \div \frac{56·571}{2} = 1·4$ ft.

(*b*) Force exerted by the cylinder = wt. of the cylinder

$= 62·5 \times 0·7 \times 56·571$ lb.-wt.

When the cylinder is raised 1 inch, *i.e.* when $\left(1·4 - \frac{1}{12}\right)$ or $\frac{7·9}{6}$ ft. of its axis is under water, the buoyancy of water = wt. of the water displaced $= 62·5 \times$ vol. water displaced in cu. ft. $= 62·5 \times$ vol. of cylinder immersed

$$= 62·5 \times \left(\frac{56·571}{2} \times \frac{7·9}{6}\right) \text{ lbs.-wt.}$$

But the buoyancy when the cylinder was floating with 1·4 ft. under water $= 62·5 \times 56·571 \times 0·7$ lbs.-wt. $\therefore$ The force required to raise the cylinder by 1 inch

$$= 62·5 \times 56·571 \times 0·7 - 62·5 \times \frac{56·571}{2} \times \frac{7·9}{6}$$

$$= 62·5 \times 56·571\left(0·7 - \frac{7·9}{12}\right) = 147·32 \text{ lbs.-wt.; or } = (147·32 \times 32) \text{ poundals.}$$

8. A solid displaces $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of its volume respectively when it floats in three different liquids. Find the volume it displaces when it floats in a mixture formed of equal volumes of the aforesaid three liquids. (*Pat. 1943*)

Let V be the volume and d the density the solid, and let d_1, d_2, and d_3, be the densities of the three liquids.

When a body floats, its volume × its density = the volume immersed × density of the liquid. ∴ We have, (*i*) $Vd = V/2 \times d_1$; or $d_1 = 2d$. (*ii*) $Vd = V/3 \times d_2$; or $d_2 = 3d$; (*iii*) $Vd = V/4 \times d_3$; or $d_3 = 4d$.

The mixture is formed by taking, say, v c.c. of each liquid, then the total volume $= 3v$ c.c. and its total mass $= v(d_1 + d_2 + d_3) = v(2d + 3d + 4d) = 9vd$.

$$\text{Density of the mixture} = \frac{\text{mass}}{\text{volume}} = \frac{9vd}{3v} = 3d.$$

Now, if x c.c. be the volume of the mixture displaced, we have, $Vd = x \times 3d$; or $x = \frac{V}{3}$, *i.e.* it displaces $\frac{1}{3}$ of its volume.

9. *A cylinder of wood, whose specific gravity is 0·25, has another cylinder of metal (specific gravity 8·0) attached to one end. The cylinders are 2 in. in diameter, they have the same axis, and are respectively 20 in. and 1 in. long. If the whole is placed in water, find how much of it will be above the surface.* (*C. U. 1935*).

Volume of wood $= \pi \times 1^2 \times 20 = 20\pi$ cu. in. ; Volume of metal $= \pi \times 1^2 \times 1 = \pi$ cu. in. ;

Their total volume $= 20\pi + \pi = 21\pi$ cu. in ; Sp. gr. of wood $= 0·25$;

∴ Mass of 1 cu. ft. of wood $= (62·5 \times 0·25)$ lb. Hence mass of 20π cu. in. of wood $= \left(\frac{20\pi}{1728} \times 62·5 \times 0·25\right)$ lb. ; So mass of metal $= \left(\frac{\pi}{1728} \times 62·5 \times 8\right)$ lb.

Their total mass $= \left\{\frac{\pi \times 62·5}{1728}(5 + 8)\right\}$ lbs.

This is equal to the mass of the displaced water, whose volume $= \pi \times 1^2 \times h \times d$ $= \left(\frac{\pi}{1728} \times h \times 62·5\right)$; ∴ $\frac{\pi \times 62·5}{1728} \times h = \frac{\pi \times 62·5}{1728} \times 13$; or $h = 13$ inches.

Hence the height above the surface $= 21 - 13 = 8$ in.

120. Principle of Hydrometer.—Different methods for the accurate determination of the specific gravity of a liquid are given in Art. 122, but for commercial purposes none of these methods is quite suitable. For these purposes we need a method which should be simple and quick although the results may not be very accurate. Thus instruments, called *Hydrometers*, are used, which depend upon the principle of flotation, *i.e.* when a body floats in a liquid the weight of the body is equal to the weight of the displaced liquid. There are two types of hydrometers in use. One is called the *Variable Immersion* (or constant weight) type; and the other the *Constant*

Immersion (or Variable weight) type. In the former case, the volumes of liquids displaced vary in different liquids, while in the latter volumes are kept constant. In practice, however, the Variable Immersion type is used.

The Variable Immersion Hydrometer.—The principle of this type of hydrometer may be understood by taking an ordinary flat-bottomed uniform test tube T (Fig. 70) and loading it with a suitable amount of sand or lead shot so that it floats vertically in a liquid. Paste inside the test tube a strip of mm. squared paper, marked off in centimetres measured from the bottom, and close the tube with a cork. Now float the tube in a jar of water and observe the depth immersed (d_1). Take out the tube, wipe it and float it in a jar of a different liquid. Again observe the depth immersed (d_2).

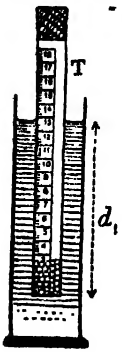


Fig. 70

Let W be the weight of the hydrometer, *i.e.* the test tube with load, a, its area of cross section, and d the density of the liquid, then, since the weight of the hydrometer is equal to the weight of the displaced liquid in each case,

we have, $W = a \times d_1 \times 1 = a \times d_2 \times d$, or $d/1$ (or sp, gr.) $= d_1/d_2$. Since d_1/d_2 is a ratio, the depths can be measured either in inches or in centimetres.

The experiment can be varied with different amount of lead shot in the tube and a graph can be drawn with h_1 and h_2. [Vide Art. 122 (2)]

Examples.—*1. The density of sea-water is 1 025 gm. per c.c., and the density of ice is 0 917 gm. per c.c. Find what portion of an ice-berg is visible above the water surface, when it is in sea-water, and when in fresh water.*

Let v be the volume of ice immersed, and V the total volume of ice.

Then $(V-v)$ is the volume which is visible above the surface of the sea.

We have, from Art.115(b), $\dfrac{v}{V} = \dfrac{\text{density of ice}}{\text{density of sea-water}} = \dfrac{0{\cdot}917}{1{\cdot}025}$;

or
$$1 - \frac{v}{V} = \frac{V-v}{V} = \frac{1{\cdot}025 - 0{\cdot}917}{1{\cdot}025} = \frac{0{\cdot}108}{1{\cdot}025} = \frac{1}{9{\cdot}5}.$$

Therefore the portion of the ice-berg which is above the water surface is $1/9{\cdot}5$ of the total volume.

In fresh water, the density of which is 1 gm. per c.c., we have,

$$\frac{V-v}{1}=\frac{1-0{\cdot}917}{1}=\frac{0{\cdot}083}{1}=\frac{1}{12}\text{ (approx.)}$$

2. *A variable immersion hydrometer is prepared by taking a test-tube about 15 cms. long and 3 cm. wide. The test-tube which is assumed to have a uniform cross-section is weighted with a few lead shots to make it float upright. A narrow piece of graph paper is pushed into the test tube to serve as a scale. The tube is then placed in glycerine of specific gravity 1·25 and then it is placed in water. The scale reading, which increases upwards, is 1·6 cm. for the level of the glycerine surface and 2·8 cm. for the level of the water surface. The scale reading, when the test-tube is placed in a solution of copper sulphate, is 2·5 cm. What is the specific gravity of the latter ?*

Let V be the volume of the portion of the test-tube below the zero mark and V' the volume of 1 cm. of the tube. Then

In glycerine the immersed volume $=(V+1{\cdot}6\ V')$c.c.

$\therefore$ The upthrust in glycerine = wt. of this volume $=1{\cdot}25\ (V+1{\cdot}6\ V')$

Similarly the upthrust in water $=1\times(V+2{\cdot}8V')$

Since each upthrust = wt. of the test-tube ; $V+2{\cdot}8\ V'=1{\cdot}25\times(V+1{\cdot}6V')$

or $0{\cdot}25\ V=0{\cdot}8\ V'$; or $V=3{\cdot}2V'$. Again, if S be the sp. gr. of the copper sulphate solution, $S(V+2{\cdot}5V')=(V+2{\cdot}8V)$

$$\text{or}\qquad S=\frac{V+2{\cdot}8V'}{V+2{\cdot}5V'}=\frac{3{\cdot}2V'+2{\cdot}8V'}{3{\cdot}2V'+2{\cdot}5V'}=\frac{6}{5{\cdot}7}\quad\therefore\quad S=1{\cdot}05.$$

(**Note** that this example explains the principle of preparing and graduating a variable immersion type of hydrometer.)

3. *The stem of a common hydrometer is cylindrical, and the highest graduation corresponds to a specific gravity 1·0 and the lowest to 1·3. What specific gravity corresponds to a point exactly midway between these divisions ?* (*Pat. 1944*)

Let l be the total length of the stem from the lowest to the highest graduation, a the area of cross-section of the stem, V the volume of the bulb up to the lowest graduation and W the weight of the hydrometer. Then

$(\text{V}+la)\times1=W$, and $\text{V}\times1{\cdot}3=W$; ... ... (1)

$\therefore\ \ \text{V}+la=1{\cdot}3\text{V}$; or $0{\cdot}3\text{V}=la$ ... ... (2)

Again, $(\text{V}+l/2\times a)\ S=W$, where S is the required sp. gr.

or from (1) and (2), $(\text{V}+\frac{0{\cdot}3}{2}\text{V})S=1{\cdot}3\text{V}$; or $S=\frac{2{\cdot}6}{2{\cdot}3}=1{\cdot}13$.

121. Sp. Gr. of Solids.

(1) By Direct Measurement—In the case of a solid having some regular form (*e.g.* spherical or cylindrical), the volume of the solid can be calculated by measuring its linear dimensions. The body is

then weighed. Let the weight of the body be W gm., and let its volume be V c. c., then

density of the body $= W/V$ gm, per c.c.,
and sp. gr. ,, $= W/V$.

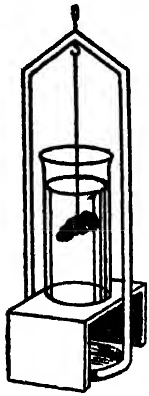


Fig. 71

(2) By Hydrostatic Balance.—

(a) Solid heavier than water.

Let the weight of the solid in air $= W_1$ gm.
and the wt. in water $= W_2$ gm.

To take the weight of the body in water, it is suspended by means of a fine thread from the hook of the left pan and made to sink completely in water contained in a beaker. The beaker is placed on a small wooden bridge, which is put across the pan in such a way that the bridge, or the beaker, does not touch any part of the pan of the balance (Fig. 71).

The weight of the same volume of water $= (W_1 - W_2)$ gm.

$$Sp.\ gr. = \frac{wt.\ of\ the\ body\ in\ air}{wt.\ of\ an\ equal\ volume\ of\ water} = \frac{W_1}{W_1 - W_2}.$$

(b) Solid lighter than water.

Let the weight of the solid in air $= W_1$ gm.

Take another heavy body, called a *sinker*, such that two tied together will sink in water.

Let the weight of the solid and sinker both in water $= W_2$ gm.
and the weight of the sinker alone in water $= W_3$ gm.

∴ The weight of solid in air + the weight of sinker in water— the weight of solid and sinker in water = the weight of water whose volume is the same as that of the solid $= (W_1 + W_3 - W_2)$ gm.

Hence, $$\text{Sp. gr.} = \frac{W_1}{W_1 + W_3 - W_2}.$$

Otherwise thus :—

wt. of the solid in air $= W_1$

wt. of solid in air + sinker in water $= W_2$

wt. of solid and sinker both in water $= W_3$

∴ wt. of displaced water $= W_2 - W_3$

Hence, $$\text{Sp. gr.} = \frac{W_1}{W_2 - W_3}$$

(c) **Solid soluble in water.**—The specific gravity of a solid soluble in water can be found by immersing the solid in a liquid of known specific gravity, but in which the solid is insoluble.

Determine the specific gravity of the solid relative to the liquid. *Then the actual specific gravity of the solid will be obtained by multiplying this value with the specific gravity of the liquid.* For, we have,

$$\text{Sp. gr. of the solid} = \frac{\text{weight of solid in air}}{\text{weight of the same volume of water}}$$

$$= \frac{\text{weight of solid in air}}{\text{weight of the same volume of liquid}} \times \frac{\text{weight of the same volume of liquid}}{\text{weight of the same volume of water}}$$

$$= \frac{\text{weight of solid in air}}{\text{weight of the same volume of liquid}} \times \text{sp. gr. of the liquid} = \frac{W_1}{W_1 - W_2} \times \rho,$$

where W_1 = wt. of solid in air ; W_2 = wt. of solid in the given liquid ; ρ = sp. gr. of the liquid.

(3) **By Specific Gravity Bottle.**—It is a glass bottle fitted with a ground glass stopper having a narrow central bore. The bottle is filled to the top of the neck with any liquid, and the surplus liquid overflows through the hole in the stopper when the stopper is pushed into its position (Fig. 72). Shake the bottle to remove air bubbles. The bottle holds a definite quantity of liquid. *This bottle is used to find out the specific gravity of a solid in the form of powder or small fragments, and of liquids also.*

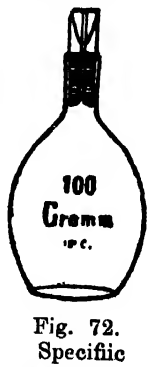


Fig. 72. Specific gravity bottle.

Let the weight of the empty bottle = W_1 gm.

The weight of the bottle + powder put inside = W_2 gm.

∴ The weight of the powder = $(W_2 - W_1)$ gm.

The weight of the bottle + powder + water to fill the bottle = W_3 gm.

Now throw out the contents of the bottle and fill it up with pure water taking care to drive out air bubbles from inside.

Let the weight of the bottle full of water $= W_4$ gm.

Then the weight of an equal volume of water

$$= (W_4 - W_1) - (W_3 - W_2) \text{ gm.}$$

$$\text{Hence, Sp. gr.} = \frac{W_2 - W_1}{(W_4 - W_1) - (W_3 - W_2)}.$$

N. B. To determine the specific gravity of a powder **soluble in water,** a liquid is taken in which the solid does not dissolve or chemically act. Then the Sp. gr. so found is multiplied by the Sp. gr. of the liquid at the observed temperature.

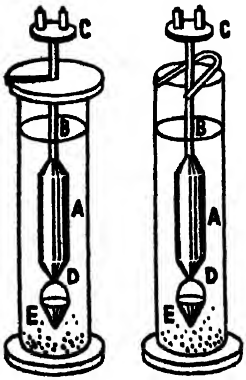


Fig. 73.—Nicholson's hydrometer

(4) By Nicholson's Hydrometer—This is a constant immersion type hydrometer.

It consists of a cylindrical hollow vessel *A* to which is attached a thin stem *B* at the top of which there is a small scale-pan C (Fig.73). Below the vessel is attached, by the curved metallic hook *D*, a conical pan which is so weighted with lead shots or Hg. that the hydrometer may float vertically in a liquid. There is a scratch mark on the stem up to which the instrument is always made to sink in the liquid. The hydrometer is placed in water contained in a glass cylinder. A slotted card board (left hand figure) or a bent wire (right hand figure) is so placed across the mouth of the cylinder that the upper pan is arrested before sinking into water in the cylinder. All the points in the hydrometer are air tight. Weights are placed on the upper pan of the hydrometer to make it sink upto the mark on the stem. Let the weight be W_1 gm.

Remove the weights and place the solid on the upper pan. Add weights again on the upper pan to make the instrument sink to the mark. Let it be W_2 gm. Then the weight of the body in air $= (W_1 - W_2)$ gm.

Now remove the weights and place the body in the lower pan, which is in water.

Again, find the weights necessary to bring the hydrometer up to the mark. Let this weight be W_3 gm.

Then the weight of the body in water $= (W_1 - W_3)$ gm.

The weight of displaced water $=(W_1 - W_2) - (W_1 - W_3)$ gm.
$=(W_3 - W_2)$ gm.

$$\therefore \quad \text{Sp. gr.} = \frac{W_1 - W_2}{W_3 - W_2}.$$

[**Note.**—It is evident that the method depends on Archimedes' principle.

If the solid be lighter than water, tie it on to the lower pan and proceed exactly as above]

122. Specific Gravity of Liquids.—

(1) By Balance.—

Let the weight of a solid body, which is heavier than the liquid but which is not chemically acted upon by it $=$ W_1 gm., and the weight of the solid immersed in water $= W_2$ gm. ; and that in liquid $= W_3$ gm.

Then $(W_1 - W_3)$ represents the weight of a volume of liquid equal in volume of the solid ; and $(W_1 - W_2)$ is the weight of the same volume of water.

$$\therefore \quad \text{Sp gr.} = \frac{W_1 - W_3}{W_1 - W_2}.$$

(2) By Common (or Variable Immersion) Hydrometer—This is a glass instrument (Fig. 74) which floats vertically in different liquids with parts of the stem above the surface of the liquid. In order that the instrument may float vertically, the small lower bulb *B* contains mercury or lead shot. The weight of the liquid displaced by the hydrometer is equal to the weight of the hydrometer itself, which is always constant. But mass $=$ volume $\times$ density ; hence mass being constant, volume is inversely proportional to density. So the volume of the liquid displaced increases as the density of the liquid diminishes, hence it sinks deeper into the lighter liquid than into the heavier one. The stem *S* can thus be graduated so that the specific gravity of a liquid can be read off directly. The number of the division on the scale fixed in the tube, which is in level with the surface of the liquid, gives the specific gravity of the liquid.

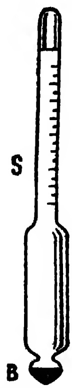


Fig. 74—Common hydrometer.

(a) Graduation of a Common Hydrometer.—To graduate the instrument float it in water and put a mark on the stem which is in line with the surface of the liquid, and similarly put another mark on the stem when it is floated in another liquid of known density (d). Let the lengths of the stem exposed above the surface of the liquid in

the two cases be l_1 and l_2 respectively, Then, if W be the weight and V the volume of the whole instrument, and a the area of cross-section of the stem, we have

$$W=(V-l_1a)\times 1=(V-l_2a)\times d, \text{ the density of water being } 1,$$

$$\therefore\ l_1=\frac{1}{a}(V-W), \text{ and } l_2=\frac{1}{a}\left(V-\frac{W}{d}\right);\ \text{or } (l_2-l_1)=\frac{W}{a}\left(1-\frac{1}{d}\right).$$

Now, if l be the length of the stem exposed in a liquid of density d', we have

$$(l-l_1)=\frac{W}{a}\left(1-\frac{1}{d'}\right);\qquad \therefore\ \frac{(l-l_1)}{(l_2-l_1)}=\frac{1-1/d'}{1-1/d}.$$

For different values of d' the corresponding values of l can be calculated from the above relation and the instrument can thus be graduated.

It is so graduated that, when the hydrometer is put into water, the scale reading is 1000, which means sp. gr. = 1·000. In another liquid it might be 1210, *i.e.* the sp. gr. = 1·21.

This type of hydrometer is generally used in different industries for finding the densities of liquids, and these hydrometers are named according to the use to which it is put ; for example, it is called a **lactometer** when it is used to find the density of milk (which is generally between 1·029 and 1·033), an **alcoholimeter** when used to find the density of alcohol, and a **sacharometer** to determine the sugar content of a solution.

The determination of density by means of a lactometer, however, is not a conclusive test as to the *purity of the milk*, for the density of skimmed milk is less than that of unskimmed milk, so by adding water to the skimmed milk the density can be brought to its normal value. So the amount of fat should be determined along with density in order to test the quality of the milk.

(3) **By Nicholson's Hydrometer.**—

In this experiment the principle that a floating body displaces its own weight of the liquid in which it is floating is utilised by immersing the hydrometer each time up to the same index mark in the liquid and in water.

Let the weight of the hydrometer = W_1 gm.

It is then floated in the liquid contained in a glass cylinder and weights are added on the upper pan to make it sink up to the mark. Let this weight = W_2 gm.

$\therefore$ The total weight of the displaced liquid $=(W_1+W_2)$ gm.

Similarly, let the weight required on the upper pan to bring it up to the mark when placed in water $= W_3$ gm.

$\therefore$ The weight of the displaced water, whose volume is the same as that of the displaced liquid $=(W_1+W_3)$ gm.

$$\therefore \quad \text{Sp. gr.} = \frac{W_1+W_2}{W_1+W_3}.$$

(*a*) **Alternative Method (without using a Balance).**—A piece of solid is taken which is not soluble in the liquid and also which will not react chemically with it.

Let the weight required on the upper pan to sink the hydrometer in water up to index mark when the solid is placed on the upper pan $= W_1$

The solid is then placed in the lower pan and let the wt. required to sink the instrument up to the mark $= W_2$

Then $(W_2 - W_1) =$ wt. of the same volume of water as that of the solid $=$ volume of the solid ($\because$ Sp. gr. of water $=1$)

Similarly let W_3 and W_4 be the corresponding weights when the above operations are repeated in the given liquid, then

$(W_4 - W_3) =$ wt. of the same volume of the liquid as that of the solid.

$$\therefore \quad \text{Sp. gr. of liquid} = \frac{W_4 - W_3}{W_2 - W_1}.$$

(4) **By Specific Gravity Bottle.**—

Let the wt. of empty bottle $= W_1$ gm.

It is then filled completely with water and weighed. Let this weight be W_2 gm.

The bottle is emptied out and carefully dried. It is then filled with the liquid. Let the weight be W_3 gm. Then

$$\text{Sp. gr.} = \frac{W_3 - W_1}{W_2 - W_1}.$$

(5) **By Balancing Columns.** (*U-tube*)—The densities of two different liquids, which do not mix, nor have any chemical action with

each other, can be determined by pouring them one after another in a U-tube.

Take a U-tube of glass and pour first the heavier of the two liquids taken, (say mercury and water), and note that the liquid (mercury) attains the same level in both the limbs (Fig. 75). Now carefully pour some water into the left-hand limb. The weight of water pushes the mercury down in the left limb and up in the right limb. Let C be the common surface of separation of mercury and water. Consider the horizontal level AC. The pressures at these two points must be equal because the liquids are at rest, and so the two columns AD and CL are called *balancing columns*.

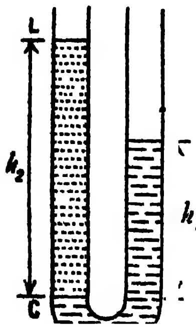


Fig. 75—Balancing columns.

Now, press. at A = force exerted on unit area at A
= P + wt. of the column AD of 1 sq. cm. base
= P + volume of the column AD of 1 sq. cm. base × density × g
= $P + h_1 \times \rho_1 \times g$; where P = atmospheric pressure, $h_1 = AD$, ρ_1 = density of mercury, and g is the acceleration due to gravity.

Similarly, press. at $C = P + h_2\rho_2 g$;
where $h_2 = CL$, and ρ_2 = density of water.

$$P + h_2\,\rho_2\,g = P + h_1\,\rho_1\,g\ ; \quad or,\ h_2\,\rho_2 = h_1\,\rho_1\ ;$$

$$or,\ \frac{h_2}{h_1} = \frac{\rho_1}{\rho_2}.$$

That is, *the heights of balancing columns are inversely proportional to the densities of the liquids.*

In this case, ρ_1/ρ_2 is the ratio of the density of the liquid (mercury) to the density of water, *i.e.* it is the sp. gr. of the liquid.

(6) **By Hare's Apparatus.**—The above U-tube method can be applied when the *liquids do not mix*, but the liquids which will mix must be kept separate, and in that case the following method, which is merely a modification of the U-tube method described above, can be adopted. By this method the relative densities of two liquids can be determined by *balancing* two liquid columns against each other.

Hare's apparatus consists of two parallel vertical tubes M and N connected at the top by a three-way tube A fitted with a piece of india-rubber tubing B and a clip P (Fig. 76). It is merely an inverted U-tube with a side tube at the top. The lower end of each tube is dipped in a liquid contained in a beaker x or y. The liquids are drawn up to suitable heights by suction through C and are kept steady by pressing the clip P. Generally water is taken as one of the liquids.

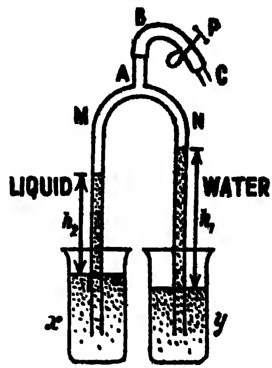


Fig. 76.—Hare's Apparatus

Let h_1 and h_2 be the heights of the liquid columns having densities ρ_1 and ρ_2 respectively. The height in each case is measured from the surface of the liquid up to the lower meniscus of the top. Let,

P = the atmospheric pressure.

p = the pressure of air inside the tube.

The pressure at $A = P = g\,\rho_1\,h_1 + p$.

The pressure at $B = P = g\,\rho_2 h_2 + p$;

$\therefore\quad p + g\,\rho_1\,h_1 = p + g\,\rho_2\,h_2$;

or $\quad h_1\,\rho_1 = h_2\,\rho_2$;

or $$\frac{h_1}{h_2} = \frac{\rho_2}{\rho_1}.$$

That is, *the densities are inversely as the heights of the liquid columns.*

Knowing one of these, the other is known.

Note. (i) It is to be noted that though cross-sections of the tubes do not come into consideration, the tubes should be of moderately wide bore in order to avoid the effects of surface tension. If, however, there is any rise of the liquid column due to capillarity, this should measured and subtracted from the corresponding height. (ii) Both the tubes may not be of the same bore, as pressure depends only on the vertical height. (iii) It should be tested whether the tubes are vertical. (iv) Take the heights after the liquid columns are steady which will not be the case if the apparatus is not air tight. (v) Draw a graph with h_1 and h_2 (which should be a straight line) and calculate h_1/h_2 corresponding to the highest point in the graph.

Examples.—(*1*) *The cross-sections of two limbs of a U-tube are 10 sq. cm. and 1 sq mm. in area respectively. The lower part of both tubes contains mercury (sp. gr. 13·6) What volume of water must be poured into the wider tube to raise the surface of mercury in the narrow tube 1 cm.?* (*Pat. 1924*)

The area of cross-section of the wider tube is 10 sq. cm., and that of the narrow tube is 0·01 sq. cm.

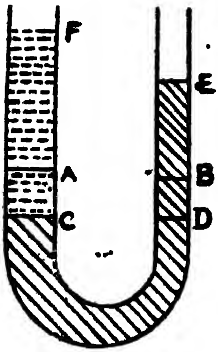


Fig. 77

In Fig. 77, let A and B be the original positions of mercury levels in the two limbs, and C and E the final positions. Let C and D be in the same horizontal level.

The volume of mercury raised in the smaller tube must be equal to the volume of $(AC \times 10)$ c.c. of water.

$\therefore$ $EB \times$ its area $= AC \times$ its area ;

or, $1 \times 0·01 = AC \times 10$; or, $AC = ·01/10$ cm.

But $AC = BD$. $\therefore$ $ED = 1 + ·01/10 = 1·001$ cm.

The press. at C = the press. at D.

or, $FC \times 1 \times g = 1·001 \times 13·6 \times g$, (density of water = 1)

$\therefore$ $FC = 1·001 \times 13·6 = 13·6136$ cm.

$\therefore$ The volume required $= 13·6136 \times 10 = 136·136$ c.c.

(2) *Mercury (denstiy 13 6) and a liquid which does not mix with water are placed in the limbs of a U-tube, and the surfaces of the mercury and the liquid are at 3 and 28 cm. respectively from their common surface. Find the density of the liquid. What change, if any, would be produced if the U-tube is immersed wholly in water so that it enters into both the limbs of the tube ?* (*Pat. 1938*).

As in Art. 122(5), $P + h_1 \rho_1 g = P + h_1 \rho_2 g$, where P is the atmospheric pressure, $h_1 = 3$ cm. ; $\rho_1 = 13·6$; $h_2 = 28$ cm. ; and ρ_2 the density of the liquid.

$$\therefore \quad \rho_2 \times 28 = 3 \times 13·6, \text{ whence } \rho_2 = \frac{3 \times 13·6}{28} = 1·457.$$

When the U-tube is immersed in water, the height of water in the limb above D (Fig. 75) will be greater than that above L ; so the pressure above D being greater, the mercury column will be depressed a little and the liquid column will be raised up.

123. Temperature Correction.—In the above experiments the specific gravity of the substance has been determined relative to water at the room temperature, but true specific gravity can be obtained by taking water at 4°C. If, however, the water is taken at the room temperature t°C., the true specific gravity of the substance would be given by the product of the actual value of sp. gr. obtained by experiment at t°C. and the sp. gr. of water at t°C. For the true

$$\text{sp. gr. at } t^\circ\text{C.} = \frac{\text{weight of any volume of a substance}}{\text{weight of an equal volume of water at } t^\circ\text{C.}}$$

$$\times \frac{\text{weight of the same volume of water at } t^\circ\text{C.}}{\text{,, \quad ,, \quad ,, \quad at } 4^\circ\text{C.}}$$

$=$ sp. gr. of the substance at t°C. $\times$ sp. gr. of water at t°C.

(N.B.—In the C.G.S. units density is numerically equal to sp. gr.)

Examples.—*1. Given a body A which weighs 7 55 gm. in air, 5'17 gm. in water, and 6'35 gm in another liquid B, calculate from these data the density of the body A and that of the liquid B.*

Wt. of A in air $= 7 \cdot 55$ gm ; wt. of A in water $= 5 \cdot 17$ gm.

$\therefore$ Wt. of the same volume of water $= (7 \cdot 55 - 5 \cdot 17) = 2 \cdot 38$ gm.

Hence, volume of $A = 2 \cdot 38$ c.c. $\therefore$ Density of $A = \dfrac{7 \cdot 55}{2 \cdot 38} = 3 \cdot 17$ gm. per c.c.

Again, loss of wt. of A in $B = (7 \cdot 55 - 6 \cdot 35) = 1 \cdot 20$ gm.

Hence, 1'20 gm. is the wt. of B whose volume is the same as that of A, which is 2'38 c.c. $\therefore$ Density of $B = \dfrac{1 \cdot 20}{2 \cdot 38} = 0 \cdot 5$ gm. per c.c.

2. A piece of metal weighs 100 grams in air and 88 grams in water. What would it weigh in a liquid of specific gravity 1'5. (*C.U. 1915*)

The weight of the volume of water equal to that of the solid $= 100 - 88 = 12$ grams. $\therefore$ Volume of the body = volume of the displaced water = 12 c.c.

$$\text{Sp. gr. of the liquid} = \frac{\text{wt. of 12 c.c. of the liquid}}{\text{wt. of 12 c.c. of water}} = \frac{\text{wt. of 12 c.c. of the liquid}}{12} = 1 \cdot 5.$$

$\therefore$ The weight of 12 c.c. of the liquid $= 12 \times 1 \cdot 5 = 18$ grams. Hence, the apparent weight of the body in the liquid $= 100 - 18 = 82$ grams.

3. A test tube is loaded with shot so that it floats in alcohol, immersed to a mark on the tube ; the tube and shot weigh 17'1 gms. The tube is then placed in water and shot added to sink it to the same mark ; the tube and shot now weigh 20'3 gms. Find the specific gravity of alcohol. (*Pat. 1922*)

The wt. of displaced alcohol whose volume is equal to that of the test tube up to the mark = 17'1 gm., and the wt. of the same volume of displaced water = 20'3 gm.

Hence, sp. gr. of alcohol $= \dfrac{17 \cdot 1}{20 \cdot 3} = 0 \cdot 84$.

4. A lump of gold mixed with silver weighs 20 grams. The specific gravity of the lump is 15. Find the quantity of gold in the lump. (Sp. gr. of gold = 19'3 ; sp. gr. of silver = 10'5).

Let W_1 be the weight of gold in the lump, and W_2 that of silver in the lump.

The volume of gold $= W_1/19 \cdot 3$ c.c. ; the volume of silver $= W_2/10 \cdot 5$ c.c.

The weight of displaced water, when the lump is weighed in water, is $\left(\dfrac{W_1}{19 \cdot 3} + \dfrac{W_2}{10 \cdot 5}\right)$ gms.

The sp. gr. of the lump $= \dfrac{\text{weight of the lump in air}}{\text{weight of displaced water}}$; or $15 = \dfrac{20}{\dfrac{W_1}{19 \cdot 3} + \dfrac{W_2}{10 \cdot 5}}$;

or, $10{\cdot}5\ W_1 + 19{\cdot}8\ W_2 = 270{\cdot}2$; and $W_1 + W_2 = 20$ gms. whence $W_1 = 18{\cdot}1$ gm.

5. A sphere of iron is placed in a vessel containing mercury and water. Find out the ratio of the volume of the sphere immersed in water to that immersed in mercury (Density of mercury = 13·6 ; density of iron = 7·8 ; density of water = 1.)

Let V_1 c.c. be the volume of the sphere immersed in mercury, and V_2 c.c. the volume immersed in water.

Then, the wt. of displaced mercury $= V_1 \times 13{\cdot}6$, and that of displaced water $= V_2 \times 1$. Now, wt. of displaced mercury + wt. of displaced water = wt. of the iron sphere ; *i.e.* $V_1 \times 13{\cdot}6 + V_2 = (V_1 + V_2) \times 7{\cdot}8$; *or*, $V_1(13{\cdot}6 - 7{\cdot}8) = V_2(7{\cdot}8 - 1)$.

Hence, $\dfrac{V_2}{V_1} = \dfrac{13{\cdot}6 - 7{\cdot}8}{7{\cdot}8 - 1} = \dfrac{29}{34}$.

6. A mixture is made of 7 c. c of a liquid of specific gravity 1·85 and 5 c c of water. The specific gravity of the mixture is found to be 1·615. Determine the amount of contraction. (C. U. 1927)

Mass of 7 c. c. of liquid of sp. gr. $1{\cdot}85 = (7 \times 1{\cdot}85) = 12{\cdot}95$ gms.

Mass of 5 c. c. of water = 5 gms. ∴ Mass of the mixture = 17·95 gms.

Volume of the mixture $= \dfrac{\text{mass}}{\text{density}} = \dfrac{17{\cdot}95}{1{\cdot}615} = 11{\cdot}11$ c. c.

Hence the amount of contraction $= (7 + 5) - 11{\cdot}11 = 0{\cdot}89$ c. c.

7. A piece of metal weighing 20 gms. has equal apparent weight with a piece of glass, when suspended from the pans of a balance and immersed in water. If the water is replaced by alcohol (density 0·9), 0·84 gm must be added to the pan from which the metal is suspended in order to restore equilibrium. Find the weight of the glass

Let m = mass of glass, ρ = sp. gr. of glass, and d = sp. gr. of the metal.

For equilibrium in water, we have, $m - \dfrac{m}{\rho} = 20 - \dfrac{20}{d}$ ... (1)

and for equilibrium in alcohol, $m - \dfrac{m}{\rho} \times 0{\cdot}9 = 20 - \dfrac{20}{d} \times 0{\cdot}9 + 0{\cdot}84$ ... (2)

Multiplying (1) by 0·9 and subtracting it from (2)

$m \times 0{\cdot}1 = 20 \times 0{\cdot}1 + 0{\cdot}84$; ∴ $m = 28{\cdot}4$ gms.

8. A ship with her cargo sinks α inches when she goes into a river from the sea. She discharges her cargo, while still on the river, and rises β inches, and on proceeding again to sea she rises by another γ inches. If the sides of the ship be assumed to be vertical to the surface of water, show that the specific gravity of sea-water is $\dfrac{\beta}{\gamma - \alpha + \beta}$.

Let x inches = the length of the ship with cargo immersed when in sea-water before going into the river.

then $x + \alpha$, " = the length immersed in river,

$x + \alpha - \beta$ " = " " " " , " (without cargo)

$x + \alpha - \beta - \gamma$ " = " " " " " sea (without cargo)

Now, if ρ = density of sea-water, and β_1 = density of river water, we have

wt. of ship + cargo = $\rho x = \rho_1 (x+\alpha)$, and

wt. of ship − cargo = $\rho (x+\alpha-\beta-\gamma) = \rho_1 (x+\alpha-\beta)$

So $\rho x = \rho_1(x+\alpha)$ ... (1)

$\rho(x+\alpha-\beta-\gamma) = \rho_1 (x+\alpha-\beta)$ ... (2)

Substracting (2) from (1), $\rho (\gamma-\alpha+\beta) = \rho_1 \beta$; $\therefore \dfrac{\rho}{\rho_1} = \dfrac{\beta}{\gamma-\alpha+\beta}$

Questions

Art 111.

1. State Archimedes' principle. How would you demonstrate its truth? (C. U. 1912, '19, '20, '24, '35, '39, '40, '46, '47 ; Cf. Pat. 1919, '23, '31, '36.)

1(a). Describe an expt. to demonstrate the buoyancy of air. (Pat. '46).

2. Explain how Archimedes' principle may be used to distinguish a metal from its alloy. (C. U. 1922, '16 ; Cf. Pat. 1928, '32)

[**Hints.**—Determine the density of the alloy and compare its value with that of the pure metal.]

(a) Why it is easier to lift a heavy stone under water than in air? (C. U. 1937)

3. Describe a hydrostatic balance and explain clearly the principle on which the working of the instrument is based. (C. U. 1932)

4. A beaker containing water weighs 300 gms., and a piece of metal whose volume is 10 c.c. and mass 88 gms. is immersed in the water, being suspended by a fine thread. Find (*a*) the upward force which must be applied to the thread to support the metal, and (*b*) the upward force necessary to support the beaker. (L. M.)

[*Ans*: (*a*) 78 gms.-wt. (*b*) 310 gms.-wt.]

4. (a) A piece of iron weighing 275 gms., floats in mercury (sp. gr. = 13·59) with 5/9 of its volume immersed. Find the volume and the sp. gr. of iron. (C. U. '46)

[*Ans*: 36·42 c.c. : 7·55]

(b) A piece of wax of volume 22 c.c. floats in water with 2 c.c. above the surface. Find the wt. and the sp. gr. of wax.

[*Ans*: 20 gms. : 0·909] (C. U. '47)

Arts. 113 & 114.

5. Why in C. G. S. units the values of density and sp. gr. are the same? (C. U. '47)

6. 144 gms. of an alloy of two metals, sp. gr. 8 and 12 respectively, is found to weigh 129 gms. when totally immersed in water. Find the proportion by weight of the metals in the alloy. (Pat. 1939)

[**Hints.** Let w_1 = wt. of metal A. $\therefore$ wt. of metal B = $(144-w_1)$.

Hence $\frac{w_1}{8}+\frac{(144-w_1)}{8}=(144-129)$, whence $w_1=72$ gms.]

7. Distinguish between density and specific gravity.
(Pat. 1921, '28; C. U. 1923, '36)

8. Describe how you will determine experimentally the density of a metal in the form of a long wire of about 5 metres in length. (Pat. 1922)

[**Hints** —Measure the diameter and hence the radius of the wire by a screw gauze, and measure the length. Volume $=\pi r^2 l$. Weigh it in a balance by turning the wire into a coil of several turns. Then density = mass/vol.

The volume can also be determined by displacement of water.]

9. The density of three liquids are in the ratio of 1 : 2 : 3. What will be the relative densities of mixture made by combining (a) equal volumes, (b) equal weights ? (L. M. B.)

[*Ans.* : 11 : 9]

Arts. 115 & 115(a).

10. Distinguish between density and specific gravity. Under what conditions do bodies float or sink in a liquid ? A piece of iron weighing 272 gms. floats in mercury of density 13·6 with $\frac{5}{8}$ths of its volume immersed. Determine the volume and density of iron. (C. U. 1930 ; Cf. Dac. '27, '32)

[**Hints.**—Let the volume of the iron piece be x c.c. Then $\frac{5}{8}x$ c.c. = volume of iron piece immersed in mercury = volume of mercury displaced by the iron piece.

Then $\frac{5}{8}x=\frac{272}{13\cdot6}$; or $x=32$ c.c. ; and density $=\frac{\text{mass}}{\text{volume}}=\frac{272}{32}=8\cdot5$ gm. per cc.]

11. Discuss the stability of equilibrium of a floating body. Apply your result to the case of a uniform sphere of a wood floating on water.
(Pat. '47)

Art. 116.

12. What is meant by Buoyancy ? Explain why an iron ship floats in water. (C. U. 1928, '37 ; Pat. '32 ; Dac. '33).

Art. 118.

13. Describe 'the Cartesian driver' and explain how it acts. Do you know of any modern appliance which is based on this principle ? (C. U. '38, '46)

14. A solid body floating in water has one-sixth of its volume above the surface. What fraction of its volume will project if it floats in a liquid of specific gravity 1·2 ?

[*Ans* : $\frac{11}{36}$.]

Art. 119.

15. You are given a piece of paraffin cut in the form of a cube ; how would you roughly determine its specific gravity without using a balance ?
(Dac. 1931)

16. The specific gravity of ice is 0·918, and that of the sea-water is 1·03. What is the total volume of an ice-berg which floats with 700 cubic yards exposed ? (C. U. 1923 ; Pat. '35)

(Let V cubic yards be the total volume of the ice-berg. ∴ Volume under water $=(V-700)$ cu. yds. The mass of the ice-berg $=(V\times 27)\times 62\cdot5\times 0\cdot918$ lbs.

The mass of the sea-water displaced $=(V-700)\times 27\times 62\cdot5\times 1\cdot03$ lbs. According to the law of flotation, the mass of the floating body = mass of the displaced liquid. ∴ $(V\times 27)\times 62\cdot5\times 0\cdot918=(V-700)\times 27\times 62\cdot5\times 1\cdot03$; or $V=6437\cdot5$ cu. yds.]

Art. 120.

17. You are provided with a hollow glass tube of uniform cross-section with a bulb blown as its lower end, and other necessary materials. State how will you proceed to construct a common hydrometer, and explain how you will graduate it. [See also Art. 122 (2)] (Pat. 1944)

Arts. 121 & 122.

18. How do you find the specific gravity of a solid lighter than water.

A piece of cork whose weight is 19 grams is attached to a bar of silver weighing 63 grams and the two together just float in water. The specific gravity of silver is 10·5. Find the specific gravity of cork. (C. U. 1925)

[*Ans* : 0·25]

19. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 28 and its density 5·6 ; if the mass of other is 36, what is the density ? (Pat. 1928)

20. Explain how you would determine the specific gravity of a solid by a specific gravity bottle.

21. Explain how you would determine the specific gravity of an insoluble powder by the specific gravity bottle.

A specific gravity bottle weighs 14·72 grams when empty, 39·74 grams when filled with water, and 44·85 grams when filled with a solution of common salt. What is the specific gravity of the solution ? (C. U. 1934)

[*Ans* : 1·2]

22. Describe an experiment to find the specific gravity of a solid soluble in water in the laboratory. (C. U. 1944)

23. If the specific gravity of a metal is 19, what will be the weight in water of 20 c.c. of the substance ? (C. U. 1917)

[*Ans* : 360 grams].

24. Explain the principle of a Nicholson's hydrometer. How will you use it to determine the specific gravity of a substance lighter than water ?

25. How would you determine the specific gravity of a liquid ?
(C. U. 1915, '16, '18 : Pat. 1928)

26. Describe a Nicholson's hydrometer, and explain how you would determine the specific gravity of a liquid with its help.

27. 60·8 gms. have to be placed on the pan of a hydrometer to sink it to the mark in water and 6·8 gms. only in alcohol. If the hydrometer weighs 200 gms. what is the specific gravity of alcohol ? (C. U. 1931)

[*Ans* : 0·79]

28. 1 c.c. of lead (sp. gr. 11·4) and 21 c.c. of wood (sp. gr. 9·5) are fixed together. Show whether the combination will float or sink in water.
(C. U. 1933)

[**Hints.**—Mass of 1 c.c. of lead = 11·4 gms : mass of 21 c.c. of wood = 21 × 0·5 = 10·5 gms. ∴ The total mass of the combination = 11·4 + 10·5 = 21·9 gms. Their total volume = 21 + 1 = 22 c.c. So the combination floats with 21·9 c.c. of it being immersed in water and keeping the rest (i.e. 0·1 c.c.) above the surface of water.]

29. Explain clearly how you would determine the specific gravity of a liquid by a Nicholson's hydrometer without using a balance.

30. You are given a specific gravity bottle, enough kerosene and water, heating arrangements and a table of densities of water at various temperatures. How would you find the density of kerosene at 50°C., the room temperature being 30°C. ? (Pat. 1932)

CHAPTER X

Atmospheric Pressure

124. The Atmosphere. (Its weight).—The atmosphere is the envelope of air surrounding the earth. It is principally a mixture of 4 parts of nitrogen and 1 part of oxygen by volume besides which it contains a small quantity of carbon dioxide and also aqueous vapour.

If the whole atmosphere is supposed to be divided into a number of layers of air, one above another, over the surface of the earth, then it is evident that the surface of the earth, or any other layer over it,

has got to bear the weight of the layers of air above it, and thus exposed to a pressure which is called the **atmospheric pressure**. It is measured by the weight of a column of air of unit cross-section and height equal to that of the atmosphere above that surface. The value of this pressure is 15 lb.-wt. per sq. inch, or 1,013,961 dynes per sq. cm.

Air has weight.—Take a fairly large flask fitted with a rubber cork through which passes a glass tube. To this is attached a piece of rubber tubing with a clip. Put a little water in the flask and boil it after opening the clip. After sometime close the rubber tubing with the clip and also remove the flame. Weigh the flask when it is cooled. Now open the clip; air rushes in; weigh again. The difference between the weights is the weight of the air that has entered the flask.

125. Air exerts Pressure.—

Experiments.—(*a*) Tie a piece of india-rubber sheet *R* over the rim of an open glass tumbler *G* (Fig. 78). Press the other end of the tumbler on the plate *B* of an air-pump after it is well greased. On exhausting the air through the tube *T* by means of a pump it will be found that the rubber sheet will be depressed more and more until finally burst with a loud report. This shows the action of the atmospheric air on the rubber sheet.

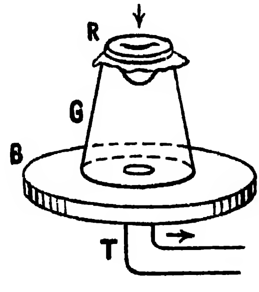


Fig. 78

(*b*) **Magdeburg Hemispheres**.—These are two tightly fitting metal hemispheres which can be easily separated ordinarily. But when the air from the interior of the hemispheres is pumped out, great force is required to separate them (Fig. 79).

(*c*) A small rubber balloon, partially filled with air, is placed inside the receiver of an air-pump. When the air is pumped out, the balloon extends and finally bursts as the external pressure is removed.

(*d*) Fill a tumbler completely with water and cover its mouth with a piece of stout paper. Press the palm of your right hand on the paper and with the other hand slowly invert the glass. On withdrawing the right hand from the paper, the water will be found to remain in the inverted tumbler. This is due to the pressure of the atmosphere acting on the surface of water in the upward direction. The paper helps the mass of water to present a flat surface, otherwise the mass of water would be divided, air will enter and thus the

water would fall. The experiment will not be succesful if there is much air within the tumbler.


Fig. 79

(*e*) **Syringe and Pipette.**—Dip the lower end of a syringe in water keeping the piston up. Thus any air which may have been below the piston is much rarefied and consequently the pressure of air inside the syringe is reduced. The greater air pressure on the outside now forces water into the barrel of the syringe. (See Art. 148).

Dip a **pipette** into water and lift it out of the water after placing your finger over the top. The water inside the pipette does not run out although the bottom is open, as the air pressure outside is greater.

The drinking of a liquid by sucking through a hollow straw or tube and thus creating a partial vacuum inside is explained exactly in the same manner as above.

(*f*) **Torricelli's Experiment**.—Torricelli, an Italian physicist and a pupil of Galileo, carried out an experiment in 1643 which laid the foundations of our knowledge of the atmospheric pressure. His experiment is as follows :—

Take a thick glass tube about a metre long and completely fill it up with mercury. Close the end with the thumb, invert it and open under a bowl of mercury. The mercury falls, and then comes to rest being supported by the pressure of the atmosphere. The height of the mercury column is generally 30 inches or 76 cms. The pressure of any fluid depends upon its vertical height and density. Here the pressure exerted by the mercury column of height equal to the vertical distance between the two surfaces of mercury is supported by the atmosphere. On slanting the tube it will be seen that the mercury surface inside will approach the top of the tube but the vertical height between the surfaces of mercury remains constant until there is no vacuous space above (Fig. 80). This constitutes a simple **mercury barometer** The space above the top of mercury in the tube, though contains a little mercury vapour,

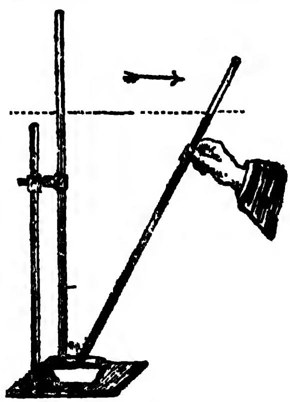


Fig. 80

is known as the **Torricellian vacuum** after the name of Torricelli. This experiment of Torricelli first taught the world that it was possible to produce a vacuum.

That the space above the mercury column is a vacuum can be tested by slowly tilting the tube until the mercury touches the top of the tube, when a sharp sound will be produced because there is no air present to act like an elastic cushion and thus deaden the sound.

126. Barometers.—*Barometers* (*baros*, weight) are instruments for measuring the pressure of the atmosphere by the weight of a column of mercury supported by it. Mercurial barometers are of two kinds:—**Cistern** and **Siphon barometers**

Fortin's Barometer.—It is a Cistern type of mercurial barometer. The barometer tube is filled with pure, dry, and air-free mercury and is inverted over a cistern of mercury, *R*, called the reservior (Fig. 81). The mercury stands at a certain height depending on the atmospheric pressure at the time. The tube is enclosed within a long brass casing *C* on the front side of the upper part of which there is a rectangular slit through which the upper level of the mercury in the tube can be seen and observed by the help of a small mirror placed on the back side of the tube. The meniscus of the mercury surface is read by the main scale *U*, graduated in inches and centimetres on either side of the slit, with the help of the vernier *V* worked by the screw *P* which is a rack and pinion arrangement.

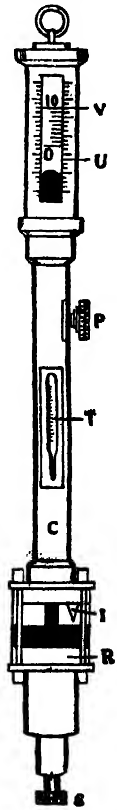


Fig. 81

The cistern has its upper part made of a glass cylinder *F* (Fig. 82) through which the surface of the mercury *M* contained in it can be seen. The glass cylinder is fitted in a box-wood cylinder *K* whose lower end is closed by a flexible leather bag *L* (usually made of Chamois leather). This bag has a wooden bottom *N* against which the point of the base screw *S* presses. The screw works through the brass casing *E* which surrounds the reservoir. By turning the base-screw the level of the mercury in the reservoir can be raised or lowered at will and finally made to touch the tip of the ivory pointer *I* which is fixed to the lid of the cistern. The tip of the pointer is in the same horizontal level as the zero of the main scale *U* (Fig. 81). The mouth of the cistern is provided with a seat *l* of leather through the

pores of which the atmospheric pressure is transmitted from the outside to the inside of the cistern. The barometer tube is of wide bore at the upper portion so that the effect of surface tension is avoided but it gradually tapers to a narrow end having a bulge G a little above, which rests on the leather seat l The end of the tube is made narrow in order that there may not be oscillations in the mercury during adjustment.

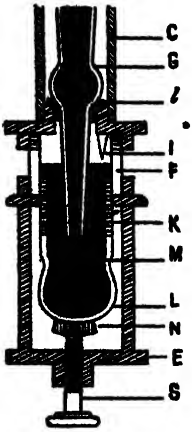


Fig. 82

To read the barometer, the screw S is worked until the Hg-surface in the cistern just touches the tip of the ivory pointer. This is called the *Zero-adjustment*. The adjustment is accomplished when the tip of the pointer appears to touch the inverted image produced in the clean Hg-surface. Keeping the eye in level with the Hg-surface, seen in the slit, the vernier V is moved along the main scale U until its lower edge (Zero mark) appears to be tangential to the convex Hg-surface, as shown in Fig. 83. The main scale reading below the zero-mark of the vernier plus the value of the vernier reading against the main scale gives the barometer height at the time of observation. The temp. of the atmosphere is given by the thermometer T placed on the casing of the instrument.

[**Note :** For corrections to be applied to the barometric height, read Art. 27 on Heat].

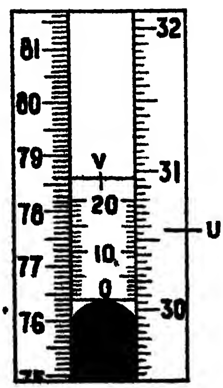


Fig. 83

(b) **Siphon Barometer.**—This is a portable type of barometer and is more convenient than a cistern type. It has no cistern but consists of a U-shaped tube having unequal limbs (Fig. 84). The longer limb AB, about a metre long, is closed at the top while the shorter limb CD serves as the cistern. To protect the Hg-surface from incidence of impurities, CD is closed at the top leaving a small opening at D to enable communication with the external air. In order that no air may enter into AB when the instrument is inclined, AB and CD are joined by means of a wooden board and one-scale is fixed on it against each of the two limbs to read the difference of levels of

Fig. 84

mercury which gives the barometric height at the time of observation.

(c) The **weather glass** or household barometer is a form of siphon barometer.

It has got a dial in front of the shorter tube, which is graduated and marked, *stormy*, *rain*, *variable*, *fair*, etc. There is some arrangement by which a pointer moves over the graduated dial and indicates the pressure in inches, and also the probable state of the weather in the immediate future. For this reason it is called *weather glass.*

(d) **The Aneroid Barometer.**—(Fig. 85). The word "aneroid" means "no liquid", so the name indicates that no liquid is used in this barometer. It consists of a cylindrical box which is partially exhausted, and closed with a thin elastic metal diaphragm, which is corrugated in order to yield more easily to external pressure. The variations of atmospheric pressure cause the diaphragm to yield proportionally, and the change is indicated by an index connected to a lever.

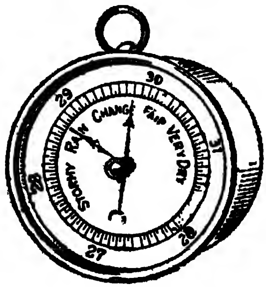


Fig. 85—Aneroid Barometer

Barograph.—

Continuous record of pressure can be obtained by means of a type of barometer, known as the **Barograph** (Fig. 86). This is also an aneroid barometer, where indications are recorded on a piece of squared paper by means of a pen attached to a long lever. The squared paper on which a continuous record of the changes in atmospheric pressure is left is wound on a cylinder which is rotated by clock-work.

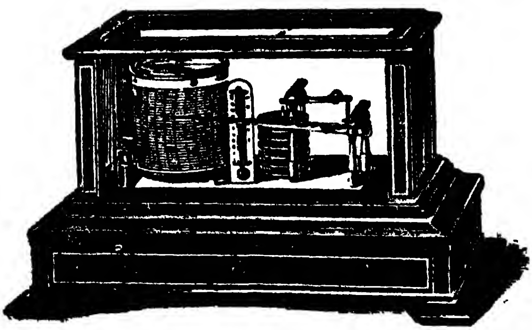


Fig. 86—Barograph

127. *Corrections of Barometer Reading.—A number of correc-

tions, as given below, should be applied to the barometer reading as described in Art. 126(a).

(1) *Correction for Temperature.*—Since the pressure exerted by a given height of mercury depends upon its density, a correction is to be applied for the expansion, and the consequeut change in density, of mercury. Further, a correction is also to be made for the expansion of the metal scale (which is supposed to be correct at 0°C.) with rise of temperatue (see Ch. III, Part II).

(2) *Correction for Height above Sea-Level.*—As the value of the attraction of the gravity diminishes with the height above the sea-level, the length of a column of mercury which produces the standard pressure increases with the height of a place above the sea-level and so a correction is to be applied so that the observed reading is corrected for altitude by being *reduced to the sea-level.*

(3) *Correction for the Latitude.*—For the reason stated above the value of gravity varies from place to place. It is less at the equator than at the poles. For correcting the above two effects, the value of gravity at latitude 45° and at the sea-level is taken as the standard. *The height reduced to sea-level in latitude* $45^\circ = H\{1 - 0{\cdot}00257 \cos 2\lambda - 1{\cdot}96h \times 10^{-9}\}$, where H = observed height reduced to 0°C. ; λ = latitude of the place, and h = height (in cm.) of the place above the sea-level.

The reading corrected for (1), (2) and (3) would represent the height of the mercury column which would be supported by the atmospheric pressure at a standard temperature (*i.e.* 0°C.) and at a standard place (*i.e.* at the sea-level in latitude 45°).

128. Diameter of the Barometer tube.—The *height* of the mercury column supported by the pressure of the atmosphere is *not affected by the width* of the barometer tube, for, let

a = area of cross-section of the tube ; h = vertical height of the column ; d = density of mercury ; g = acceleration due to gravity,

then, the wt. of the mercury column = $a\ h\ d\ g$ = the upward force due to the atmosphere by which it is supported.

Now, if the area is doubled (*i.e.* $2a$), the upward force due to the atmosphere acts on twice the area, and so is also doubled. This force becomes = $2\ a\ h\ d\ g$ = the weight of the mercury column which it has to support.

129. Forecasting of Weather : Why the Barometric Height Varies ?—Some amount of water vapour is always present in the air. Contrary to the popular belief that moist air is heavy, water vapour is actually lighter than dry air, the density of water vapour being $\frac{5}{8}$

the density of dry air. Hence, when there is a considerable amount of water vapour present in the air, the density of atmosphere, and therefore the pressure exerted by it, is less, which causes the mercury column in the barometer to fall slightly. This is the reason of the variations in the height of the barometer. The presence of *much water vapour* in the air indicates that *rain* is imminent. For this reason, the barometer is used for forecasting the weather. A *low barometer* indicates the presence of much water vapour in the air, which, again, indicates a fall of *rain* in the near future, and a rapid fall in barometric height is usually accompanied by *stormy conditions*. On the other hand, a *high barometer* indicates *dry weather*. It should be noted, however, that the barometer is by no means an infallible guide in forecasting weather conditions.

Uses of Barometers.—So we find that a barometer can be used for the following purposes,—(a) *Measurement of the atmospheric pressure ;* (b) *forecasting of weather ;* (c) *determination of the altitude of a place* (see Art. 83, Part II).

130. Value of the Atmospheric Pressure.—The *normal* or *standard atmospheric pressure is the pressure exerted by 76 cms.* (or 30 inches) of mercury at 0°C. at sea-level and at latitude 45°. So, the value of this pressure is the force exerted on an area of one square centimetre by a column of mercury of height 76 cm., or that exerted on an area of one square inch by a column of mercury of height 30 inches.

C. G. S. units :

Atmospheric pressure = weight of 76 (= 76 × 1) c.c. of mercury ;

$= 76 \times 13{\cdot}6 \times g$ dynes per sq. cm.

$= 1{,}013{,}961$ dynes per sq. cm.

(density of mercury = 13·6 grams per c.c., and $g = 981$ cm. per sec.^{2}.)

When reduced to 0°C. at latitude 45°, this becomes = 1013231

$= 1{\cdot}013231 \times 10^6$ dynes per sq. cm.

So, it is approximately equal to one megadyne, i. e. 10^6 *dynes.*

The unit of pressure used in *meteorology* is 1,000,000 (or 10^6) dynes per sq. cm., which is called a **bar**, one thousandth part of which is called a **millibar**. Thus *the value of the standard atmospheric pressure becomes 1013·23 millibars.*

F. P. S. units :

Again, taking 30 inches of mercury as the height of the barometer, atmospheric pressure = weight of 30 (= 30 × 1) cubic inches of mercury.

We know, that 1 cu. ft. of water weighs 62·5 lbs. So 1 cu. in. will weigh $\frac{62·6}{12 \times 12 \times 12}$ lbs.

$\therefore$ The weight of 1 cu. in. of mercury $= 13·6 \times \frac{62·5}{1728}$ lbs.

or *Atmospheric pressure* $= 30 \times 13·6 \times \frac{62·5}{1728}$ lbs.-wt. per. sq. in.

$= 14·7$ lbs.-wt. per sq. in.

$=$ *15 lbs -wt. per sq. in.* (*roughly*)

[So, 30 inches being the height of a mercury barometer, the height of *water barometer* will be 30 in. $\times$ 13·6 $=$ 34 ft. (about).

Similarly, to get the height of a *glycerine barometer* (h), we have, ht. of water barometer $\times$ density of water $=$ ht. of glycerine barometer $\times$ density of glycerine or $34 \times 1 = h \times 1·26$ (density of glycerine $= 1·26$).

$\therefore$ $h = \frac{34}{1·26} = 27$ ft. (about)]

[**Note.**—Pressure is sometimes measured in **atmospheres**. The pressure exerted by a column of mercury 76 cms. high is universally accepted as the **normal pressure** of the atmosphere, and this is taken as a standard. '*One atmosphere*' means the pressure equal to that of a column of mercury 76 cms. high at sea-level at latitude 45° and at the temperature of melting ice (0°C.), which is taken as the normal temperature.]

131. Why Mercury is a convenient Liquid for Barometers ?— A column of mercury only 30 inches high is able to support the pressure of the atmosphere, whereas to support the same pressure, a column of water 34 ft. high, or a column of glycerine 27 ft. high, will be necessary. For this reason (*i.e.* due to the *high specific gravity*) mercury is used for barometers as a matter of convenience. Besides this, mercury *does not wet glass* and *does not evaporate rapidly*. But the advantage in the case of lighter liquids is that a small variation in barometric height can be read more accurately due to much greater alteration in the level. For this reason glycerine barometer is sometimes used. Water is not suitable as it has a considerable vapour pressure, and in some countries it cannot be used in winter when it will freeze.

Example.—*The pressure exerted by the atmosphere on a circle, whose diameter is 4·5 ft., is equal to 33800 pounds. Calculate the height of the mercury barometer, if the density of mercury is 13·6 and the weight of 1 cu. ft. of water 62·5 pounds.*

Let h be the height of the barometer. Then the total pressure exerted on the circle $=$ the weight of a column of mercury of height h standing on the

circle. The volume of this mercury column $= \pi \times \left(\frac{4'5}{2}\right)^2 \times h = 15'9\ h$ cu. ft.

One cu. ft. of water weighs 62'5 lbs. ; hence 15'9 h cu. ft. of water will weigh 15'9 $h \times 62'5 = 993'7\ h$ lbs.

But mercury is 13'6 times heavier than water, so, 15'9 h cu. ft. of mercury will weigh 993'7 $h \times 13'6 = 13514'32 \times h$; and this $=33800$ lb.

or $13514'32 \times h = 33800$; *or* $h = 2'51$ ft.

133. Variation in the Atmospheric Pressure at Different Altitudes.—As we ascend through the atmosphere with a mercury barometer, the amount of air pressing upon the exposed surface of it is reduced and consequently the height of the mercury column supported by the air becomes less and less as we ascend more and more ; on the other hand, as we descend below the sea-level, say, down the shaft of a a mine, the amount of air pressing upon the surface is increased and so the mercury column is pushed higher and higher. It has been found that *for low altitudes there is a variation of 1 inch in the barometric height for a vertical rise or fall of 900 ft., but for greater heights this is not strictly true.* Hence, from the variations in the readings of a barometer the altitude of a place, or the depth of a mine, can be ascerrained (see also Art. 83, Part II).

Aneroid barometers are often made as watches and these can be conveniently used for determining the heights of mountains. It is also a part of the necessary equipment of an aeroplane or airship. It is by means of this instrument that an airman knows at what height he is flying. At a height of about $3\frac{1}{2}$ miles the pressure of the atmosphere is about 30 cm., and the pressure at a height of 20 miles is approximately 7 mm.

Altimeters of aeroplanes are aneroid barometers which are so graduated as to give direct reading of altitudes to which they rise.

The following table shows how the atmospheric pressure at different places in India changes with altitude, *i.e.* their heights above the sea-level.

Place	Altitude	Mean Atmospheric Pressure
Calcutta	21 ft.	762'4 mm.
Bombay	38 ft.	759'3 ,,
Simla	7238 ft.	586'5 ,,
Darjeeling	7425 ft.	580'2 ,,

134. Homogeneous Atmosphere—It is very difficult to tell as to what height the atmosphere extends, but knowing the fact that the weight of a column of air of 1 sq. cm. cross-section and reaching from the sea-level up to the extreme limits of the atmosphere will support a column of mercury 76 cm. high, we can calculate the extent of the atmosphere taking it to be homogeneous, *i. e.* of the same density throughout, beginning from the earth's surface. Taking the density of dry air at the earth's surface to be 0·00129 gm. per c.c., and the density of mercury 13·6, the height of the homogeneous atmosphere would be

$$\frac{76 \times 13{\cdot}6}{0{\cdot}00129} = 8{\cdot}012 \times 10^5 \text{ cm.} = 8{\cdot}012 \text{ km.}$$

$$= 8{\cdot}012/1{\cdot}609 \text{ miles, } (1 \text{ mile} = 1{\cdot}609 \text{ km.})$$

$$= 4{\cdot}98 \text{ miles or 5 miles approximately.}$$

But actually the atmosphere extends up to several miles, and its density gradually diminishes. For example, the density of air at a height 6 miles above is $\frac{1}{4}$, and 15 miles above is $\frac{1}{30}$, of its value at the sea-level. Opinions vary as to how high the atmosphere reaches; some put it as high as 200 miles or more.

Temperature of the Atmosphere.—So far as the temperature of the atmosphere is concerned it may be taken to be divided into two regions; in the lower of which, called the "*troposphere*", there is a fairly rapid fall of temperature with height at the rate of about 5°C. per kilometre, and in the upper region, called the "*stratosphere*", there is no material change of temperature with height. The temperature of stratosphere is about −55°C. The level where the stratosphere begins is not fixed. It changes with the seasons and the geographical position of the place. The temperature of Mont Blanc (15,781 ft.) is 3°F. and that of Mount Everest −27°F.; so at very high altitudes the aviators and mountaineers may be frozen to death unless special precautions are taken.

135. Pressure on the Human Body.—We know that the magnitude of the pressure of the atmosphere is 15 lbs. on every sq. inch of everything on the surface of the earth. The surface of the human body is always exposed to this pressure. The total surface area of the body of a man of middle size is about 16 square feet. Therefore, the pressure which a man supports is (16 × 144 × 15) lbs. or about 15 tons. It might seem impossible how this enormous pressure can be borne without any serious damage or, at least, considerable discomfort; but the explanation can be found in the fact that a bottle full of air is not crushed by the weight of the atmosphere acting on its sides, however thin the wall may be, because the pressure of the air inside the bottle counterbalances the pressure acting on the outside.

If, however, the air is pumped out from inside the bottle, the outside pressure would immediately crush the bottle. Our bodies are not closed vessels. The lungs are full of air at atmospheric pressure and the air is carried at that pressure throughout the whole system. Thus, our bodies can easily bear the enormous surface pressure as it is counterbalanced by the internal pressure. Indeed, the discomfort is experienced when the outside pressure is partially removed. In experiment (c), Art. 125, it was the outside pressure which was removed by the pump, and the pressure of the inside air then burst the balloon. Thus, in rarefied regions at high altitudes the *mountaineers and aviators experience difficulty in breathing*, and sometimes the internal pressure forces out the blood through the tissues of the nose and the ear. On the other hand, *the divers have to work under high atmospheric pressure* —sometimes of several atmospheres, when it is found that air dissolves in the blood-stream. The oxygen of the air is absorbed by the blood but the nitrogen tends to separate as bubbles in the tissues, which causes severe pains and sometimes death. This is the reason why *the diver should ascend slowly* in order to allow sufficient time for the nitrogen to come out of the tissues and escape through the lungs. For example, about two hours or more would be necessary to bring a diver safely to the surface from a depth of about 200 ft. It has been found by experiment that it is better to supply the divers with air containing helium, instead of nitrogen, as helium dissolves much less than nitrogen in the blood and it is also got rid of much quickly, and so the diver can come to the surface in much less time.

136. Balloon, Airship, and Parachute.—It follows from the principle of Archimedes that if the weight of a body is less than that of the air displaced by it, the body will be forced up, or buoyed up as it is called, and will rise in the atmosphere. The difference between the weight of the body, and that of the air displaced by it, is called the "*lifting power*' of the body. This principle is applied in a balloon or airship, which contains some gaseous substance like hydrogen, or helium, which is lighter than air. The combined weight of the gas, engine, passengers, etc., must be less than the weight of the displaced air in order that the balloon may rise.

At greater heights the pressure of the air is small, so a balloon displaces a smaller weight of air.

An *airship*, which has a light rigid framework, and also engines for propulsion, works on the same principle as the balloon. Though either hydrogen or helium may be used in airship, hydrogen has the great disadvantage that it is highly imflammable whereas helium is not, so *with helium risk of accident is much reduced*. The advantage with *hydrogen* is that it is much *lighter* and *cheaper*.

Parachute.—The parachute is a device like that of an umbrella, which resists the falling of a body by putting air resistance, *i.e.* it acts as an "air-brake" to a falling body.

(*a*) **Lifting Power of a Balloon.**—If d be the density of the air, d' the density of the gas in the balloon, V the external volume of the balloon, which is the volume of the displaced air, and V' the volume of the gas, the weight of the air displaced, *i.e.* the force due to buoyancy of air $= Vd$ and the weight of the gas in the balloon $= V'd'$; the *total lifting power* $= (Vd - V'd')$. In practice V is very nearly equal to V', so the total lifting power reduces to $V(d-d')$, part of which is used to raise the balloon itself, and the remainder is required to raise its passengers and cargo. The density of hydrogen $= 0{\cdot}0694 \times$ density of air. So for a balloon filled with hydrogen, *the lifting power* $= V(d - 0{\cdot}0694 \times d) = 0{\cdot}9306\ Vd$. For a balloon filled with helium for which $d' = 0{\cdot}1388 \times d$, lifting power $= V(d - 0{\cdot}1388d) = 0{\cdot}8612 \times Vd$. Thus, it is found that though helium is twice as dense as hydrogen, the lifting power of a balloon filled with helium is almost equal (93 per cent.) to that of a similar hydrogen-filled balloon.

Examples.—*1. A spherical balloon 4 metres in diameter is filled with hydrogen gas (density $\frac{1}{13}$ that of air). The silk envelope of the balloon weighs 250 gms. per square metre. How much hydrogen is required to fill it and what weight can it support, the weight of a litre of air being 1·293 gm.*

The volume of the balloon $= \frac{4}{3}\pi \times 2^3 = 33{\cdot}51$ cubic metres, and the surface of balloon $= 4\pi \times 2^2 = 50{\cdot}265$ sq. metres. (The wt. of 1 litre of air is 1·293 gm.)

$\because$ The wt. of one cubic metre of air $= 1{\cdot}293$ kilogram.

The wt. of air displaced by the balloon $= 33{\cdot}51 \times 1{\cdot}293 = 43{\cdot}328$ kgms. and the wt. of hydrogen filling the balloon

$$= \frac{1}{13} \times \text{wt. of the same volume of air} = \frac{1}{13} \times 43{\cdot}328 = 3{\cdot}333 \text{ kgms.}$$

The wt. of the silk envelope is 250 gms. per sq. metre

$$= \frac{250}{1000} \text{kgms per sq. metre.}$$

$$\therefore \text{ The wt. of the envelope of the balloon} = 50{\cdot}265 \times \frac{250}{1000} = 12{\cdot}566 \text{ kgms.}$$

Hence the wt. of hydrogen in the balloon + its envelope $= 12{\cdot}566 + 3{\cdot}333$ kgms. So the wt. which the balloon can support $= 43{\cdot}328 - (12{\cdot}566 + 3{\cdot}333)$ $= 27{\cdot}429$ kgms. This is the lifting power of the balloon.

2. A litre of hydrogen and a litre of air weigh about 0·09 gramme and 1·3 grammes respectively at a certain temperature (t) and pressure (p). What would be the capacity of a balloon, weighing 10 kilogrammes, which just floats when filled with hydrogen having the same pressure (p) and the same temperature (t) as the air. (*C. U. 1912*)

Let V litres be the volume of the balloon. Mass of hydrogen enclosed in the balloon $= V \times 0{\cdot}09$ gms. Mass of air displaced by the balloon $= V \times 1{\cdot}3$ gms.

When a body just floats in a fluid, the wt. of the body is equal to the wt. of the displaced fluid. Hence wt. of balloon + wt. of hydrogen in it = wt. of air displaced by the balloon.

or $10 \times 1000 + V \times 0{\cdot}09 = V \times 1{\cdot}3$; *or* $V = \frac{10000}{1{\cdot}21} = 8264{\cdot}46$ litres (nearly).

137 Manometers or Pressure Gauges—The instrument used for measuring the pressure of a gas is known as a **manometer** or **pressure gauge**. The simplest form of manometer is a U-tube containing a liquid of known density, such as mercury or water (Fig. 87) One end of the U-tube is connected to the vessel V in which the pressure is to be measured. The other end is exposed to the atmospheric pressure. If the pressure of the gas in the closed vessel is the same as the atmospheric pressure, there will be no difference of level, and, in other cases, there will be a difference of level h between the surfaces A and B of the liquid in the two limbs of the U-tube. If d be the density of the liquid, the pressure of the enclosed gas at $V = gdh +$ atmospheric pressure.

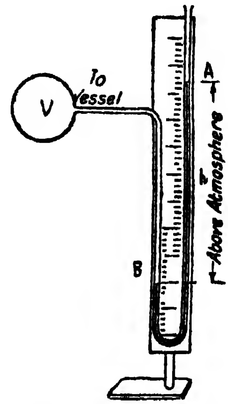


Fig. 87.—Manometer

Thus **the height h of the liquid column in the open tube measures the difference between the pressure of the enclosed gas and the pressure of the atmosphere.**

If the level in the open limb A is *higher* than the level in the closed limb B, as in Fig. 87, the pressure of the gas in V is *greater* than the atmospheric pressure, and if the level in A is *lower* than that in B, the pressure of the gas is *less* than the atmospheric pressure.

138. Boyle's Law.—Robert Boyle (1527-1691), an Irishman, first established the exact relation between the pressure of a confined mass of gas and its volume when they are varied at a constant temperature, and the law named after him is stated as follows :

The temperature remaining constant, the volume of a given mass of gas varies inversely as its pressure

Thus, if P be the pressure and V the volume of a gas,

we have, $P \propto \frac{1}{V}$; *or* $P = K\frac{1}{V}$, where K is a constant whose value depends on the mass of the gas taken and its temperature.

Thus, $PV = K$.

If the pressure P be changed to P_1, and the corresponding volume becomes V_1, we have, $P_1 V_1 = K$. But $PV = K$.

$$\therefore \quad P_1V_1 = PV$$

Thus, if a given mass of gas at constant temperature has volumes V_1, V_2, V_3, etc., under pressures P_1, P_2, P_3, etc., respectively, then, by Boyle's Law, we have, $P_1 V_1 = P_2 V_2 = P_3 V_3 \ldots\ldots = \text{a constant}$.

138(a) Pressure and Density.—Boyle's Law may also be expressed in terms of pressure and density of a gas. Thus, let a given mass of gas at a given temperature have a volume V_1 and density d_1 under a pressure P_1, and a volume V_2 and density d_2 under a pressure P_2, then, since the mass m of the gas remains the same, we have

$$m = d_1 V_1 = d_2 V_2\ ; \quad \text{or} \quad d_1/d_2 = V_2/V_1 \ldots\ldots\ldots\ldots(1)$$

Thus, *the density of a given mass of gas is inversely proportional to its volume.* But, by Boyle's Law, $P_1 V_1 = P_2 V_2$

or $$\frac{V_2}{V_1} = \frac{P_1}{P_2}\ ; \qquad \therefore \quad \frac{d_1}{d_2} = \frac{P_1}{P_2} \ldots\ldots \text{from (1)}$$

or *the density of a gas at constant temperature is directly proportional to its pressure.*

$\therefore$ P_1/d_1, *i.e.* the ratio of pressure to density of a gas is a constant.

139. To verify Boyle's Law. (*Experiment*).—Boyle's Law can be verified by a *Boyle's Law tube* (Fig. 88), which consists of a glass tube AB of uniform bore, connected by means of a flexible rubber tubing P to one end of another glass tube T of somewhat wider bore and open at the top. The closed tube is partly filled with dry air, and the rubber tubing with some portion of both the glass tubes is filled with clean mercury. The tubes are mounted on a vertical adjustable wooden board provided with a graduated scale to read the heights of mercury level in the two tubes.

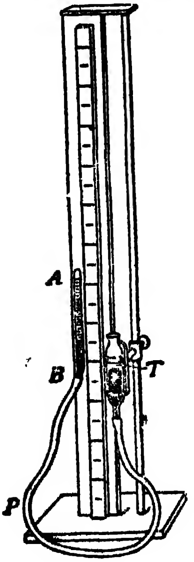


Fig. 88—
Boyle's Law tube

When the mercury levels in both the tubes are the **same**, the pressure of the enclosed air is *one atmosphere.* If the level of mercury in the wider tube is **higher** than that in the other tube, the *pressure* of the enclosed air is *one atmosphere plus the difference of the two levels of mercury* ; and if it is **lower**, then the pressure is *one atmosphere minus the difference of the two levels.*

In this way, the volumes of the enclosed

air and the corresponding pressures are read off in different positions.

To verify the law only *for pressures greater than the atmospheric pressure,* all the readings for pressure should be taken by keeping the *level* of mercury in the wider tube *higher* than that in the other tube; and *for pressures lower than the atmospheric pressure,* the *level* of mercury in the wider tube should always be *lower* than that in the other.

Plot the results on a squared paper where the abscissæ represent the pressures, and the ordinates the corresponding volumes. The curve will be a *rectangular hyperbola* (Fig. 89).

But as P is proportional to $1/V$, *a straight line* is obtained if P is plotted against $1/V$.

(a). *Verification by Graph.*—If a straight line results on plotting P against $1/V$, this is a verification of the law. (It should be known that a straight-line graph is a far more conclusive evidence than a curved line for the verification of some law.)

The law can be verified graphically even without a knowledge of the atmospheric pressure. To do this plot the excess of pressure, *i.e.* pressure above the atmospheric pressure, against $1/V$, when, again, a straight line will be obtained; for, we have $PV=K$, a constant;
$\therefore (H+X)\ V=K$; or $H+X=K/V=KY$ ...(1)
where H represents the atmospheric pressure, X the excess of pressure and Y stands for $1/V$.

This is an equation of a straight line. So, if the graph of X, the excess of pressure, and Y, *i.e.* $1/V$, gives a straight line, the law is verified.

Atmospheric Pressure.—This graph also provides a method of knowing the value of H. For when $1/V$ is 0, the intercept on the pressure axis gives the value of X (see Eq. 1). Hence H is found.

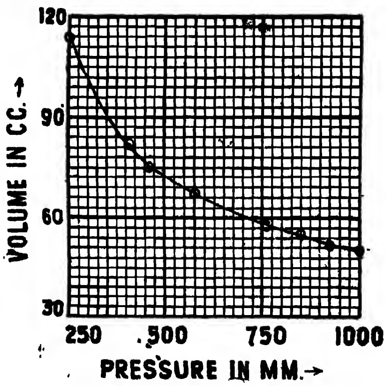


Fig. 89.—Boyle's Law Curve

The expansion of a gas at constant temperature is said to be *isothermal* (Gk. *Isos*, equal, *thermos*, heat) expansion and the curve by which the relation between pressure and volume at constant temperature is represented is said to be an **isothermal curve**. So the curve, as in Fig. 89, is an *isothermal curve* or *simply an isothermal.*

Deviations from Boyle's Law.—It should be noted that for all practical purposes Boyle's Law is true for *permanent gases* like oxgen, nitrogen, hydrogen, etc. The **permanent gases** obey Boyle's Law very nearly under moderate pressures at ordinary temperatures. But for large changes of pressure almost all gases deviate from the law more or less. A gas obeying Boyle's Law accurately for all pressures is called a **perfect gas** but *no such gas exists.*

Precautions.—For the success of the experiment with a Boyle's Law tube, the following precautions should be taken ; (*a*) The air enclosed in the tube AB must be perfectly dry.

(*b*) The scale by which mercury levels are read must be accurately vertical.

(*c*) The volume of the enclosed gas must be changed *slowly* so that that the heat developed by compression and the cold produced by expansion may be dissipated into the surrounding space.

(*d*) In order to account for any variation in the atmospheric pressure during the experiment, the barometer should be read at the beginning and at the end of the experiment, and the mean of the two readings should be taken.

139(b) Other Method.—Boyle's Law can be verified more simply by taking a glass tube AB about a metre long having a uniform bore of about 2 mm., closed at one end A and open at the other end B. The tube contains a mercury index DC about 25 cm. long which encloses a column of dry air AD (Fig. 90).

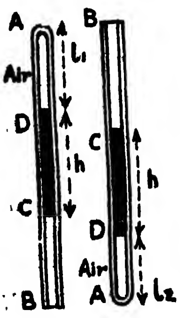


Fig. 90

Procedure.—Read the barometer and let P be the correct atmospheric pressure. Hold the tube vertically with the open end downwards. The atmospheric pressure in this case presses upwards on the mercury column, so the pressure of the enclosed air is $(P-h)$, where h is the length of the mercury column. Measure h and l_1, the length of the air column AD.

Now clamp the tube with the open end upwards. The pressure of the enclosed air now is $(P+h)$. Measure h and l_2, the length of the air column now.

If a is taken to be the cross-section of the tube, the volumes of the air enclosed in the two cases are al_1 and al_2. Now by assuming Boyle's Law to be true, we have $(P-h)al_1=(P+h)al_2$

or $\dfrac{P-h}{P+h}=\dfrac{l_2}{l_1}$, from which P can be calculated.

The result can be *checked* by measuring length of the air column when the tube is kept horizontal. The pressure in this case is P which can easily be calculated. Thus by this method we can approximately determine the *atmospheric pressure.*

In order to have more readings for the verification of Boyle's Law by the above method, the tube can be clamped at various angles with the open end up or down. In these cases h will be difference of the two vertical heights of the two ends, C and D, of the mercury column, which can be measured by using a plumb line. The pressure of the enclosed air in these cases will be $P \pm (h_2 - h_1)$ according as the open end is up or down. All the results obtained in various positions of the tube can be tabulated, and it will be seen that the product $(P \times V)$ will be proportional to $P \times l$, (where l is the length of the air column), and will be constant.

140 Faulty Barometer.—A barometer containing some air in the tube will always give faulty readings as the air will expand and depress the mercury column. To *test* whether the barometer tube contains air or not, incline the tube sufficiently, or screw up the bottom of the cistern in the case of Fortin's barometer, until the whole tube will be filled with mercury, if there is no air in it. But if there be any air in the tube it will always be left in the tube, however much the tube may be inclined or the bottom may be screwed up.

As the mercury rises and falls, this air obeys Boyle's Law, and hence it is possible to determine the correct atmospheric pressure with such a faulty barometer by the application of Boyle's Law.

Determination of correct pressure.—Let h_1 be the height of the mercury column and l_1 the length of the air column in the tube (Fig 80). Now raise or depress the barometer tube in the cistern so that the air column is about double or half of what it was. Read the new height h_2 of the mercury column and the length l_2 of the air column. If P be the correct atmospheric pressure, we have, by applying Boyle's Law, $(P - h_1) \times al_1 = (P - h_2) \times al_2$, where a is the cross-section of the tube. From this P is determined.

Example—*A faulty barometer reads 76 cm. with a space 9 cm. in length above the mercury in it when the true barometer reads 77 cm. What is the correct atmospheric pressure when this faulty barometer gives a reading of 75 cm.?*

It should be noted that the volume of air in the beginning is $(9 \times \alpha)$ c.c., and when the barometer reads 75 cm. instead of 76 cm., the volume of air above the mercury surface becomes $\{9 + (76 - 75)\}\alpha = 10\alpha$, where α is the cross-section of the tube. Hence, as explained above, we have

$$(77 - 76) \times (\alpha \times 9) = (P - 75) \times \{\alpha \times (9 + 1)\} \text{ ; or } 9 = 10P - 750, \text{ i.e. } P = 75{\cdot}9 \text{ cm.}$$

141. Explanation of Boyle's Law by the Kinetic Theory of Gases.—All substances are composed of very minute particles called molecules. According to the Kinetic theory, the molecules of gases are free to move about at random inside the vessels which contain them. In 1 c.c. of a gas there are no less than 25×10^{18} molecules. The molecules are themselves smaller in size than the spaces that separate them. *This explains why gases can be easily compressed.* Just as a stream of water particles from a hose exerts a force on the wall on which it strikes, the innumerable molecules of a gas strike against the walls of the containing vessel and these blows constitute a continuous force tending to push out these walls. (The magnitude of this force exerted by air against the walls of an ordinary room, as we have found before, is about $(16)^6$ dynes per sq. cm., or 15 lbs. per sq. inch. This may be increased to as much as 100 lbs. per sq. inch in a tyre of an automobile, or 250 lbs. per sq. inch in the boiler of an engine.) *When a molecule, proceeding with a certain velocity, hits a wall of the containing vessel it rebounds, and, therefore, undergoes a change of momentum, and the rate of change of momentum, according to Newton's second law of motion, is proportional to the force exerted on the walls. If we consider the total force exerted by the gas molecules on one square centimetre of the wall of the vessel, we shall get the pressure exerted by the gas.* If the gas is compressed to half its original volume, the number of molecules in one cubic centimetre of the vessel is doubled, *i.e.* the density of the gas is doubled, and so the number of molecules striking against one square centimetre of the wall in one second, *i.e.* the rate of striking blows against the wall, is doubled. Hence the pressure of the gas is doubled, if the temperature remains constant. Thus the Kinetic theory accounts in this simple way for Boyle's Law. When a gas is heated at constant volume, the velocity of the molecules increases and so its pressure increases.

During the *change of state* of a substance the temperature does not rise; so the heat supplied must be utilised in further separating the molecules from one another against their forces of attraction without increasing their velocity.

Examples.—*1. What volume does a gramme of hydrogen occupy at 0°C when the height of the mercurial barometer is 740 mm.? (1 c.c. of H weighs 0·00008958 gram. at 0°C. and 760 mm.)* *(C. U. 1918)*

0·00008958 gram of hydrogen at N.T.P. occupies 1 c.c.

$\therefore$ 1 gram of hydrogen at N.T.P. occupies 1/0·00008958 c.c.

If V c. c. be the volume of 1 gram of hydrogen at 0°C. and 750 mm. we have, by Boyle's law,

$$V \times 750 = \frac{1}{0{\cdot}00008958} \times 760 \; : \; or \;\; V = \frac{1}{0{\cdot}00008958} \times \frac{760}{750} = 11{\cdot}38 \text{ litres (nearly).}$$

2. What is the depth in water where a bubble of air would just float, when the height of the water barometer is 34 ft ? Given that the mass of 1 cubic foot of water is 62·5 lbs., and that of air is 5/4 oz.

Let h ft. be the depth at which the bubble would just float, when the density of air is d, and let d' be the density at the atmospheric pressure, then, we have,

by Art. 138(a), $\dfrac{34}{h+34}=\dfrac{d'}{d}$. But since at this depth the bubble of air just floats, the density of air is just equal to the density of water, so

$$\frac{34}{h+34}=\frac{d'}{\text{density of water}}=\frac{5/4}{(62\cdot5\times16)}$$

or $h=27165$ ft. $=9055$ yds. $=5\cdot14$ miles.

3. At what depth in a lake will a bubble of air have one half the volume it will have on reaching the surface ? The height of barometer at the time is 76 cms., and the density of mercury 13·6. (*All. 1928*)

Let the volume of air bubble at the surface be V c.c., and the depth below the surface at which the volume of the bubble is $V/2$ be x cms.

x cm. of water exerts the same pressure as $x/13\cdot6$ cm. of mercury.

Hence the total pressure on the bubble at bottom $=76+\dfrac{x}{13\cdot6}$ cms.

Then, by Bayle's Law we have $\left(76+\dfrac{x}{13\cdot6}\right)\times\dfrac{V}{2}=76V$;

or $\dfrac{x}{13\cdot6}=76$; $\therefore$ $x=76\times13\cdot6=1033\cdot6$ cms.

4. A barometer reads 30 inches and the space above mercury is 1 in. If a quantity of air which at atmospheric pressure would occupy 1 in. of the tube is introduced, what will be the reading of the barometer ? (*All. 1931*).

Let a be the area of cross section of the tube, so the volume of air occupying 1 inch of the tube $=a\times1$, and the pressure of the above air, before it is introduced in the tube $=30$ inches.

When air is introduced, let the mercury column comes down by x inches, which, then, is the pressure of the introduced air, the volume of which is $(x+1)\times a$ cu. inches. $\therefore$ By Boyle's Law, $30\times a\times1=x\times(x+1)\times a$;

$$\text{or}\quad x^2+x-30=0\,;\qquad\text{or}\quad (x-5)(x+6)=0\,;\qquad\text{or}\quad x=+5\ \text{or}\ -6.$$

According to the first value of x, the reading of ths barometer will be $30-5=25$ inches; and according the other value, the reading will be $30-(-6)=36$ inches. But as the pressure, after air is introduced. cannot be greater than the original pressure, the second value is not admissible.

5. A siphon barometer which a little air in its 'vacuum' indicates a pressure of only 72 centimetres, and on pouring some more mercury in the open limb until the vacuum is diminished to half its former bulk, the difference of the level becomes 70 centimetres. What is the true height of a proper barometer ? (*Pat. 1922*)

Let V be the volume of air in the tube and p the pressure exerted by this volume of air before mercury was poured in

∴ The true height of the baremeter $=72+p$. Then $V/2$ is the volume of this air after mercury was poured in. Let the pressure exerted by this volume of air be p_1. ∴ The true height $=70+p_1$

Then, we have, by Boyle's Law, $pV=p_1 \times V/2$ ∴ $p_1=2p$

But the true height of the barometer before pouring in mercury $=72+p$, and after pouring in mercury $=70+p_1$ ∴ $72+p=70+p_1=70+2p$,

∴ $p=2$. Hence the true height of the barometer $=72+2=74$ cms

6. *A tube 6 feet in length closed at one end is half filled with mercury and is then inverted with its open end just dipping into a mercury trough. If the barometer stands at 30 inches, what will be the height of the mercury inside the tube?* (C. U. 1931)

Let x ft = height of mercury inside the tube when inverted. The initial volume of air occupies $\frac{6}{2}$ or 3 ft length, and the initial pressure $\frac{30}{12}$ ft, the final volume $=(6-x)$ ft in length, and the final pressure $=(\frac{30}{12}-x)$ ft. Then, by Boyle's Law, $3\times\frac{30}{12}=(6-x)(\frac{30}{12}-x)$, or $\frac{15}{2}=(6-x)(\frac{5}{2}-x)=15-6x-\frac{5}{2}x+x^2=15-\frac{17}{2}x+x^2$

∴ $2x^2-17x+15=0$, or $(x-1)(2x-15)=0$,

Hence $x=1$ ft., or $\frac{15}{2}$ ft, or $7\frac{1}{2}$ ft

The second root is not admissible as the height cannot be $7\frac{1}{2}$ ft, i.e. longer than the tube. The required height $=1$ foot

7. *The height of the mercury barometer is 30 inches at sea level and 20 inches at the top of a mountain. Find approximately the height of the mountain if the density of air at sea level is 0.0013 gm per c.c., and of mercury 13.5 gm per c.c.*

By Boyle's Law, $\dfrac{\text{the density of air at top of mountain}}{0.0013}=\dfrac{20}{30}=\dfrac{2}{3}$,

∴ Density of air at top $=\frac{2}{3}\times 0.0013=0.00086$

∴ Mean density $=\frac{1}{2}(0.0013+0.00086)=0.00108$

The difference of pressures at the two points is equal to the weight of $(30-20)$ inches of mercury standing on one square inch, i.e. of 10 cubic inches of mercury.

Now considering the atmosphere to be homogeneous having its density equal to 0.00108, it can be found what column of this air will be equal in weight to a column of mercury 10 inches high. Hence, if h be the length of the air column, we have

$$\frac{h}{\text{length of mercury column}}=\frac{\text{density of mercury}}{\text{density of air}}; \quad \text{or} \quad \frac{h}{10}=\frac{13.5}{0.00108}$$

$$\therefore h=\frac{13.5}{0.00108}\times 10=125000 \text{ inches}=10416{\cdot}66 \text{ ft nearly}$$

8. *A bubble of air rises from the bottom of a lake and its diameter is doubled on reaching the surface. Find the depth of the lake.*

Volume of a sphere $= \frac{4}{3}\pi\,(\text{radius})^3 = \frac{1}{6}\pi\,(\text{diameter})^3$

$\therefore\ \dfrac{\text{Vol. of air bubble at bottom}}{\text{,, ,, ,, surface}} = \dfrac{\frac{1}{6}\pi\,(\text{diameter})^3}{\frac{1}{6}\pi\,(2\times\text{diameter})^3} = \frac{1}{8}$

$\therefore$ Volume at surface $= 8$ times volume at bottom.

Therefore, if V be the volume at bottom, P its pressure, and P' the pressure of the atmosphere, we have, by Boyle's Law, $PV = P' \times 8V$; $\therefore\ P = 8P'$

That is, the pressure at the bottom is 8 times the atmospheric pressure.

Now, taking the height of the water-barometer as 34 ft., the pressure increases by 1 atmosphere for every 34 ft. descent into the lake. The difference of pressures on air bubble at the surface and bottom is $(8-1)$ or 7 atmospheres.

$\therefore$ The depth of the lake $= 34 \times 7 = 238$ ft.

9. *An empty beaker floats in water with its bottom upwards and is gradually pushed down in that position. Show that after reaching a certain depth, the beaker loses all buoyancy and sinks of itself.* **(*Pat. 1941*)**

Let V c.c. be the capacity of the beaker; so it is the volume of air within the beaker at atmospheric pressure. When the beaker is forced down, the pressure on the enclosed volume of air increases and the volume of air within the beaker is diminished. At any moment the quantity of water displaced by the beaker $=$ volume of air within it. Let h cm. be the required depth below the surface when the beaker loses all its buoyancy and sinks of itself, then at this depth the weight of the beaker

$=$ wt. of water displaced; *or* $W = v + W/d$, where W is the mass of the beaker, d, the density of the material of the beaker and v, the volume of air compressed at depth h.

$$\text{or} \qquad v = W - W/d \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

Now, if H be height of water barometer, we have, by Boyle's Law,

$$VH \text{ (at the surface)} = v\,(H+h) \text{ at the depth } h$$

$$VH = (W - W/d)\,(H+h) \qquad \ldots \qquad \ldots \quad \text{from (1)}$$

$$\text{or} \quad (H+h) = \frac{dVH}{Wd - W}; \quad \therefore \quad h = \frac{dVH}{W(d-1)} - H.$$

Questions

Art 124

1. Explain what you mean by the atmospheric pressure. Give a brief description of any form of barometer. Find the height of a glycerine barometer when the water barometer stands at 32 ft. (Sp. gr. of glycering $= 1{\cdot}27$).
(C. U. 1928)

[*Ans.*: 25·19 ft.]

Art. 125.

2. Describe an experiment to prove that air exerts pressure. How is the pressure measured ? (C. U. 1917, '18, '19, '26, '37 ; Pat. '28 ; Dac. '34)

3. Prove that the pressure of air can be measured by means of a long tube containing mercury inverted over mercury in a trough. (C. U. 1940)

4. What is Torricelli's Vacuum ? Is it really a vacuum ? In performing a Torricelli's experiment, it was suspected that some little air has entered. What will you do to ascertain whether it was really so ? (C. U. 1925)

[**Hints.**—It contains a little mercury vapour ; so, strictly speaking, it is not a vacuum. (See Art. 125 (f). On depressing the tube further in the mercury trough if the tube be ultimately filled completely with mercury, there is no air inside. If there be air, the surface of mercury will not reach the closed end.]

5. State what happens in the following cases giving reasons. (a) A glass tube 20 inches long closed at one end and entirely filled with mercury is inverted over a trough of mercury, (b) a narrow glass tube open at both ends is partially dipped in a vessel containing water. The upper end is closed by the thumb and the tube taken out of water. (C. U. 1922)

Art. 126.

6. Write a short note on the different kinds of barometers which you have seen in the laboratory. How is it possible to get continuous records of the barometric height at a place ? (Pat. 1927)

7. Describe a good form of standard barometer. Can a tube of any diameter be used as a standard barometer tube ? If not, why ? Why is a thermometer always attached to a barometer ? (All. 1932)

(See also Art. 128).

8. Describe, with a neat sketch and an index of parts, a good barometer, mentioning the precautions necessary in setting up each part. Why is it necessary to have a good vacuum on the top of the mercury ?
(Pat. 1921 ; cf. 1922, '26, '39)

9. Describe briefly the process of constructing a Fortin's barometer. On what factors does the reading of a barometer depend ? Obtain an expression for reducing a barometer reading to that at the normal temperature.
(Pat. 1932)

10. Describe any form of barometer you have used in your laboratory. Give the directions necessary for reading the atmospheric pressure, and point out the precautions necessary for its use. (C. U. 1921, '29, '35)

11. Describe a Fortin's Barometer and explain how you will use it.
(All. 1930 ; Dac. '31 ; C. U. 1939, '45 ; Pat. 44)

11(a). Write a short note on Aneroid barometer. (All. 1946 ; Pat. '46)

Art. 130.

12 (a). Explain fully the meaning of the statement,—the atmosphere exerts a pressure of 15 lbs. per square inch nearly.

How would you verify the statement experimentally ? (C. U. 1919, '41)

12(b) Express the normal pressure of air in absolute units. (C. U. 1947)

13. A hollow glass sphere is weighed on a sensitive balance. The sphere is broken and the fragments are carefully collected (none being lost) and weighed. Would you expect any difference in the two weights ? Would your answer have to be modified if the weighings are carried out in vacuo ? Give reasons in each case. (Pat. 1930)

14. In order to determine whether air has weight Voltaire weighed a flexible bladder first when it was inflated with air and afterwards when it was deflated. He found both weighings to be equal and concluded that air had no weight. Criticise the conclusion. (C. U. 1926, '44)

[**Hints.**—Wt. of the inflated bladder in air = true wt. of the bladder (in vacuum) + wt. of the contained air − wt. of air displaced by the bladder (when inflated). Wt. of the deflated bladder in air = true wt. of the bladder (in vacuum) − wt. of air displaced by the bladder (when deflated).

In the first case, the wt. of air displaced by the bladder (when inflated) = wt. of air displaced by the bladder itself (*i.e.* the rubber portion only) + wt. of air equal in volume to the contained air. Therefore, ultimately the first wt. is equal to the second wt., *i.e.* the wt. of the bladder in air remains constant.]

15. Describe an experiment showing that Archimedes' principle applies to bodies immersed in a gas.

Criticise the statement 'A pound of feathers weighs less than a pound of lead'. (C. U. 1944)

Art. 133.

15 (a). Why there is difference in the reading of a barometer at Puri and at Derjeeling ? (C. U. 1947)

Art. 135.

16. What is the effect of the pressure of the atmosphere on the weight of a body ? Give reasons for your answer, and describe an experiment by which this effect can be demonstrated. (C. U. 1934)

Art. 136.

17. As a balloon rises to greater and greater altitude, what changes are found in (a) the atmospheric pressure, (b) the density of air, and (c) the lifting power of the balloon, by a person in it ? Explain the changes. (Pat. 1940)

18. A balloon, weighing 150 kg., contains 1,000 cu. m. of hydrogen and is surrounded by air density 0·00129. Calculate the additional weight it can lift. Also explain why the balloon will float in stable equilibrium at a constant altitude. (Density of hydrogen = 0·00009 gm./cu. cm.) (Pat. 1941)

[**Hints.**—Density of H per cu. m. $= 0{\cdot}00009 \times 10^6 = 90$ gm. ∴ The wt. of 1000 cu. m. of $H = 90$ kg. So total wt. $= (150+90) = 240$ kg. Wt. of 1000×10^6 cm. of air $= 1290$ kg. ∴ Lifting power $= 1290 - 240 = 1050$ kg. It will be in stable equilibrium because at a constant altitude the acceleration due to gravity, and also density of air, remain constant.]

Art. 138 & 139.

19. State Boyle's Law and describe an experiment to verify it.
(C. U. 1911, '13, '15, '16, '20, '23, '25, '31, '44 ; Pat. 1922, '28, '36, '38, '47).

20. State Boyle's Law and describe with a neat diagram the experiment you would make to verify it for pressures lower than the atmospheric pressure.
(C. U. 1941, '47)

21. The height of a barometer is 75 cm. of mercury and the evacuated space over mercury surface has a volume of 10 c.c. One cubic centimetre of air at atmospheric pressure is introduced into the evacuated space. What is the new reading of the barometer ? The cross-section of the tube is unity.
(C. U. 1921, '29)

[*Ans :* 70 cms.—because the other value 90 is inadmissible.]

22. Find the pressure exerted by a gramme of hydrogen in a vessel of 5·55 litre capacity at 0°C., assuming that the mass of a cubic centimetre of hydrogen at 0°C. and a pressure of 760 mm. of mercury is 9×10^{-5}gms.
(Dac. 1930)

[*Ans :* 152·6 mm.]

23. Assuming the water barometer stands at 33⅓ ft., find the length of a cylindrical test tube in which the water rises 1 inch if the tube is vertical and pressed mouth downward into water until the base of the tube is level with the surface of the water.
(C. P.)

[*Ans :* 21 in.]

Art. 139(b).

24. A column of air is enclosed in a fine tube by a thread of mercury 25 cm. long. The air column is 5 cm. long when the tube is held vertically with its open end uppermost. On inverting the tube, the air column measures 80 cm. Find the atmospheric pressure. (O. M. B.)

[**Hints.**—$(P+25) \times 5 = (P-25) \times 10$; $\therefore$ $P = 75$ cm.]

25. State and explain Boyle's Law.

A narrow tube with uniform bore is closed at one end, and at the other end is a thread of mercury of known length. The tube is held vertical with the closed end (i) up, (ii) down. Show how the barometric height can be determined from the positions of the thread, assuming that Boyle's Law holds.
(Pat. 1938, '47)

Art 140.

26. How would you test whether the space above the mercury column in a barometer tube contains air or not ? Show how a correction for the reading of a barometer containing some air above the mercury column may be found when no other barometer is available. (M. U. 1937)

27. A barometer whose cross-sectional area is one sq. cm. has a little air in the space above the mercury. It is found to read 77 cm. when the true

height is 78 cm., and 71 cm. when the true height is 71·8 cm. Determine the volume of the air present in the tube measured under normal conditions.

(C. U. 1937)

[Hints.—$(78-17)V = 71{\cdot}8 - 71\{V + (77-71)\}$; whence $V = 2{\cdot}463$ c.c.]

Art. 141.

28. How do you account for the pressure of a gas in a closed space and on what factor does it depend ?

CHAPTER XI

Applications of Air Pressure

(Air and Water Pumps, Siphon, Diving-bell)

Valves. - **A valve** is a trap door, hinged in such a way that when air presses on one side, it opens a little way and allows air to pass through, but it shuts up the opening when air presses on the other side. Thus the valves allow the passage of a fluid in *one direction only*. They are made in many forms according to the purposes for which they are required. They may be *conical*, *ball-shaped cup-shaped*, or *flat*, but they all fulfil the same purpose.

142. Air-Pump (or **Exhaust-Pump**.—The term 'air-pump' generally means a pump for exhausting air. The simplest type of exhaust-pump was invented by Otto Von Guericke about 1650. It consists of a barrel *AB* fitted with an air-tight piston *P* having a valve *b* opening upwards. The barrel AB is connected by means of a pipe CD to the plate EF, called the plate of the pump, on which the Receiver R (to be exhausted) is placed. There is another valve *a* opening upwards at the mouth of the connecting tube below the barrel (Fig. 91). The side tube M connected to a manometer gives the pressure within R at every stage.

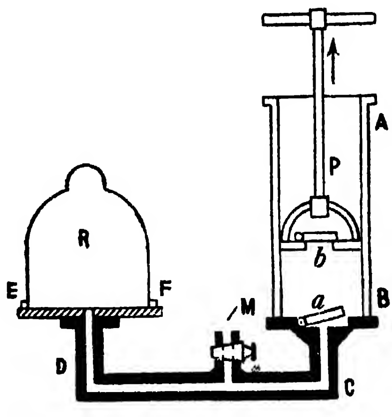


Fig. 91 —Exhaust-Pump

Action.—Suppose the piston *P* is at the bottom of the barrel. During the up-stroke of the piston, the piston valve *b* closes due to the pressure above, and the pressure of air within the barrel below the piston falls. So air from the vessel *R* lifts the valve *a* of the pipe and expands in the barrel, and thus the pressure of air in *R* is reduced.

When the piston moves down, the valve a is closed, and the air inside the barrel being compressed opens the piston valve b and escapes into the atmosphere The operation goes on until the air in the vessel R is too rarefied to lift the lower valve a.

Double-barrelled Air-Pump.—The action in the single-barrelled pump is *intermittent*. To make the action continuous, a double-barrelled pump is to be used. In such a pump, the two pistons in the two barrels are simultaneously worked by a rack and pinion arrangement such that the pistons move up and down alternately, that is, as one goes up, the other comes down. A tube coming from the two barrels is connected to the common receiver. The action in this case is very rapid, and the rate of evacuation is twice that obtained in a single-barrelled pump.

Note.—It is to be noted that a perfect vacuum can never be reached though the pressure of air in the vessel can be greatly reduced.

142(a). To calculate the Density of Air left in the Receiver.—

Let V = volume of the receiver R up to the lower valve a of the barrel.

V^1 = volume of the barrel between the two valves, when the piston is at the top of its up-stroke. When the piston occupies the lowest position in the barrel, the volume of air in the receiver is V, and for the piston occupying the highest position, the volume of air is $(V+V^1)$.

Let d = density of air originally in the receiver.

d_1 = density of air at the end of the first up-stroke At the end of the first upstroke of the piston, the volume V of air of density d occupies the volume $(V+V^1)$, and is of density d_1.

Then, we have, since the mass remains constant,

$$Vd=(V+V^1)d_1, \qquad \therefore \quad d_1=\left(\frac{V}{V+V^1}\right)d \ldots \qquad \ldots\ldots(1)$$

Now, if d_2 be the density of air at the end of the second up-stroke, we have, as before, $d_2=\left(\frac{V}{V+V^1}\right)d_1=\left(\frac{V}{V+V^1}\right)^2 d$, from (1)

Proceeding in this way it can be shown that $d_n=\left(\frac{V}{V+V^1}\right)^n d.$

If Boyle's Law holds good. *i.e.* the pressure varies directly as the density, the temperature remaining constant during the working of the pump, we have, $P_n=\left(\frac{V}{V+V^1}\right)^n P=\left(\frac{V}{V+V^1}\right)^n$ atmospheres,

where P is the original atmospheric pressure and P_n the pressure after n strokes.

[**Note.**—*From the result it is seen that* d_n *can never become zero : so a perfect vacuum is not possible.*]

143. Filter Pump (or Water Jet Pump).—It is a simple air-pump ordinarily made of glass and is used when the degree of vacuum required is not lower than 7 mm. Its special feature is that it needs no attention.

The pump is shown in Fig. 91 (a). The side tube *B* is connected with a rubber tubing to the vessel intended for evacuation. The upper end of the vertical tube *A* which tapers and ends in the nozzle *N* is connected to the water mains, the pressure of which should remain constant. As a strong jet of water forces out of the nozzle with a very high speed, some air from around the nozzle is also entangled and carried down the tube. The draught produced thereby draws out the air from within the vessel at the same rate.

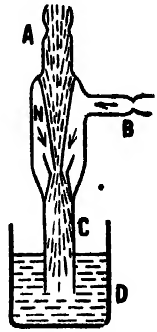


Fig. 91(a)

144. The Condensing (or Compression Pump).—This is an air-pump for compressing air into another vessel, as used for inflating foot-balls etc. It consists of a barrel *AB* in which a piston *P* works. The barrel is connected with a vessel, known as the receiver, where air is to be compressed. Both the piston and the end of the barrel contain valves *b* and *a* respectively opening towards the receiver. So it is like an exhaust pump with the valves reversed. There is a stop-cock at the mouth of the receiver which may be closed after a required amount of compression is attained (Fig. 92).

Fig. 92
Compression Pump.

Action—Suppose the piston is at the end *B* of the barrel. At the backward stroke of the piston, the pressure of air in the barrel below the piston falls ; the valve *a* is closed by the pressure of air in the receiver ; the atmospheric air acting on the other side of the piston opens the piston valve *b* ; and the barrel is filled up with air at the atmospheric pressure. At the forward stroke the piston valve *b* is closed and the air in the barrel enters the receiver by forcing the valve *a* open. This process is repeated several times.

145. The Density and Pressure of air in the Receiver after a number of strokes.—

Let V = volume of the receiver and the connecting tube ;

V_1 = volume of the barrel between the higher and the lower valves.

d = density of atmospheric air ; d_n = density of air in the receiver after n strokes. The mass of air originally in the receiver = Vd.

At each down-stroke, a volume V_1 of air at atmospheric density d enters the receiver. Hence after n complete strokes mass of air in the receiver $=(V+nV_1)d$. But its volume is V.

$$\therefore \quad \text{Its density } d_n = \frac{\text{mass}}{\text{volume}} = \left(\frac{V+nV_1}{V}\right)d = \left(1+n\frac{V_1}{V}\right)d \quad \ldots \quad (1)$$

If Boyle's Law holds good, then pressure will be directly proportional to density.

If P_n be the pressure in the receiver after n strokes and P the original pressure, we have

$$\frac{P_n}{P} = \frac{d_n}{d} = \left(1+\frac{nV_1}{V}\right), \text{ from (1), } i.e.\ P_n = \left(1+\frac{V_1}{V}\right)\text{atmospheres.}$$

146. Difference between the Compression and the Exhaust Pump.—*In the compression pump* a quantity of air whose volume is the same as that of the barrel is forced into the receiver at each stroke and as air from outside easily enters the cylinder on every backward stroke, the density of air which is forced into the receiver is always the same as that of the outside air, and consequently the mass of this air is constant. But in the *exhaust pump* the density of air extracted from the receiver diminishes with each stroke though the volume may be the same, and hence the mass of that air also diminishes.

147. Compression Pump in different Forms. (*The Bicycle Pump.*)—Ordinary bicycle pump (Fig. 93) is an example of the simplest kind of compression pump. It is made of a vulcanite or metal cylinder with a piston P inside, which is fitted with a cup-shaped leather washer W, the rim of the cup being directed towards the bottom of the pump During the up-stroke, the cup collapses inwards and air from above passes readily between the washer and the wall of the cylinder into the lower part of the barrel, and during the down-stroke the exit is closed, and by the increased pressure of air the leather is pressed air tight against the walls of the cylinder and so no inside air can pass out. As the piston comes down, the pressure becomes greater and it is sufficient to overcome the resistance of the inlet valve which is in the air-tube of the bicycle tyre.

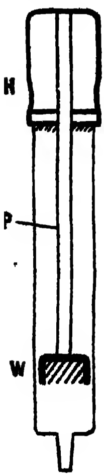


Fig. 93

The connector of the pump screws on to this *valve* which consists of a narrow metal tube entirely closed except for a small hole on one side over which a thin rubber tubing is stretched very tightly. During the *up-stroke* of the piston, the pressure of the air in the tyre presses the rubber tube to the valve, closing the small side hole, and

so the air cannot flow back from the tyre into the pump. During the *down-stroke*, the compressed air in the pump forces its way through the small hole and enters the tyre.

A *foot-ball inflator* is also a similar type of pump which has a nozzle preventing any return of air from the foot-ball.

The *Soda-water machine* also acts as a compression pump in forcing carbon dioxide gas into a bottle containing water. The water absorbs the gas and is said to be *aerated.* This water is ordinarily called *soda water*.

An *air gun* may also be regarded as a compression pump without any valve. At each stroke more and more air is forced in the barrel and becomes compressed. When the compressing force is suddenly released, the air expands with a great force and the effect is that of an explosion. This force is used in *air-guns*, where the released air works upon a spring with a great force and throws out the bullet.

If the air in the above case is released slowly then a steady force may be applied against a surface such as is used in **'Westing house' automatic brake** employed on some trains.

Compressed air is also employed in working *drills* used in quarrying, street repairs, etc.

Examples.—*1. The barrel and receiver of a condensing pump have capacities of 75 c.c. and 1000 c.c. respectively. How many strokes will be required to raise the pressure of the air in the receiver from one to four atmospheres?* (*C. U. 1925*)

Pressure after n strokes, $P_n = \left(1 + n\frac{V_1}{V}\right)$ atmospheres :

where V_1 = volume of the barrel, and V = volume of the receiver.

$\therefore \quad 4 = \left(1 + n\frac{75}{1000}\right)$; $\qquad$ *or* $120 = 3n$; *or* $n = 40$.

2. If the pressure in a pump were reduced to $\frac{1}{3}$ of the atmospheric pressure in 4 strokes, to what would it be reduced in 6 strokes? (*Pat. 1931*)

Pressure P_4 after 4 strokes, $P_4 = \left(\frac{V}{V+V'}\right)^4 P$ where P = original pressure,

V = volume of the receiver, and V' = volume of the barrel.

But $P_4 = \frac{1}{3}P$; $\therefore \frac{P}{3} = \left(\frac{V}{V+V'}\right)^4 \times P$. $\therefore \left(\frac{V}{V+V'}\right)^4 = \frac{1}{3}$; *or* $\frac{V}{V+V'} = \frac{1}{\sqrt[4]{3}}$.

After 6 strokes, $P_6 = \left(\frac{V}{V+V'}\right)^6 P = \left(\frac{1}{\sqrt[4]{3}}\right)^6 P = \frac{1}{(3)^{\frac{3}{2}}} P = \frac{P}{3\sqrt{3}}.$

That is, the pressure is reduced to $\frac{1}{3\sqrt{3}}$ of the original pressure.

148. Water Pumps.—These are instruments for raising water from a lower to a higher level, most of which depend on the principle that the atmospheric pressure is capable of supporting a column of water whose height does not exceed the height of the water barometer. This principle will be clear by considering the action of an ordinary syringe.

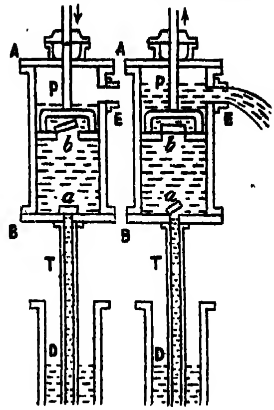
The Syringe.—It is an instrument the working of which depends on the atmospheric pressure. It is the simplest type of water pump. It consists of a hollow glass or metal cylinder ending in a nozzle and provided with a watertight piston. When the piston is drawn up from its lowest position in the cylinder (the nozzle being dipped under a liquid), a partial vacuum is created within the cylinder below the piston. So the atmospheric pressure acting on the liquid surface becomes greater than the pressure inside the cylinder and thus the liquid is pushed up into the cylinder. After sufficient liquid has been drawn into the syringe it is removed, when owing to the greater external pressure, the liquid cannot escape through the nozzle. When the piston is pushed down, the pressure inside becomes greater and so the liquid is forced out. This is the underlying principle of all the pumps, which are described below, in which the water is said to rise by *suction*. The drinking of water by drawing it through a tube is also a familiar example of the principle of suction.

Pen-filler.—The ordinary pen-filler used for fountain pens, which consists of a rubber bulb fitted at one end of a piece of glass tubing drawn out to a jet, works on the above principle. On compressing the bulb some air from inside the tube is driven out, and when the jet is now placed in the ink and the pressure inside is released, ink rises up into the tube due to the external pressure on the ink surface being greater than the pressure inside the tube.

In the *self-filling* fountain pen, the filler is inside the pen. It consists of a long rubber bag which is compressed by pulling out a metallic lever in the side of the barrel of the pen. The lever presses a metal strip against the bag and this drives out some air. On replacing the lever after immersing the nib in ink, the pressure is released and so some ink is drawn up into the pen.

149. Common or Suction Pump (Tube-Well Pump).—This is

like an ordinary syringe with an extended nozzle T terminating beneath the surface of water D which is to be raised (Fig. 94). The nozzle, or the pipe, is connected below a barrel, or cylinder, AB in which a piston P works. Two valves or trap doors a and b opening upwards are fitted, one at the bottom of the barrel, and the other within the piston. There is a spout E at the top of the barrel.



Down-stroke. Up-stroke.
Fig. 94—Common Pump

Action.—When the piston is *raised* up-stroke, the pressure inside the barrel below the piston falls : the valve a opens due to the pressure of the air outside the pipe T and the valve b closes due to the atmospheric pressure acting from above. The pressure on the surface of water in the pipe is thus less than the atmospheric pressure acting upon the water outside the pipe. So the water is forced up into the pipe.

As the piston comes *down* down-stroke, the valve a is closed, and the water in the barrel being compressed, escapes through the valve b. Further pumping will raise more water into the barrel, and finally water will rush through the valve b at the down-stroke and flow out by the spout E at the up-stroke.

One disadvantage of this pump is that it gives only an intermittent discharge on up-strokes only.

Limitation of the Suction Pump.—It should be noted that water is raised in the tube by the atmospheric pressure only, and the atmospheric pressure can support a vertical column of mercury 30 inches in length, and a column of water (30 × 13·6) in., or 34 ft. long, so the '*head of water*' *above the water surface, i.e. the distance between the valve a and the surface of water D must not exceed the height of the water barometer, that is to say, 34 feet.* In practice, however, the height is less than 34 feet (practically about 25 ft. only) as the valves have got weight and the pump is not absolutely air-tight. This kind of pump is now being widely used in tube wells.

Examples.—*(1) What is the discharge of a pump having a diameter of 1 foot, a stroke of 2 feet, and worked at the rate of 20 strokes per minute ?*

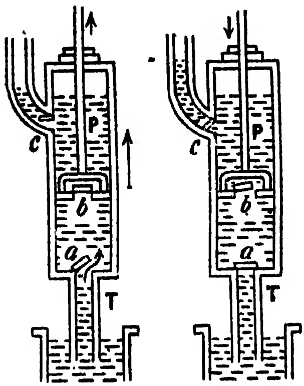
The volume of the barrel of the pump $= \pi \times (\frac{1}{2})^2 \times 2 = 1·5708$ cu. ft.

In a single acting pump, the half of the number of strokes per minute is only effective in discharging water. Hence the volume of liquid discharged per minute $= 1·5708 \times \frac{20}{2} = 15·708$ cu. ft.

(2) The piston of a suction pump is 10 ft. above the level of water in a well, and the height of the water column above the piston is 4 ft. If the diameter of the

barrel is 6 inches, find in lbs-wt. the force necessary to sustain the piston in its position. (1 cu. ft. of water weighs 62·5 lbs)

Let h be the height of the water barometer. Then the pressure exerted under piston $=(h-10)$ ft. of water, and the pressure over the piston $=(h+4)$ ft. of water. The difference of these two pressures $=(h+4)-(h-10)=14$ ft. of water. Hence the necessary force = the wt. of the column of water of ht. 14 ft. and radius $\left(\frac{6}{2\times 12}\right)$ ft. $=\pi\times(\frac{1}{4})^2\times 14\times 62{\cdot}5$ lbs. $=171{\cdot}87$ lbs.



Up-stroke. Down-stroke.
Fig. 95.—Lift Pump.

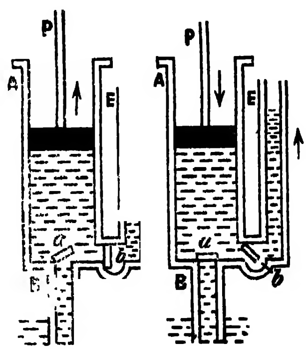
150 The Lift Pump.—This pump is only a modification of the *common* or *suction pump* used to raise water to a desired height. Here the spout is turned upwards and fitted with a valve C (Fig. 95) opening outwards. At every up-stroke of the piston the valve of the spout opens, and if the top and collar of the pump are water-tight, it would be possible to lift water up the spout. At the down stroke the valve C closes owing to the pressure of the water collected on it, the valve b opens upwards and the valve a also closes owing to the pressure on it.

[**Note.**—If the piston be strong enough, water can be raised to any height, *but the head of water below the piston cannot exceed the height of the water barometer (i.e. 34 ft.)*, if water be the liquid that is being pumped.]

151. The Force Pump.—The difference between this pump and the common pump is that here the piston is solid and the outlet is fitted to the bottom of the barrel with a valve b opening outwards. (Fig. 96).

Action.—At each up-stroke the water rises into the barrel AB and at each down-stroke the valve a closes, and the water of the barrel is forced out, opening the valve b through the delivery pipe E.

Arrangement for Continuous flow.—The *disadvantage* of this type is that the *delivery is intermittent, i.e.* on down-stroke only. To overcome this, an air-chamber R (Fig. 97) is placed in the vertical portion of the delivery pipe beyond the valve b. At the time of the discharge of water on the down-stroke, some water is



Up-stroke. Down-stroke.
Fig. 96.—Force Pump.

collected into the air-chamber which compresses the inside air. On the up-stroke the compressed air expands and forces water below it to flow up the pipe of the air-chamber and hence a continuous flow is obtained.

[**Note.**—Applying sufficient force to the handle of the piston, water can be raised to *any height* if the machine be very strong. If the height be very great, then water can be collected by one pump in a reservoir at a certain height from which it can be raised again by *another pump*.]

Fire-Engines.—These are used for extinguishing fire and are merely machine driven force pumps where a continuous flow of water is obtained with the help of an air-chamber as just described.

In the present form of the fire engine the continuity of the flow is maintained more efficiently by means of two force pumps connected to a common air-chamber and working with alternating strokes.

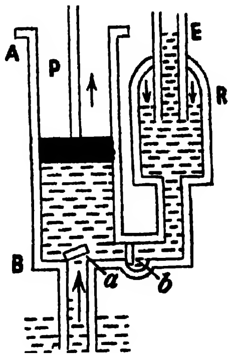


Fig. 97

Points to be Remembered.—The **suction pump** depends on the atmospheric pressure for its working, and the height to which it can raise water is therefore limited to 34 ft. theoretically,—much less in actual practice.

In a **force pump**, pressure is directly applied to the liquid by means of a piston, and this pump is not dependent on the atmospheric pressure. The height to which water can be raised by such a pump depends on the strength of its parts and the power supplied (hand, steam, or electric). The maximum distance through which it is safe to raise water in this way is, however, about 300 ft. There is no valve in the piston of a force pump.

152. The Siphon.—It consists of a bent tube with one of the arms AB longer than the other CD (Fig. 98). It is used for transferring a liquid from one vessel to another at a *lower level.*

The tube is first filled with the liquid to be drawn off; the two ends are temporarily closed with fingers, and then the shorter leg is placed in the vessel, below the level of the liquid, to be emptied. On opening the two ends, the liquid begins to flow.

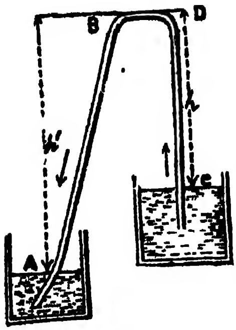


Fig. 98 —Siphon.

Let P = atmospheric pressure, d = density of the liquid, h, h' = vertical heights of D and B above the respective liquid surfaces.

The pressure p_1 at D urging, the portion of the liquid at D to the left $= P - h\,d\,g$.

The pressure p_2 at B urging the portion of the liquid at B to the right $= P - h'dg$.

$\therefore \quad p_1 - p_2 = (h' - h)d\,g$. But $h' > h$.

$\therefore$ The pressure at $D >$ the pressure at B.

Hence the water flows from D to B, and water from the vessel will be raised by the atmospheric pressure to D for filling up the vacancy so caused. Thus the flow will be maintained.

Conditions for the Working of the Siphon.—(1) The end A of the tube must be *below* the level C of the liquid in the vessel to be emptied ; otherwise the pressure of the liquid at A (which is ordinarily equal to atmospheric pressure) + wt of the column $(h' - h)$ will be less than the atmospheric pressure, and the liquid will not flow.

(2) *The height h must be less than the height of the corresponding liquid barometer* ; otherwise the pressure of the atmosphere will not be able to raise the liquid to D. So the greatest height of h, in the case of water, is 34 ft.

(3) The siphon would not work in vacuo, for the atmospheric pressure is non-existent there.

(a) **Effect of making Holes in the Siphon**.—When a hole is made at any point in the longer arm AB (Fig. 98) *above* the surface C of water in the vessel in which the shorter leg is placed, air will enter because the pressure there is less than the atmospheric pressure and so the working of the siphon will stop.

If, however, a whole is made at a point in AB *below* the surface at C, the remaining portion above that point will still form a siphon through which the liquid will continue to flow.

Example.—*The two arms of a siphon having an internal diameter of 2 inches are respectively 12 and 8 inches in length. The shorter arm is immersed in a liquid to a depth of 2 inches. Calculate the velocity of flow of the liquid and also the amount of the liquid discharge through the siphon in one second.* ($g = 32{\cdot}2$ *ft./sec*^{2}.)

The flow of the liquid depends upon the height $(h' - h)$ (see Fig. 98). So, we have, from the law of falling bodies, the velocity of flow per sec., $v = \sqrt{2g(h' - h)}$.

Here $h' = 12$ inches $= 1$ ft. ; h, i.e. the actual height over the level of water $= (8-2) = 6$ inches $= 0.5$ ft.

$$\therefore \quad v = \sqrt{2 \times 32.2(1 - 0.5)} = 5.66 \text{ ft. per sec.}$$

The amount of liquid discharged in one sec. = velocity of flow × area of the tube $= 5.66 \times \left\{\pi \times \left(\frac{2}{12 \times 2}\right)^2\right\}$ cu. ft. $= 0.123$ cu. ft.

153. Intermittent Siphon —Fig. 99 represents an intermittent siphon, which is an example of the application of the principle of a siphon. The vessel is at first empty, but as any liquid is poured into it, and the level of the liquid gradually reaches the top of the bend, the liquid will begin to flow to O. If the supply of the liquid is discontinued, or the liquid escapes faster than it is supplied to the vessel, the flow will cease as soon as the shorter branch no longer dips in the liquid. But the flow will, however, resume when the level of the liquid reaches the bend again on the supply being restored.

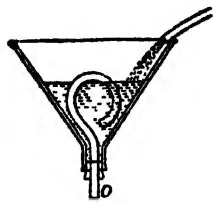


Fig. 99

Tantalus Cup —The above principle is applied in the toy siphon, known as the *Tantalus Cup*, where the siphon is concealed inside the figure of Tantalus, placed inside a vessel.

Automatic Flushes.—The same principle is also applied in automatic flushes fitted in public latrines, etc., where a siphon is fitted inside a tank, which is emptied as soon as water fills the bent pipe.

154. The Diving-bell.—When a tumbler is inverted and lowered vertically under water, we notice that there is a slight rise of water in the tumbler. As it is depressed further into the liquid, the enclosed air is under a pressure which increases, and there is only a little water rising into the vessel. This principle is applied in the diving-bell which is a heavy hollow cylindrical metal vessel open at its lower end. It is lowered into water to enable divers to go to the bottom of deep water to do some work (Fig. 100). The tension in the chains which supports the bell is equal to the weight of the bell *minus* the weight of water displaced by it, and this tension increases as the bell sinks more and more and the weight of displaced water becomes less.

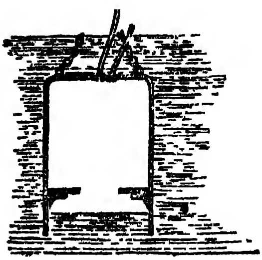


Fig. 100—Diving-bell.

Taking 34 ft. as the height of water barometer, the pressure of air within the bell at a depth of 34 ft. will be 2 atmospheres ; consequently the volume of air is halved, and the water would rise half-way up the diving-bell. As this is obviously inconvenient for the workmen inside the bell, a constant supply of fresh air is pumped into the bell through a pipe in order to prevent water from entering the chamber and also to enable the workmen to breathe.

Examples. – (*1*) *A bottle whose volume is 500 c.c. is sunk mouth downwards below the surface of a tank containing water. How far must it be sunk for 100 c.c. of water to run up into the bottle? The height of a barometer at the surface of the tank is 760 mm. and the sp. gr. of mercury is 13·6.* (*Pat. 1928*)

The volume of the air inside the bottle, when 100 c.c. of water rushes in $=500-100=400$ c.c.

If P be the pressure in cms. when the volume of the enclosed air is 400 c.c., then, by Boyle's Law, $P\times 400=76\times 500$; or $P=\frac{76\times 500}{400}=95$ cms.

∴ The pressure exerted by water only $=95-76=19$ cms. of mercury

i.e. $=19\times 13\cdot 6$ cms. $=258\cdot 4$ cms. of water. (∵ atmos. press. $=76$ cms.)

∴ The bottle must be sunk below 258·4 cms. of water.

(*2*) *Find to what depth a diving bell must be lowered into water in order that the volume of air contained may be diminished by one quarter, the length of the bell being 3 metres, atmospheric pressure 760 mm. of mercury, and the sp. gr. of mercury 13·6.* (*Pat. 1943*)

Length of the bell $=3$ metres $=300$ cms.

If P be of the total pressure in cms. when the bell is lowered into water in order to diminish its volume by one quarter, we have, by Boyle's Law,

$(300\times\alpha)\times 76=(\frac{3}{4}\times 300\times\alpha)\times P$, where α is the area of the base of the bell ;

∴ $P=\frac{76\times 4}{3}$ cms. of Hg. $=\frac{76\times 4\times 13\cdot 6}{3}$ cms. of water.

∴ The pressure exerted by water only

$$=\left\{\frac{76\times 4\times 13\cdot 6}{3}-\left(76\times 13\cdot 6\right)\right\}=\frac{76\times 13\cdot 6}{3}\text{ cms. of water.}$$

The volume of air inside being diminished by one quarter, the height of water inside the bell $=\frac{1}{4}\times 300=75$ cms., and so length of air inside $=(\frac{3}{4}\times 300)$ $=(3\times 75)$ cms.

∴ The depth to which the bell is lowered, i.e. the height of water from the surface up to the top of the bell $=\left\{\frac{76\times 13\cdot 6}{3}-(3\times 75)\right\}$ cms. $=119\cdot 53$ cms.

Questions

Art. 142.

1. Describe in detail an air-pump giving a diagram.
(C. U. 1923; Pat. '25, '29, 38; All. '25, '27).

After four strokes the density of the air in the receiver of an air-pump is found to bear to its original density the ratio of 256 to 625. What is the ratio of volume of the barrel to that of the receiver? (C. U. 1923)

(See also Art. 143) [*Ans :* 1 : 4]

2. Describe briefly the action of air-pump in its simplest form and explain how the degree of rarefaction produced by a given number of strokes can be approximately calculated. Can the apparatus you describe create perfect volume? If not, why? (Pat. 1931, '38, '41)

Art. 142(a).

3. If the cylinder of an air-pump is one-third the size of the receiver, what fractional part of the original air will be left after 5 strokes? What will a barometer within the receiver read, the outside pressure being 75 cms.?
(Pat. 1929)

[Hints. $d^5/d = (3/4)^5$; again, $P_4 = (3/4)^5 \times 76$].

4. Compare the pressure in the receivers of a condensing and exhausting air-pump after the same number of stroke in each case and account for the fundamental difference in form of the two expressions. (Pat. 1931).

(See also Art. 146)

5. Describe a double-barrelled air-pump and explain its action.
(C. U. 1938, '47)

6. A mercury barometer is in the receiver of an air-pump, and at first its height is 76 cms. After two strokes the height is 72 cms. What will it be after ten strokes. (Neglect the volume of the barometer.) (Pat. 1937)

[*Ans :* 57 cms.]

7. Explain the mode of action of a foot-ball inflating pump. (Pat. 1929)

Art. 143.

7(a). Write a short note on 'Filter pump'. (All. '46)

Art. 144.

8. Describe in detail with a diagram a condensing pump and its mode of action. (C. U. 1925)

Art. 147.

9. Describe, with the help of a neat sketch, the working of an ordinary bicycle-pump, and the action of the valve in the bicycle tube. (Pat. 1944)

10. Describe a suction pump. Water cannot be raised to a height much greater than 30 ft. by means of such a pump. State the reason for this and describe a laboratory experiment by which you prove your explanation to be correct. (C. U. 1930, '24; Dac. 1932)

11. Describe in detail with a diagram a common pump and its mode of action. Is there any limit to the depth from which it can raise water?
(C. U. 1924 ; Pat. '33 : Dac. '32.)

Arts. 150 & 151.

12. Explain clearly the working of the usual types of Lift or Force pumps.

A lift pump is used to pump oil of sp. gr. 0·8 from a lower into an upper tank. What is the maximum possible height of the pump above lower tank, when the pressure of the atmosphere is 76 cms. of mercury? Is this height practically obtained? Give reasons for your answer. (Pat. 1920)

[**Hints.** $h \times a \times 0{\cdot}8 \times g = 76 \times a \times 13{\cdot}6 \times g$; $\therefore$ $h = \dfrac{76 \times 13{\cdot}6}{0{\cdot}8} = 1292$ cm.]

Art. 152.

13. Explain the action of a siphon.
(C. U. 1926, '37 ; Pat. '21 ; Dac. '26 ; All. '46)

14. A siphon is used to empty a cylindrical vessel filled with mercury. The shorter limb of the siphon reaches the bottom of the vessel which is 45 inches deep, but it is found that mercury ceases to run before the vessel is empty. Explain this observation, and calculate what fraction of the volume of the vessel will remain full of mercury. The barometric height may be taken as 30 inches. (Pat. 1935 ; cf. C. U. 1926 ; Dac. ' 30).

[*Ans* : $\frac{1}{3}$.]

15. Explain the principle and use of the siphon, and state how the principle is used in Tantalus cup. (C. U. 1928)

(See also Art. 153).

PART II

HEAT

CHAPTER I

Heat, Temperature, Thermometry

1. **What is Heat ?**—When we touch a lump of ice we feel cold. When we stand in the sun we feel hot. The cause of these sensations is called **heat**. But what is its real nature ? What does a body gain or lose when it becomes hot ? Until less than a hundred years ago it was thought that when bodies rose or fell in temperature they took in or gave out a weightless fluid called *caloric*. This theory has, however, been completely abandoned. The explanation now given is that a rise of temperature is caused by the quickening of the movements of molecules composing the body, and a fall in temperature is caused by a slowing down of the molecular movements.

Every one knows that friction produces heat. When we rub a piece of. wood against another, we increase the movements of their molecules which produce heat. This molecular movement is brought about because they are gaining energy at the expense of the mechanical work we do at the time of rubbing. Again, when we pump up a bicycle tyer, heat is produced at the expense of work done, that is, the heat is due to the energy spent in forcing the air into the tyre. The heat is not due to friction, as it will be seen that very little heat will be produced by only pumping up and down without attaching it to the bicycle tyre. When coal burns in air, the quantity of chemical energy lost is received by a body put in it as the energy of heat.

Heat is, therefore, a form of energy and may be defined as the energy possessed by a bodu as a result of the internal vibrations of the molecules of which the body is composed. Thus, to heat a body is to increase its store of energy and to cool a body is to diminish its store of energy.

2 **Temperature.**—When on touching two substances A and B we feel A hotter than B, we express it by saying that the temperature of A is higher than that of B. *Temperature is*, therefore, *a measure of*

the hotness, or coldness, of a body. **It is the thermal state of a body which determines whether it will give heat to, or receive heat from, other bodies.** Heat tends to flow from a body at a higher temperature to a body at a lower temperature just at water flows from higher to lower levels. Thus *heat* and *temperature* are analogous to *water* and *level.*

3. **Heat and Temperature.**—It is no doubt that the terms *heat* and *temperature* are closely related, but it should not be thought that they convey the same meaning. They should be distinguished by considering the following.—(*a*) If a red-hot iron ball is put in a bucket of warm water, the ball will lose some of its heat, and water will gain it. In this case, the quantity of heat contained in water is likely to be much greater than that in the ball, but it is temperature, and not the amount of heat, which determines the flow of heat from one body to another. (*b*) *Temperature* is no more *heat* than the *level of water* in a tank is the *water* itself (see Art. 2). (*c*) Temperature is the *thermal condition* of a body, and is quite different from the amount of *heat energy* in the body. (*d*) Two bodies may have the same temperature, but different quantities of heat. A spoonful of a sweet liquid taken from a larger quantity will be as sweet as the larger, though it does not contain as much sugar. So the temperature of boiling water in a tea cup may be the same as that in a big bucket, but the quantity of heat in the water of the bucket will be much greater, because the quantity of heat contained in a body depends on its *temperature* as well as on its *mass.*

4. **Effects of Heat** :—

(1) *Change of State.*—When water is heated for some time it changes into steam (a vapour). When water is cooled, *i.e.* heat is withdrawn from it, it may change into ice (a solid).

(2) *Change of Temperature* (without change of state).—When a body absorbs heat its temperature rises, and when it gives out heat its temperature falls, except when it is not changing its state, as water into steam, or water into ice.

(3) *Change of Dimension.*—Every body, whether a solid, a liquid, or a gas, expands on heating and contracts on cooling.

(4) *Change of Composition.*—(*Chemical change*).—The composition of many substances, when heated, is changed altogether. Sugar, for example, when heated in a test tube is turned into carbon, which is

left at the bottom of the tube, and water vapour, which condenses at the top of the tube.

(5) *Change of Physical properties.*—Many bodies when heated show weakness possibly due to some internal rearrangement of their molecules. Thus, iron when heated to redness differs materially from iron at ordinary temperatures, and ordinarily glass when heated becomes weakened.

(6) *Electrical effect.*—When by heating one of the junctions of two dissimilar metals, say copper and iron, a difference of temperature is produced between the junctions, an electric current flows into the wires. This is known as *thermo-current* (see Ch. V. Part VII).

5. Measurement of Temperature. (*Thermometers*).—We can have an idea about temperature, *i.e.* the degree of hotness, by our sense of touch. But the measurement of temperature by our sense of touch often gives unreliable and inaccurate results. This sensation depends upon, (*i*) *the amount of heat transferred* to the skin of the body from the substance touched, when the temperature of the substance is higher than that of the body ; or from the skin to the substance, when the temperature of the substance is lower than that of the body, and on (*ii*) *the conductivity of the substance*, that is, on the rate at which heat is transferred. As this sensation is not a safe guide in the correct and numerical measurement of temperature, instruments, called *thermometers*, are devised for the purpose, where a change in temperature is recorded by a change in volume of a suitable substance or changes in some other property of the substance.

6. Choice of a Thermometric Substance.—In choosing a substance for preparing a thermometer it is necessary to see that (*a*) *the substance always shows the same temperature for the same hotness ;* (*b*) *the temperature changes continuously with the change of the degree of hotness ;* (*c*) *the substance is convenient to use ;* (*d*) *the expansion of the substance is fairly large.*

Liquids are suitable as thermometric substances as their expansions are moderate ; whereas solids expand little, and gases expand very much. Of all liquids *mercury* has been found to be the best on account of its many advantages (Art. 11).

It should be noted that the most reliable way of measuring temperature is by making use of the expansion of gases (see Art. 35).

7. Construction of a Mercury Thermometer.—A thick-walled capillary glass tube of *uniform bore* with a bulb *B* blown at one end

is taken. At C, near the open end, the tube is heated and drawn out so as to make a narrow neck there.

A small funnel E is fitted at the open end by means of a piece of rubber tubing (Fig. 1). Some pure and dry mercury is put in the funnel E, but the mercury cannot get into the tube owing to the contained air and the fineness of the bore. The bulb is heated gently to drive out some of the air in it, which, on cooling, contracts in volume, and so mercury from the funnel passes down the tube into the bulb. *This process of alternate heating and cooling* must be repeated several times till sufficient mercury has entered to fill the bulb and the lower part of the tube. The funnel is taken away and the bulb is strongly heated until the mercury fills the whole of the tube, which is then sealed at C by a blow-pipe flame. Mercury having filled the entire tube, the tube is free from air. On cooling, the mercury contracts, and, at ordinary temperatures, fills the *bulb* and a part of the *stem*. The rest of the tube contains only a negligible quantity of mercury vapour.

Fig. 1.

Three points are to be observed regarding thermometer construction.—

(1) The size of the bulb and the bore of the tube will depend upon the sensitivity of the thermometer and the number of degrees and their subdivisions which the thermometer is to register ; that is, a thermometer to read to 1/5th$^\circ$ or 1/10th$^\circ$ must have a longer tube with a finer bore than a thermometer reading only to 1°.

(2) The quantity of liquid used should be small so that it might take as little heat as possible from the source whose temperature is being recorded, otherwise it will result in a lowering of that temperature. Thus the bulb should be smaller in size.

(3) The *bulb* of the thermometer should be made thin so that the heat from the source may quickly pass through to warm up the liquid, and thus the thermometer will be quick in action.

8. Graduation.—The tube being filled with mercury and sealed, it should be left out for several days to enable it to recover its original volume. Then, for *graduation*, the two *fixed points* are to be determined.

Lower-fixed point (or **Ice point**)—It is the temperature at which pure ice melts under normal atmospheric pressure. Since its variation with pressure is very small, it is determined under the ordinary atmospheric pressure and no correction is necessary. The funnel *F* (Fig. 2) contains powdered ice washed with distilled water. A hole is made in this ice and the bulb of the thermometer (*T*) is inserted in it while the thermometer is held vertically in it by means of a stand. The mercury column descends and after sometime takes a *stationary stand*, when the position of its top is marked on the glass. This gives the lower-fixed point.

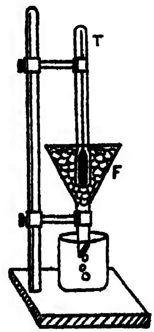


Fig. 2

Upper fixed point (or **Steam point**).—It is the temperature at which pure water boils under the normal atmospheric pressure. It is usually determined under the ordinary atmospheric pressure and correction is then made remembering that the boiling point of pure water varies directly by 1 C. as the superincumbent pressure changes by 27 mms.

The thermometer *T* is inserted into the inner chamber of a *hypsometer* (Fig. 3) leaving the upper part projecting out above the cork *C*. The boiler *D* contains water upto a level below the bulb of the thermometer. It is heated and the steam generated by boiling water heats up the mercury of the thermometer. The heating is regulated in order that the pressure of steam may be equal to the atmospheric pressure outside, which is indicated by the equality of the Hg-levels in the manometer *M*. When the Hg-top in the thermometer is observed to have become stationary, it is marked. The thermometer is held in the steam, and not in the water, because the temperature of the latter may be higher than the boiling point corresponding to the atmospheric pressure due to any dissolved impurity.

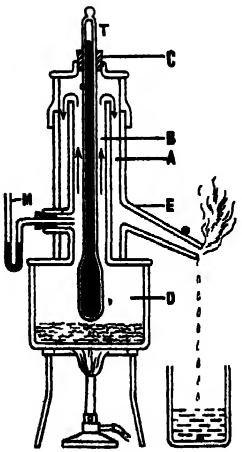


Fig. 3 – Hypsometer

After locating the positions of the two fixed points on the stem, the interval between the points is divided into the appropriate number of equal parts, called **degrees**, in some desired scale of temperature.

This method of marking assumes that the *bore of the tube is uniform* and that *the liquid expands uniformly*.

Should the Bore of tube be uniform ?—Unless the bore is uniform, equal rise of Hg. in the tube will not indicate equal rise of temperature and so the graduation shall have to be done point to point throughout the bore. Such action being tedious and costly, a tube of uniform bore is selected for commercial success.

8 (a) Two Sources of Error.—(*1*) *Non-uniformity of the bore.*—Each degree of a thermometer represents equal change of temperature, which is measured by a change of volume of a certain mass of the thermometric liquid contained in the bulb. When the temperature rises, the liquid column moves along the stem of the thermometer, and the movements of the liquid column due to corresponding changes of volume of the liquid will be uniform only if the bore is uniform ; otherwise each small part of the stem will not indicate equal rise of temperature.

(*2*) *Temperature of the Exposed Column.*—At the time of using a thermometer for recording a temperature, a part of the stem remains outside the substance, the temperature of which is different from that of the bulb and the rest of the stem below it. So the temperature recorded will be lower than the actual temperature ; and thus it is desirable to include as much of the stem as possible inside the substance whose temperature is to be taken.

8. (b) The Hypsometer.—Hypsometer is a Greek word meaning a 'measurer of height'. The apparatus (Fig. 3) is used for the determination of the boiling point of a liquid from which measurement of the height of the place can be made. It consists of a brass vessel having the internal chamber *B* and the chamber *A* external to it closed at the bottom, *B* being in communication with the boiler *D*. The manometer *M*, open at both ends, indicates the pressure of the vapour inside. The vessel is closed at the top by means of a cork *C* through which the thermometer *T* is inserted such that its bulb is above the level of liquid in the boiler. The vapour rises along the internal chamber and passes down the external chamber and finally escapes through the exit tube *E* into the atmosphere. The liquid formed by condensation is received by the receptacle as shown in Fig. 3. The outer chamber containing the heated vapour protects the vapour in the inner chamber from condensation. When the level of liquid in the manometer is the same in both the limbs, the pressure is atmospheric.

9. Scales of Temperature.—There are three scales of temperature in use ; **Fahrenheit, Centigrade,** and **Reaumur.**

Anders Celcius (1701—1744), a Swedish astronomer, introduced a scale by taking 0° as the boiling point of water and 100°C. as the melting point of ice.

Then at about 1742 Linne introduced the *Centigrade scale* by reversing the above with melting point of ice at 0° and boiling point of water at 100°C.

Fahrenheit scale was devised by Fahrenheit, a German philosopher (1686—1736), at about 1709 in which a temperature of the freezing mixture of snow and common salt (which is much below the melting point of ice) was taken as the zero of his scale. The melting point of pure ice, according to this scale, was taken as 32°, and the boiling point of water as 212°, under normal atmospheric pressure. This scale is generally used by doctors and meteorologists and engineers.

Reaumur scale was introduced by Reaumur (1683—1757), a French philosopher in 1731 in which the melting point of ice was taken as 0° and the boiling point of water, under normal atmospheric pressure, as 80°.

The Fahrenheit scale is generally used in England for household purposes. It is also used in constructing clinical thermometers. The Centigrade (from L. *Centum*, a hundred + *gradus*, step) scale is used in scientific works all over the world. The Reaumur scale is used in some parts of the continent and Russia.

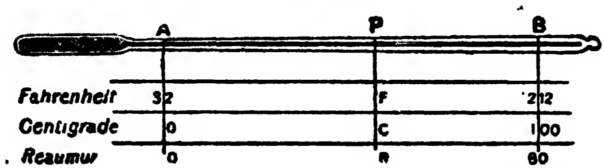


Fig. 4

The distance between the two fixed points of a thermometer is called the **Fundamental Interval**, which is 180 for Fahrenheit, 100 for Centigrade, and 80 for Reaumur scale (see Fig. 4 and also Fig. 27)

Scale	Symbol	Freezing Point	Boiling Point	No. of Divisions between Fixed Points
Fahrenheit ...	F	32°	212°	180
Centigrade ...	C	0°	100°	100
Reaumur ...	R	0°	80°	80

Comparison.—We find that, $100°C. = 180°F. = 80°R.$

or, $1°C. = \frac{9}{5}$ of $1°F. = \frac{4}{5}$ of $1°R.$

Let P (Fig. 4) be the position of the top of the mercury thread, and let F, C, R, be the readings of the temperatures on the three scales,

then, $\frac{AP}{AB} = \frac{F-32}{180} = \frac{C-0}{100} = \frac{R-0}{80}$, *or,* $\frac{F-32}{9} = \frac{C}{5} = \frac{R}{4}$

Remember that 1 Centigrade degree is nine-fifths of a Fahrenheit, and 1 Fahrenheit degree is five-ninths of a Centigrade degree

Examples 1 —*Calculate the temperature which has got the same value on both the Centigrade and the Fahrenheit scales.*

Let x be the value required, then, $\frac{x-32}{2} = \frac{x}{5}$, or, $5x - 160 = 9x$,

or, $4x = -160$ · i.e. $x = -40$ Thus, $-40°C$ when converted to the Fahrenheit scale will also be $-40°$, *or,* $-40°C. = -40°F$

2. *The same temperature when read on a Centigrade and a Reaumur thermometer gives a difference of 1°. What is the number of degrees indicated by each thermometer ?*

Let x = Centigrade temperature, and let y = Reaumur temperature

Then, we have, $x - y = 1$ ... ... (1)

Now, $x°C.$ transformed into Reaumur degrees $= x \times \frac{4}{5} = y$

$\therefore$ from (1), $(1+y)\frac{4}{5} = y$, $\therefore$ $y = 4°R.$

But $4°R. = 4 \times \frac{5}{4} = 5°C.$ $\therefore$ The required temperatures are $5°C.$ and $4°R.$

3 *Find out the temperature when the degrees of the Fahrenheit thermometer will be 5 times as the corresponding degrees of the Centigrade thermometer.*

Let x = Fahrenheit temperature and y = Centigrade temperature

then $x = 5y$ ... (1)

But $x°F.$ transformed into Centigrade degree $= (x-32)\frac{5}{9} = y.$

$\therefore$ from (1), $(5y+32)\frac{5}{9} = y$; or, $16y = 160$, $\therefore$ $y = 10°C.$

And $10°C. = (10 \times \frac{9}{5}) + 32 = 50°F.$

Hence the required temperatures are $10°C.$ and $50°F.$

4. *Two thermometers A and B are made of the same kind of glass and contain the same liquid. The bulbs of both the thermometers are spherical. The internal diameter of A is 7·5 mm , and the radius of cross-section of its tube is 1 25 mm. . the corresponding figures of B being 6·2 mm. and 0·9 mm. Compare the length of a degree of A with that of B.*

Let l_1 and l_2 be lengths corresponding to 1° rise in temperature for A and B respectively and γ the apparent coefficient of expansion of the liquid. Increase in volume of the liquid in the bulb of A for 1° rise $= \frac{4}{3}\pi\left(\frac{7{\cdot}5}{2}\right)^3 \times \gamma \times 1$; and this must rise in the tube, the volume being $\pi(1{\cdot}25)^2 l_1$; $\therefore \frac{4}{3}\pi\left(\frac{7{\cdot}5}{2}\right)^3 \times \gamma \times 1 = \pi(1{\cdot}25)^2 l_1$. Similarly for B, $\frac{4}{3}\pi\left(\frac{6{\cdot}2}{2}\right)^3 \times \gamma \times 1 = \pi(0{\cdot}9)^2 l_2$;

$$\therefore \quad \frac{l_1(1{\cdot}25)^2}{l_2(0{\cdot}9)^2} = \frac{(7{\cdot}5)^3}{(6{\cdot}2)^3} \text{ ; whence } \frac{l_1}{l_2} = \frac{1{\cdot}00}{1{\cdot}09}.$$

10. Corrections for Thermometer Readings.—The temperature at which water boils depends upon the atmospheric pressure. It is 100°C. when the atmospheric pressure is normal, *i.e.* 760 mm. The temperature at which water boils increases or decreases with the increase or decrease of the atmospheric pressure. For small deviations from the normal pressure there is *a change of 1°C. in the boiling point of water for a change of about 27 mm. in the atmospheric pressure, and so a change of about two-thirds of a degree Fahrenheit for 10 mm. change of pressure.* The effect of change of pressure is, however, negligible on the freezing point of water, which is lowered only by about 0·0073 of a degree Centigrade for one atmosphere increase of pressure.

So the fixed points of a thermometer can be corrected at any time by reading the height of the barometer. This will be clear from the following observations :—

Atmospheric pressure $= 754{\cdot}96$ mm.

$\therefore$ Difference from the normal pressure $= 760 - 754{\cdot}96 = 5{\cdot}04$ mm.

There is a variation of 1°C. for a change of 27 mm. in the atmospheric pressure. $\therefore$ The required correction $= 5{\cdot}04 \div 27 = 0{\cdot}186°$.

But as the observed atmospheric pressure is less than the normal pressure, the steam point will be less than 100°C. Thus the true steam point $= (100 - 0{\cdot}186) = 99{\cdot}814$°C.

Observed steam point $= 99{\cdot}6$°C. $\therefore$ Error at steam point $= 99{\cdot}6 - 99{\cdot}814 = -0{\cdot}214$°C. $\therefore$ Correction at steam point $= +0{\cdot}214$°C.

If for the above thermometer the freezing point is 0·5° above zero, the error is +0·5°C., and the correction to be applied is −0·5°C. Thus plotting these two points on a squared paper, the straight line (Fig. 5) joining these two points will indicate the corrections at intermediate temperatures. From the graph it is evident that no correction would be required at 70°C.

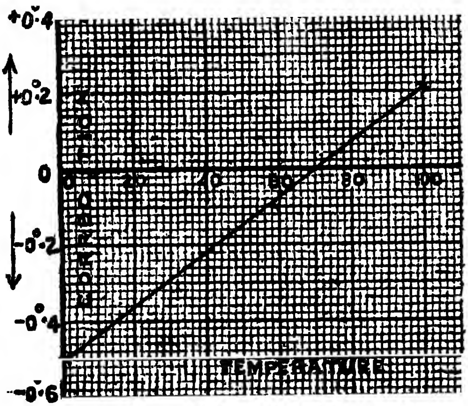


Fig. 5

Examples—*1. The stem of a Fahrenheit thermometer has a scale upon it which is graduated in equal parts. The reading of the ice-point is 30 and of the steam-point 300. What is the reading indicated by the thermometer (a) when placed in steam at a pressure of 73 cm. of mercury, and (b) in water at 50°F.*

(*a*) Here $300-30=270$ scale divisions are equivalent to 180°F.

$\therefore$ 1 scale division $=(2/3)$°F.

The difference of pressure $(76-73)=3$ cm. For 10 mm., *i.e.* 1 cm. change in pressure, the boiling point is changed by 2/3°F. $\therefore$ For a change of 3 cm. in pressure, the change in boiling point $=3\times\frac{2}{3}=2$°F. $\therefore$ The true steam point $=212-2=210$°F. (-2 is taken because the pressure is below normal).

Now, 2°F. is equivalent to $2\div\frac{2}{3}=3$ scale divisions of the thermometer.

Hence the reading indicated by the thermometer $=300-3=297$.

(*b*) The temperature of water is 50°F. $=(32+18)$°F.

$\therefore$ The reading is 18°F. above the ice-point, which is 30 on the scale.

Now, 18°F. is equivalent to $18\div\frac{2}{3}=27$ scale divisions. $\therefore$ The reading is $30+27=57$.

2. If, when the temperature is 0°C., a mercury thermometer reads +0·5°C., while at 100°C., it reads 100·8°C., find the true temperature when thermometer reads 20°C., assuming that the bore is cylindrical and the divisions are of uniform length. (*C. U. 1926*).

The thermometer reads 0·5°C. for 0°C. and 100·8°C. for 100°C. So there are $(100·8-0·5)=100·3$ divisions between the two fixed points of this thermo-

meter. ∴ Each division of the above thermometer $= \frac{100}{100 \cdot 3}$ of a true Centigrade division. When the thermometer reads 20°C., there are $(20 - 0 \cdot 5)$ or $19 \cdot 5$ divisions above the freezing point. Hence the true temperature of the thermometer when it reads $20°C. = \frac{100 \times 19 \cdot 5}{100 \cdot 3} = 19 \cdot 442°C.$

3. When the fixed points of a Centigrade thermometer are verified, it reads 0·5°C. at the melting point of ice and 99·2°C. at the boiling point of water at normal pressure. What is the correct temperature when it reads 15°, and at what temperature is its reading exactly correct? (*Pat. 1944*)

The fundamental interval $= 99 \cdot 2 - 0 \cdot 5 = 98 \cdot 7$ divisions. Let x be the correct temperature, then, we have $\frac{15 - 0 \cdot 5}{98 \cdot 7} = \frac{x}{100}$, when normal boiling point $= 100°C.$, whence $x = 14 \cdot 7°C.$

Again, let the reading be exactly correct at t°C., then $\frac{t - 0 \cdot 5}{98 \cdot 7} = \frac{t}{100}$;

or $100t - 50 = 98 \cdot 7t$; or $t = 38 \cdot 5°C.$

11. Advantages of Mercury as a Thermometric Substance.—

(1) It can be used through a very wide range, *i.e.* from $-39°C.$ to 350°C.

(2) It does not require much heat to raise its temperature; so the temperature of the body with which it is in contact is very slightly affected.

(3) It expands uniformly.

(4) It quickly transmits heat throughout its substance, so it readily assumes the temperature of the substance in which it is placed.

(5) It can readily be obtained pure.

(6) It is a shining opaque liquid and does not wet glass.

12. Different Forms of Thermometer.—

(1) **Mercury-in-Glass Thermometer.**—These have been dealt with before. (See Arts. 7 & 11).

(2) **Alcohol Thermometer.**—Alcohol is sometimes used as a thermometric substance instead of mercury. Its advantages and disadvantages are given below :—

Advantages.—(*a*) For the measurement of low temperatures

alcohol is used as the thermometric substance instead of mercury, because it has the advantage of a much *lower freezing point* (–130°C.) compared to mercury (–39°C.).

(*b*) The expansion of alcohol is high compared with that of mercury, so alcohol is more sensitive as regards expansion.

(*c*) It is a light liquid which wets glass, and thus a thread of alcohol can move smoothly in a tube of very fine bore.

Disadvantages.—(*a*) Alcohol cannot be used for high temperatures as it boils at 78 C., whereas mercury boils at 357°C.

(*b*) The expansion of alcohol is not as uniform as that of mercury. and for this reason alcohol thermometers are usually graduated by direct comparison with a mercury thermometer, both being placed side by side in the same bath.

(*c*) It is a bad conductor of heat in comparison with mercury.

(*d*) It is highly volatile and so it begins to distil and readily collects at the top of the stem, which being colourless is less likely to be noticed.

(*e*) Mercury is opaque and so it can be easily seen in glass, but alcohol has to be coloured with a dye.

(*f*) Alcohol wets glass, so a film of alcohol sticks to the side of glass when the temperature falls.

(3) **Water Thermometer.**—Water has almost all the disadvantages of alcohol and its advantages are very few. Besides this, it

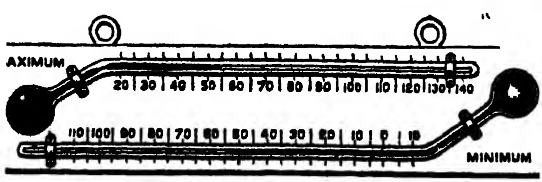


Fig. 6. Maximum and Minimum thermometers

cannot be used as a thermometric substance due to its peculiar behaviour between 0°C. and 10°C., which has been discussed in Art. 26.

(4) **Gas Thermometer.**—These have been dealt with in Art. 35.

(5) **Maximum and Minimum Thermometers.**—*The maximum thermometer* (Fig. 6) registers the *highest temperatures* attained during a period.

It is an ordinary *mercury thermometer* with a constriction in the tube just above the bulb. When the thermometer gets heated, mercury rises in the tube, and, when cooled, the mercury column breaks at the constriction. The mercury left in the tube registers the highest temperature. To use it again, the mercury is driven into the bulb by giving a few jerks.

The **Clinical Thermometer** (*Doctor's thermometer*) is a maximum thermometer constructed on the above principle (Fig. 7). It is used for measuring the temperature of the human body. It is graduated from 95°F. to 110°F. which give the maximum limits of human temperature.

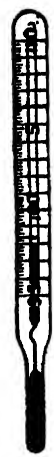


Fig. 7 Clinical Thermometer

In the *minimum thermometer* alcohol is used as the thermometric substance. There is a dumb-bell shaped glass index inside the alcohol, which is carried back to the lowest point by means of the *surface tension* of alcohol (see Art. 100, Part I). To set the thermometer, it is tilted until the index touches the meniscus. Fig. 6 illustrates Rutherford's maximum and minimum thermometers, both of which are used in a horizontal position.

Six's Thermometer (Fig. 8) is a combined form of maximum and minimum thermometer.

It consists of a graduated U-tube with a bulb at each end. The tube on the right-hand side of Fig. 8 and a part only of the bulb at that end contain alcohol. The upper part of the bulb contains alcohol vapour only, and so room for expansion is left there. The bent tube contains a column of mercury which merely serves as an *index*, as its movement indicates expansion or contraction of alcohol which is above it, and also in the other tube which is completely full of alcohol. The alcohol in the left-hand tube and the bulb constitute the real thermometric part of the instrument. A small steel index fitted with a spring (shown on the side of Fig. 8) is inside the tube at each end of the mercury column. Each index can be brought into contact with the mercury head at each end by means of a magnet applied outside the tube.

When the temperature rises, the alcohol in the left-hand tube expands and so the mercury thread on the right-hand tube rises up,

pushing the index before it. When the temperature falls, that mercury thread comes down leaving the index in its position (as it is prevented from returning by the spring), but the mercury thread in the left-hand tube rises up pushing the index before it, which remains there when the alcohol expands again due to rise in temperature. Thus, the lower end of the index in the left-hand tube shows the minimum temperature, while that in the right-hand tube shows the maximum temperature.

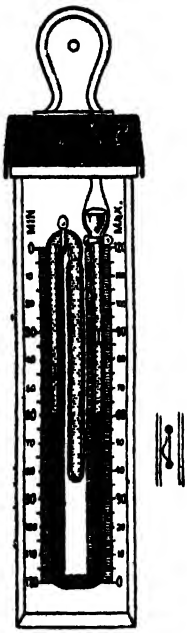


Fig. 8—Six's Thermometer

(a) Measurement of High Temperatures.—Mercury thermometers are ordinarily used up to 350°C., but they can be used for higher temperatures by introducing an inert gas like nitrogen or argon in the tube above mercury. This gas prevents mercury from boiling by exerting pressure on it. (The reason for this will be explained in a later chapter.) In this way mercury thermometers can be used up to 600°C. or 700°C. But as glass softens above 450°C, quartz or silica must be used instead of glass. An instrument used for the measurement of high temperature is called a *pyrometer*. Generally liquid thermometers are not suitable for this purpose. A constant volume, or constant pressure, gas thermometer may be used for this purpose, if the bulb is made of porcelain, instead of glass (see Art. 34). The most reliable way of the measurement of temperature is by using gas as thermometric substance.

SOME NOTEWORTHY TEMPERATURES

	Deg. C.		Deg. C.
Sun	6000	Mercury boils ...	357
Electric arc light ...	3400	Mercury freezes ...	39
Iron melts ...	1500	Blood heat ...	37
Iron white hot ...	1300	Very great cold ...	50
Hydrogen boils,	—252 to —253	Red heat	500—1000
Hydrogen solidifies,	—256 to- -257	White heat, above	1000
Lowest temperature obtained...... −272°C.			

Questions

Art. 4

1. Distinguish between temperature and quantity of heat. (C. U. 1934 ; Pat. 1921).

Art. 7

2. Briefly describe the process of constructing a mercury-in-glass thermometer. Why is it necessary to note the height of the barometer when determining the upper point of a thermometer ? How would you prepare a thermometer if you are in a deep coal mine ? (Pat. 1932).

[**Hints.**—See Arts. 7 and 10. Note the barometric height inside the coal mine and calculate the boiling point of water which will be the upper fixed point of the thermometer.]

3. There are two thermometers of which one has the larger bulb and the other finer bore. Explain the advantages and disadvantages in each case. (C. U. 1941)

Arts. 7 & 8

4. Describe the construction of a mercurial thermometer. Is it necessary that the tube should be of uniform bore throughout ? Give reasons for your answer. How is it graduated ? (C. U. 1926, '41, '45 ; cf. Pat. 1920, '22, '44).

Art. 9 & 10.

5. What is meant by the 'Fundamental Interval' (F. I.) of the thermometer scale in a thermometer ? Describe an experiment to determine it accurately.

A thermometer *A* has got its F. I. divided into 45 equal parts and another *B* into 100. If the lower point of *A* is marked 0 and of *B* 50, what is the temperature by *A* when it is 110 by *B* ? (Pat. 1940)

[*Ans* : 27°]

6. The boiling point of sulphur is 444·6°C. What Fahrenheit temperature does this correspond to ?

[*Ans* : 932·3F.]

7. The freezing point of a Fahrenheit thermometer is correctly marked and the bore of the tube is uniform, but it reads 76·5° when a standard Centigrade thermometer reads 25°. What is the reading of the boiling point on this Fahrenheit thermometer ? (L. M.)

[*Ans* : 210°.]

8. A thermometer, having a tube of uniform bore and divided into degrees of equal length, reads 20° in melting ice and 80° in steam at 100°C. Find what it would read at 100°F.

[*Ans* : $42\frac{2}{3}$°]

9. How would you test the accuracy of the "fixed points" of a mercury thermometer ? Explain the conditions which contribute to its sensitiveness. (C. U. 1937).

Art. 11.

10. Explain why in a thermometer it is advantageous to have the thermometric substance (1) of low specific heat, (2) a good conductor of heat.

Art. 12.

11. Describe any two forms of maximum or minimum thermometers. (Pat. 1921, cf. '29 ; All. 1916)

12. What do you mean by a maximum thermometer? Describe the clinical thermometer in detail with a diagram. (C. U. 1928, '37)

13. State the relative advantages of mercury and alcohol as thermometric substances. (C. U. 1919, '41 ; All. 1916)

CHAPTER II

Expansion of Solids

13. **Expansion of Solids.**—With rise of temperature most bodies expand. Solids in general expand on heating but different solids expand differently for the same rise of temperature. This can be shown by a simple apparatus known as *Grave sand's Ring*. It consists of a metal ball (Fig. 9) suspended by a chain, when both are at the same temperature. If the ball be placed on the ring after heating it in a Bunsen flame, it will no longer pass through the ring showing that it has expanded, but, on leaving the two together a short time, the ball falls through for, on cooling, it contracts. This can be shown by the following experiment :—

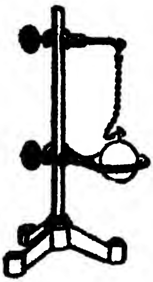


Fig. 9. Grave sand's Ring.

Expt.—Let a compound bar of brass and iron be made by riveting two strips of iron and brass. At ordinary temperature the bar is straight (Fig. 10.*a*), but, when heated edgewise, the bar bends, the more expansible brass being on the outside (Fig. 10.*b*). On cooling it in a freezing mixture of ice and salt it bends in the opposite way with the brass strip on the inside (Fig. 10.*c*). The above principle is applied to compensate the balance wheel of a good watch for changes of temperature (see Art. 19).

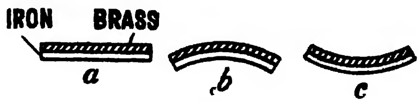


Fig. 10

Solids, when heated, expand in three ways, namely in length, called *linear expansion* ; in area, called *superficial expansion* ; in volume, called *cubical expansion*.

Liquids and gases, however, expand only in volume ; so, for liquids and gases, the term expansion is used only with reference to cubical expansion.

Note.—Solids, as a rule, expand as they are heated. Stretched india-rubber is an exception to this. It contracts when heated. In the cases of certain alloys (e.g. Invar) the expansion on heating is almost negligible.

Experiment shows that the increase in length of a metal bar is proportional to (i) the original length of the bar, (ii) the increase in temperature, and (iii) to a certain fraction, called the *co-efficient of linear expansion* of the substance concerned.

Co-efficient of Linear Expansion of a Solid.—It is the increase in length per unit length at 0°C. per unit rise of temperature.

Let l_0 be the initial length of a rod at 0°C. and let l_t be the length when heated through t°C., then the expansion of the rod for a rise of temperature $t°C. = (l_t - l_0)$, $\therefore$ Expansion for $1°C. = \dfrac{(l_t - l_0)}{t}$;

and the expansion for 1°C. per unit length $= \dfrac{(l_t - l_0)}{l_0 \times t}$.

Hence, the co-efficient of linear expansion α (pronounced "*alpha*") is given by, $\alpha = \dfrac{(l_t - l_0)}{l_0 t}$; *or* $l_t = l_0(1 + \alpha t)$.

Hence, *the co-efficient of linear expansion*

$$= \frac{\textit{Increase in length}}{\textit{Original length at } 0°C. \times \textit{Rise in temperature}}.$$

Does α depend on the Unit of length & Scale of temp. ?—

$\alpha = \dfrac{\text{Change in length per unit change of temp.}}{\text{Original length.}}$. It is to be noted that $\dfrac{\text{change in length}}{\text{original length}}$ is a ratio and has the same value whether length is measured in the C. G. S. or the F. P. S. unit of length. Therefore,

(a) Co-eff. of linear exp. has the same value both in cms. and inches, if the unit of temp. is the same.

(b) Co-eff. of linear exp. per degree Centigrade is $1\frac{4}{5}$ times larger than that per degree Frahrenheit, since $1°C. = \frac{9}{5}°F$. So, the value of Co-eff. of linear exp. depends on the scale of temp. used.

The co-efficient of linear expansion of iron for 1°C. is 0·000012 means that 1 cm. of iron rod raised in temperature by 1°C. expands by 0·000012 cm. ; or

1 yard of iron rod raised in temperature by 1°C. expands by 0·000012 yard ; or

1 foot of iron rod raised in temperature by 1°C. expands by 0·000012 foot, etc., etc.

14. Co-efficient of Expansion at Different Temperatures.—We have seen that in defining the co-efficient of linear expansion of a solid we should refer to its length at 0°C., but practically it is not

always convenient to measure the length at 0°C., and *so generally the length at the beginning of the experiment, i.e. at the temperature of the room, is taken, instead of its length at 0°C.* In the case of solids, the error made by doing so is very small and can be neglected.

The length of a rod, which is initially not at 0°C. but at some other temperature, say t_1°C., may be calculated thus :—

Let l_o, l_1 and l_2 be the lengths at 0°C., t_1°C., and t_2°C. respectively, where t_2 is greater that t_1,

then $l_1 = l_o(1+\alpha t_1)$; and $l_2 = l_o(1+\alpha t_2)$

$$\therefore \frac{l_2}{l_1} = \frac{(1+\alpha t_2)}{(1+\alpha t_1)} = (1+\alpha t_2)(1+\alpha t_1)^{-1} = (1+\alpha t_2)(1-\alpha t_1) = 1+\alpha(t_2-t_1)$$

neglecting terms containing higher powers of α.

$$\therefore \quad l_2 = l_1\{1+\alpha(t_2-t_1)\}, \quad \text{or} \quad \alpha = \frac{l_2-l_1}{l_1(t_2-t_1)}.$$

15. Measurement of Linear Expansion.—(*Pullinger's Apparatus*).—In this method the increase in length of a metal rod is measured by a spherometer. The rod which is about a metre long is surrounded by a steam-jacket having inlet and outlet tubes for steam, and two other side tubes for thermometers (Fig. 11). The rod rests with its lower end on a glass plate fixed into the base board of the apparatus ; the other end, which is free to expand upwards, reaches up to a hole in another glass plate on the top of the apparatus. This glass plate supports three legs of a spherometer, which is so placed that the central leg can be screwed down to touch the top of the rod.

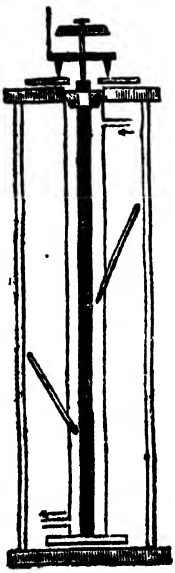


Fig. 11.—Pullinger's apparatus.

Expt.—Measure the length of the rod at the room temperature by a metre scale and place it in its proper position in the jacket. Introduce two thermometers in the side tubes and note the temperature t_1 after sufficient time has elapsed. If the readings of the two thermometers differ, then take the mean of the two temperatures. Adjust the spherometer so that its central leg just touches the top of the rod and take the reading. Now screw up the central leg to allow room for expansion, and pass steam through the steam-jacket for some time until the temperature is constant, as indicated by the thermometers ; let it be t_2°C. Screw down the central leg till contact is made and take the reading. The difference of the two spherometer

readings gives the increase in length, say x cm., of the rod for the rise of temperature $(t_2^\circ - t_1^\circ)$.

The correct result of the experiment depends upon the accuracy of measuring the total expansion of the rod, that is, on the accuracy of the spherometer readings. Then,

if l be the original length of the rod at t_1°C., we have, $\alpha = \dfrac{x}{l(t_2 - t_1)}$.

16. Substances not affected by Change of Temperature.—There are a few substances, like fused *quartz*, fused silica, and invar, which are very little affected by change of temperature. Vessels made of fused *silica*, or fused quartz, expand or contract very little when their temperatures are changed. In the laboratory the crucibles can be made red-hot and then suddenly cooled without any risk of cracking.

A measuring rod made of *invar*, which is an alloy of nickel and steel, containing 36 per cent. of nickel (invented by the French metallurgist M. Guillaume) shows very little change of length with change of temperature, its co-efficient of linear expansion (0·0000009) being almost negligible.

Note.—It may be remembered that glass and platinum expand almost equally, and the expansion for the alloy invar is very small.

16. (a) Superficial and Cubical Expansions.—The *co-efficient of superficial expansion* is the increase *in area* per unit area of surface per unit rise of temperature.

If S_o and S_t be the initial and final areas of a body ; t° the rise in temperature, then the co-efficient of superficial expansion β (pronounced "*beta*") $= \dfrac{S_t - S_o}{S_o t}$; or $S_t = S_o(1 + \beta t)$ ... (1)

Relation between α and β.—Consider a square surface of a homogeneous isotropic solid, each side of which is l_o at 0°C. and l_t at t°C. The area of the surface at 0°C., $S_o = l_o^2$, and at t°C., $S_t = l_t^2$.

But $l_t = l_o\,(1 + \alpha t)$, where α is the co-efficient of linear expansion,

$\therefore$ $S_t = \{l_o(1 + \alpha t)\}^2 = l_o^2(1 + 2\alpha t + \alpha^2 t^2)$. Since α is very small, terms containing α^2 or higher powers of α can be neglected.

$$\therefore \quad S_t = l_o^2(1 + 2\alpha t) \qquad \ldots \qquad \ldots \quad (2)$$

Again from (1) $\qquad S_t = S_o(1 + \beta t) \qquad \ldots \qquad \ldots \quad (3)$

$\therefore$ from (2) and (3) $1 + \beta t = 1 + 2at$ ($\because$ $S = l_o^2$) ; or $\beta = 2\alpha$

i.e. Co-efficient of area expansion = 2 × its linear co-efficient.

Note.—The error due to neglecting $\alpha^2 t^2$ can be seen as follows :—

Let us take the case of iron, where $\alpha = 0·000012$, and $\beta = 0·000024$.

The part neglected is $\alpha^2 = (0·000012)^2$

$\therefore$ Percentage error in value for co-efficient of superficial expansion for $1°C. = \frac{(0·000012)^2}{0·000024} \times 100 = 0·00006$. (This is a negligible error).]

17. *The Co-efficient of Cubical Expansion* is the increase *in volume* per unit volume per unit rise of temperature.

Thus, if V_o, V_t be the volumes at $0°C.$ and $t°C.$ respectively, and γ (pronounced "*gamma*") the co-efficient of cubical expansion, then

$$\gamma = \frac{V_t - V_o}{V_o \times t}; \quad or \quad V_t = V_o(1 + \gamma t).$$

As in Art. 14, it can be proved that $\gamma = \frac{V_2 - V_1}{V_1 (t_2 - t_1)}$, where V_2 is vol. at t_2 and V_1, vol. at t_1.

Relation between α and γ.—Consider a cube of a solid each side of which is l_o cm. at $0°$C., and l_t cm. at $t°$C. Then, we have, as before, $V_o = l_o^3$, and $V_t = l_t^3$, where $l_t = l_o (1 + \alpha t)$.

$\therefore V_t = \{l_o(1 + at)\}^3 = l_o^3(1 + 3at + 3\alpha^2 t^2 + \alpha^3 t^3) = l_o^3(1 + 3\alpha t) = V_o(1 + 3\alpha t).$

(neglecting the terms containing α^2 and α^3). But $V_t = V_o (1 + \gamma t)$;

Hence, we have, $1 + \gamma t = 1 + 3\alpha t$; whence $\gamma = 3\alpha$.

i.e. Co-efficient of cubical expansion = 3 × its linear co-efficient.

Examples.—(1) *A glass rod when measured with a zinc scale, both being at 20°C., appears to be one metre long. If the scale is correct at 0°C., what is the true length of the glass rod at 0°C.? The coefficient of linear expansion of glass is 8×10^{-6}, and that of zinc 26×10^{-6}.* (*Pat. 1920*)

At 0°C. each division of the zinc scale is 1 cm. and at 20°C. each division $= (1 + 0·000026 \times 20) = 1·0052$ cm.

$\therefore$ 1 metre or 100 cms. of the zinc scale at 20°C. $= 100 \times 1·00052$ $= 100·052$ true centimetres.

Hence, the correct length of the glass rod at 20°C. = 100·052 cm.

The true length of the glass rod at $0°C. \times (1 + 0·000008 \times 20) = 100·052$.

$\therefore$ The true length of the glas at $0°C. = \frac{100·052}{1 + 0·000008 \times 20} = 100·086$ cms.

(2) ***A steel scale reads exact millmetres at 0°C. The length of a platinum wire measured by this scale is 621, when the temperature of both of them is 17°C. Find the exact length in millimetres of the platinum wire. What would be the exact length of the wire at 0°C.?***

(a) Coefficient of linear expansion of steel $= 0.000012$.

At 17°C. one scale division of the steel scale which is correct at 0°C. is not exactly 1 mm, but a little greater than 1 mm

1 scale division at 17°C. would contract to 1 mm. at 0°C.

$\therefore$ 621 scale divisions at 17°C. would contract to 621 mm. at 0°C.

$\therefore$ The exact length in mm of 621 scale divisions at 17°C.

$$= 621\ (1 + 0.000012 \times 17) = 621.127$$

(b) Co efficient of linear expansion of platinum $= 0.000008$.

$\therefore$ Length of platinum wire at 0°C $\times (1 + 0.000008) \times 17 = 621.127$.

$\therefore$ Length of platinum wire at $0°C = \dfrac{621.127}{1.000136} = 621.042$ mm.

(3) *An iron bar whose sectional area is 4 sq cm is heated from 0°C. to 100°C. What force would it exert if it were prevented from expanding? The modulus of elasticity for iron $= 2 \times 10^{12}$ dynes, the mean co efficient of linear expansion of iron $= 0.0000122$*

If l_0 be the length of the bar at 0°C, and l its length at 100°C, then $(l - l_0)/l_0 = 100 \times 0.0000122 = 0.00122$ ... ... (1)

But we know that if l_0 be the natural length of the bar, and l its length when stretched with a force of F dynes, then the Young's modulus (Art. 97.)

$Y = \dfrac{Fl_0}{s(l - l_0)}$, $\therefore F = \dfrac{Ys(l - l_0)}{l_0}$ (where s = sectional area of the bar)

from (1) $= 2 \times 10^{12} \times 4 \times 0.00122$, $= 976 \times 10^7$ dynes.

(4) *A cube whose sides are each 103 cms at 0°C. is raised to 100°C. If the sides become each 101 cms. find the co efficients of linear and cubical expansion.*

Original length $= 100$ cms, final length $= 101$ cms.

The co-efficient of linear expansion $= \dfrac{101 - 100}{100 \times 100} = .0001$

and the co efficient of cubical expansion $= .0001 \times 3 = .0003$.

(4) *An iron clock pendulum makes 86405 oscilations one day, at the end of next day the clock has lost 10 seconds, find the change in temperature. The co-efficient af linear expansion of iron is 0.0000117).* [*Pat* 1924].

For a pendulum, $t = 2\pi\sqrt{\dfrac{l}{g}}$. Similarly, if t' be the period of the pendulum when the length is increased to l' for any change of temperature θ,

$$t' = 2\pi\sqrt{\frac{l'}{g}}, \quad \therefore \frac{t'}{t} = \sqrt{\frac{l'}{l}} = \sqrt{\frac{l(1+\alpha\theta)}{l}} = \sqrt{1+\alpha\theta} \quad \ldots \quad (1)$$

(α = co-efficient of linear expansion of the metal)

There are $24 \times 60 \times 60 = 86,400$ seconds in a day. So a correct seconds pendulum will make 86,400 swings in a day.

In the example, $t = \dfrac{86400}{86405}$ sec. ; $t' = \dfrac{86400}{86395}$ sec.

$\therefore$ from (1) $\dfrac{t'}{t} = \dfrac{86405}{86395} = \sqrt{1+\alpha\theta} = (1+\alpha\theta)^{\frac{1}{2}} = (1+.0000117\theta)^{\frac{1}{2}}$

$= 1 + \frac{1}{2}(.0000117\ \theta) = 1 + .00000585\ \theta,$ whence $\theta = 19.8°C.$

(6) *A clock with a brass pendulum beats seconds at 0°C. What will be the difference in its rate per day when the temperature is 30°C.? (Co-efficient of linear expansion of brass is 0 000019).*

The time of vibration of a pendulum is proportional to the square root of its length. So, if t and t_o represent periods corresponding to lengths l at 30°C. and l_o at 0°C., we have

$$\frac{t}{t_o} = \sqrt{\frac{l_o(1+\alpha\theta)}{l_o}}$$ (where α = co-efficient of linear expansion of brass ; θ = change in temperature)

$= \sqrt{1+0.000019\times 30} = 1.00028.$ $\therefore$ $t = 1.00028$ ($\because$ $l_o = 1$ sec).

But the number of seconds in a day $= 24\times 60\times 60 = 86400$ = number of swings of the pendulum at 0°C., when it beats true seconds.

$\therefore$ The number of swings per day at $30°C. = \dfrac{86400}{1.00028} = 86375.8.$

So the clock will lose $(86400-86375.8) = 24.2$ seconds per day.

(7) *A clock which keeps correct time at 25°C. has a pendulum rod made of brass. How many seconds will it gain per day when the temperature falls to the freezing point? (Co-efficient of linear expansion of brass 0.000019)* (*C. U. 1931*).

Let l_o = length at 0°C. ; l_{25} = length at 25°C.

t_o = period corresponding to the length l_o ; t_{25} = period corresponding to the length l_{25}. Then, we have, $\dfrac{t_{25}}{t_o}\sqrt{\dfrac{l_{25}}{l_o}} = \sqrt{\dfrac{l_o(1+0.000019\times 25)}{l_o}}$

$$= (1+0.000475)^{\frac{1}{2}} = (1+\tfrac{1}{2}\times 0.000475) \text{ approx.} = 1.0002375.$$

But because the pendulum keeps correct time at 25°C., the value of

$$t_{25} = 1 \text{ second., } t_o = \frac{1}{1.0002376} \text{ sec.}$$

There are 86400 seconds in a day. So the pendulum makes 86400 swings at 25°C., when it keeps correct time, *i.e.* when $t_{25} = 1$. $\therefore$ when period

$= \dfrac{1}{1.0002375}$ sec., the number of swings $= 86400 \div \dfrac{1}{1.0002375} = 86420.52.$

$\therefore$ The pendulum gains $(86420.52 - 86400) = 20.52$ seconds.

18. Practical Examples of Expansion of Solids.—In many cases precautions have to be taken on account of the expansion of metals by heat.

(*a*) *Why in laying rails, a small gap is left in between ?*—

Railway lines are laid by leaving a space of about a quarter of an inch between the successive rails in order to allow for expansion when heated, as otherwise the rails will force each other out of the line. Similarly, allowance is made for expansion in mounting girders for iron bridges. [The electric tram lines, however, are welded together. These lines serve as electrical conductors and are continuous. As the rails are embedded in the ground the variation of temperature is small.] The joints of gas and water pipes are made like those of a telescope in order to allow a certain amount of 'play' at the ends. (*b*) The length of metal chains used in 'surveying' has got to be corrected for variation of temperature. (*c*) In riveting boiler plates, red-hot rivets are used, which, on cooling, contract and grip the plates so tightly as to make the joints steam-proof.

The same principle is adopted in fixing iron tyres on cart wheels. The tyre is at first made somewhat smaller in diameter, and then heated until expanded sufficiently to be easily put on the wooden wheel. On cooling, the tyre contracts and binds the wheel firmly.

Fire alarms are also based on this principle. One form of this consists of a compound bar of brass and iron. When hot it bends over and completes an electric bell circuit, and thus rings the bell.

(d) *Why in drinking hot water, a thin bottomed glass is taken ?*—

Thick-bottomed drinking glasses frequently crack if hot water is poured into them. Glass is a bad conductor of heat. So it takes some time for equalisation of temperature in different portions, due to which there is unequal expansion of the inner and outer layers and hence it cracks.

For similar reasons the tightened glass stopper in a bottle may often be loosened by pouring hot water on to the outside of the neck of the bottle. In this way the neck expands before the stopper and so the stopper becomes loose.

(e) *In sealing metallic wires into glass, why platinum is used ?*—

Sometimes it becomes necessary to seal metallic wires into glass vessels. If a piece of copper is sealed through a glass vessel, the joint usually fractures on cooling due to unequal contraction of copper and glass. But platinum and glass have almost the same expansion, hence platinum can be safely used for the same purpose without fear of cracking.

Examples (*1*). *The distance between Allahabad and Delhi is 390 miles. Find the total space that must be left between the rails to allow for a change of temperature from 36°F. in winter to 117°F. in summer.* [*All. 1932*]

$36°F. = (36-32) \times \frac{5}{9} = \frac{20}{9}°C.$; $117°F. = (117-32) \times \frac{5}{9} = \frac{425}{9}°C.$

$390 \text{ miles} = 390 \times 5280 \times 12 \times 2{\cdot}54$ cms.

The total space to be left = expansion of iron rails 390 miles long for $(\frac{425}{9} - \frac{20}{9})°C.$ change of temperature $= (390 \times 5280 \times 12 \times 2{\cdot}54) \times 0{\cdot}000012 \times (\frac{425}{9} - \frac{20}{9}) = 0{\cdot}21$ mile.

19 Compensated Pendulum.—In a pendulum clock the time-keeping quality depends upon its length, that is, the distance from the point of suspension to the centre of gravity of the bob, because the period of oscillation of the pendulum changes with change of length according to the relation, $t = 2\pi\sqrt{\frac{l}{g}}$ [see Art. 74, Part I].

It is evident from the above expression that if l increases t will become greater. In order that the rate of a clock should be uniform, the length of the pendulum must not vary with temperature. *If the length increases, the period of oscillation will increase and the clock will lose, if the length decreases, the clock will gain.* So, generally in summer, the clock will lose, and in winter the clock will gain time.

In order to nullify the effect of expansion and contraction, compensated pendulums are constructed with some special device which will always maintain a constant length from the point of suspension to the centre of the bob inspite of any variations of temperature. Such a pendulum is called a *Compensated pendulum*

Fig. 12

(a) Harrison's Grid-iron Pendulum.—This is the best form of a compensated pendulum. The principle of construction is as follows.

Suppose AB and CD be two parallel rods of different metals (Fig. 12), say steel and brass, being connected by a cross bar BC. If the point A is fixed, AB will expand downwards, while CD will expand upwards for any rise of temperature. Now, if the lengths of the rods are such that the downward expansion of AB is equal to the upward expansion of CD for any rise of temperature $t°C.$, the distance AD will remain unaltered. So, if α, α' be the coefficients of expansion of AB and CD, and l, l' their lengths respectively, we have, $l.\alpha.t = l'.\alpha'.t$; or $l\alpha = l'\alpha'$

$$\text{or} \quad \frac{l}{l'} = \frac{\alpha'}{\alpha}$$

i.e. the lengths of the rods should be inversely proportional to the co-efficients of expansion. It is also evident that CD must be constructed with *more expansive metal* than AB.

Tha actual pendulum consists of a frame-work (Fig. 13) containing alternate rods of steel (shown in thick lines), and brass (thin lines). The central steel rod C, passing through holes in the lower cross bars of the frame, carries the bob B at its lower end. The arrangement is such that the steel rods expand *downwards*, while the brass rods expand *upwards*, and the centre of the bob is neither raised nor lowered, if the total upward expansion is equal to the total downward expansion. It should be *noticed* that in a Grid-iron pendulum all the bars, except the central one, are in pairs.

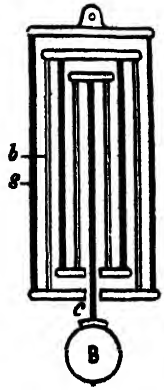


Fig. 13
Harrison's Grid-iron pendulum.

So, if there are 5 steel rods, each l_1 cm. long, and 4 brass rods, each l_2 cm. long, the effective length of the steel rods, is $3l_1$, and that of the brass rods is $2l_2$, and taking the co-efficient of linear expansion of brass to be 0·000019 and that of steel 0·000012,

we shall have $$\frac{3l_1}{2l_2}=\frac{0{\cdot}000019}{0{\cdot}000012}=\frac{19}{12}.$$

In constructing good clocks and watches precautions have to be taken to counteract the effects of expansion, in order to get a correct rate of movement of the mechanism.

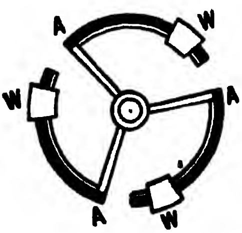


Fig. 14. Balance wheel

[**Note** —It is now usual to make the pendulum rod of a clock of **invar**, an alloy of nickel and steel, the co-efficient of expansion of which (0·0000009) is almost negligible.]

(b) Compensated Balance Wheel.—Fig. 14 illustrates a balance wheel of a watch. The time of oscillation of the wheel depends upon the distance of the circumference A from the centre, *i.e.* the radius of the wheel—the smaller the radius, the quicker the oscillation. So, an ordinary wheel oscillates quicker in winter than in summer. A compensated wheel is made of two strips (thick lines and thin lines), say brass and steel, the more expansible brass being on the outside. With the rise of temperature the radius of the wheel expands and so the clock will lose time, but the unequal expansion of brass and steel causes the strips to curve inwards. The masses of the little screws W on the rim are thus brought nearer the centre and this compensates for the expansion of the radius.

Example—*There are 5 iron rods, each 1 meter long, and 4 brass rods in a Grid-iron pendulum. What is the length of each brass rod? (The co-efficient of expansion of iron is 0·000012, and that of brass 0·000019).*

The effective length of the iron rods $= 3 \times 1 = 3$ metres,

and if l metre be the length of each brass rod, its effective length $= 2l$.

$$\therefore \quad \frac{2l}{3} = \frac{0{\cdot}000012}{0{\cdot}000019} = \frac{12}{19}\ ; \text{ or } l = \frac{3 \times 12}{2 \times 19} = 3{\cdot}947 \text{ metres.}$$

Questions

Art. 13.

1. A rod of iron and a zinc one are each 2 metres long at 0°C., and both are heated equally. At 50°C. the zinc rod is found to be longer by 0·181 cm. Find the co-efficient of linear expansion of iron when that of zinc is 0·0000298.

[*Ans:* 0·0000117]. (C. U. 1927)

2. State the laws of the simple pendulum. The pendulum of a clock is made of wrought iron and the pendulum swings once per second. If the change of temperature is 25°C., find the alteration in the length of the pendulum. (Co-efficient of expansion of wrought iron is $11{\cdot}9 \times 10^{-6}$). (Pat. 1920)

In this case, $t = 2$ secs. So $1 = \pi\sqrt{l/g} = \pi\sqrt{l}/981$; whence $l = 99{\cdot}39$ cm.

If l be the initial length of the pendulum, the length after it is increased by 25°C. $= l(1 + 0{\cdot}0000119 \times 25)$; $\therefore$ The alteration in length $= l(1 + 0{\cdot}0000119 \times 25) - l = l \times 0{\cdot}0000119 \times 25 = 99{\cdot}39 \times 0{\cdot}0000119 \times 25 = 0{\cdot}0295$ cm.

3. Define the co-efficient of cubical expansion of a solid. Does it differ when (a) the lengths are measured in centimetres or feet? (C. U. 1931)
(b) the temperature is measured in Fahrenheit or Centigrade? (C. U. 1944)

4. A brass scale reads correctly in mm. at 0°C. If it is used to measure a length at 33°C., the reading on the scale is 40·5 cm. What is the correct measurement of the length? (C. M. B.)

[*Ans:* 40·523 cm.].

5. A platinum wire and a strip of zinc are both measured at 0°C., and their lengths are 251 and 250 cm. respectively. At what temperature will their lengths be equal, and what will be their common length at this temperature? (The co-efficient of linear expansion of zinc is 0·000026) (S. C.)

[*Ans:* 234°C. ; 251·521 cm.]

6. A brass scale measures true centimetres at 10°C. The length of a copper rod measured by the same scale is found to be 100 cm. at 20°C. Find the real length of the rod at 0°C. (The co-efficient of linear expansion of copper is 0·000017 and that of of brass 0·000019).

[*Ans:* 99·985 cm.]

7. How could you show that brass expands more than iron when rods of these two metals are heated through the same temperature? (All.)

8. A railway line is laid at a temperature of 7°C. If each rail be 40 ft. long and firmly clamped at one end, calculate how much space should be left between the other end of the rail and the next one when the temperature rises to 84°C. (The co-efficient of expansion for iron is 0·0000109).

[*Ans* : 0·10464 inch.]

9. What space should be allowed per mile of engine rail to avoid stress in the rails for the variations of temperature between 25°C. and −5°C. ?

[*Ans* : 1·9008 ft.].

10. The diameter of an iron wheel is 3 ft. If its temperature is raised 400°C., by how many inches is the circumference of the wheel increased ?

[*Ans* : 0·552].

11. The co-efficient of linear expansion of brass is 0·000019 ; if the volume of a mass of brass is 1 cub. decimetre at 0°C., what will be its volume at 100°C. ?

[*Ans* : 1·0057 cub. decimetre.]

12. Two bars of iron and copper differ in length by 10 cm. at 0°C. What must be their lengths in order that they may differ by the same amount at all temperatures. (The co-efficients of linear expansion of iron and copper are 0·000012 and 0·000018 respectively).

[*Ans* : Iron 30 cm. ; Copper 20 cm.]

Art. 15.

13. Describe any method for determining the co-efficient of linear expansion of a solid. (C. U. 1913. '18, '21, '27, '31, '36 ; All. 1925)

Art. 17.

14. Define the co-efficients linear and cubical expansion. (C. U. 1915, '18)

Show that the latter is three times the former. (Pat. 1936)

15. Define co-efficient of expansion, and find out the simple but approximate relation between the co-efficients of (*a*) linear, (*b*) superficial, and (*c*) cubical expansion, of a given material. (Pat. 1940)

Art. 19.

16. How are clocks compensated for variations of temperature ? (All. 1932)

17. Describe the effect of varying temperature on the rate of a clock or watch. Explain how chronometers are constructed so as to keep accurate time inspite of changes of temperature ? (C. U. 1925)

18. Why should the time of oscillation of a clock pendulum change with rise of temperature ? What arrangement is made to make the clock give correct time both in warm and cold weather ? Given that the co-efficient of linear expansion of brass is 0·000019 and that of steel 0·000011, what must be the relative lengths of the bars of the metals used in the Grid-iron pendulum ? (Pat. 1936)

[*Ans* : 11 : 19]

CHAPTER III

Expansion of Liquids

20. Real and Apparent Expansion of Liquids.—Since liquids (or gases) have no definite shape of their own, and always take the shape of the containing vessel, the expansion or contraction in the case of liquids (or gases) is always cubical.

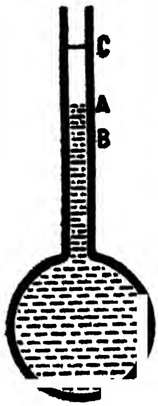


Fig. 15.

Experiment.—Take a glass bulb with a long stem. Fill the bulb and stem up to A with water. Now immerse the bulb suddenly in hot water, when it will be found that the level of water momentarily falls, say to B, and then increase up to, say, C finally (Fig. 15). This happens because the bulb becomes warm first and so expands before the heat reaches the liquid inside. In practice, however, we observe the expansion from A to C, which represents the apparent expansion of the liquid, whereas the real (or absolute) expansion is represented by BC. Therefore,

$$\text{Real expansion } (BC) = \text{Apparent expansion } (AC) + \text{Expansion of the glass vessel} (AB)$$

20(a). Relation between the Co-efficients of Real and Apparent Expansion.—Let V_o = volume of the vessel at 0°C., and thus = volume of the liquid at 0°C. ; t_o = rise in temperature ; γ = co-efficient of apparent expansion of the liquid ; γ_a = co-efficient of real or absolute expansion of the liquid ; g = co-efficient of cubial expansion of the containing vessel.

Then the real expansion of the liquid $= V_o\gamma_a t$; the apparent expansion of the liquid $= V_o\gamma t$, and the expansion of the vessel $= V_o g t$. Since real expansion = apparent expansion + expansion of vessel,

We have, $V_o\gamma_a t = V_o\gamma t + V_o g t$; or $\gamma_a = \gamma + g$ ... ... (1)

Thus,

Co-efficient of absolute expansion = Co-efficient of apparent expansion + Co-efficient of expansion of the material of vessel.

(**Note** that a hollow vessel expands as if it were solid, having the same volume, because if the hollow of the vessel were exactly solid, after expansion it would just fit with the outer vessel).

Co-efficients of Expansion.—*The co-efficient of apparent expansion of a liquid is the ratio of the observed increase in the volume produced by a rise of temperature of 1°C. to the volume of the liquid at 0°C.*

The co-efficient of real or absolute expansion of a liquid is the ratio of the real increase in volume for a rise of temperature of 1°C. to the volume of the liquid at 0°C.

21. Variation of Density with Temperature.—We know that density $= \dfrac{\text{mass}}{\text{volume}}$. Let m gm. of a substance (say, a liquid) occupy V c.c. at 0°C., then its density at this temperature, $d_o = m/V_o$ (1)

The volume occupied by the same mass at t°C. will be V_t, when the density $d_t = m/V_t$ .... (2)

But $V_t = V_o\{1+\gamma_a t\}$, where γ_a is the co-efficient of absolute cubical expansion of the liquid......... ... (3)

From (1) and (2), $\dfrac{d_o}{d_t} = \dfrac{V_t}{V_o} = \dfrac{V_o(1+\gamma_o t)}{V_o} = (1+\gamma_a t)$

or $d_o = d_t(1+\gamma_a t)$......... ...(4)

or $d_t = d_o(1+\gamma_a t)^{-1}$ i.e. $d_t = d_o(1-\gamma_a t)$ approximately ... (5)

$$\therefore \quad \gamma_a = \frac{d_o - d_t}{d_t\, t}$$

[**Note.** Compare equation (3) and (5).]

Examples.—(1) *The density of mercury is 13·59 at 0°C. What will be the volume of 30 kilograms of mercury at 100°C., the co-efficient of expansion of mercury being 1/5550 :*

Let d_{100} = density of mercury rt 100°C., d_o = density of mercury at 0°C.,

We have, $d_o = d_{100}\,(1+\gamma_a t)$

or $d_{100} = \dfrac{d_o}{1+\gamma_a t} = \dfrac{13{\cdot}59}{1+(\frac{1}{5550}\times 100)} = \dfrac{13{\cdot}59\times 5550}{5650}$.

So, the volume of mercury $= \dfrac{30\times 1000}{d_{100}} = \dfrac{30\times 1000}{\dfrac{13{\cdot}59\times 5550}{5650}} = 2247{\cdot}27$ c.c.

(2) *A glass hydrometer reads specific gravity 0·920 in a liquid at 45°C. What would be the reading at 15°C.? (Co-efficient of cubical expansion of the liquid = 0·000525, and that of glass = 0·000024)* (*Pat. 1941*)

Let V_{45}, V_{15} = volumes of the hydrometer at 45°C., and 15°C. respectively; d_{45}, d_{15} = densities of the liquid at 45°C. and 15°C. respectively; then

$d_{15} = d_{45}\ \{1+\gamma_a(45-15)\}$...(from eq. 4 Art. 21) $= d_{45}(1+0{\cdot}000525\times 30)$ $= d_{45}\times 1{\cdot}01575$; and $V_{45} = V_{15}(1+0{\cdot}000025\times 30$; $\therefore V_{15} = V_{45}(1-0{\cdot}000024\times 30) = V_{45}\times 0{\cdot}99928$.

Again the mass of V_{15} c.c. of the liquid at 15° $= V_{15}\times d_{15}$

$$\therefore \quad V_{15}\times d_{15} = (V_{45}\times 0{\cdot}99928)\times d_{45}\times 1{\cdot}01575\ ;$$

$$\therefore \quad d_{15} = \frac{V_{45}\times 0{\cdot}99928\times d_{45}\times 1{\cdot}01575}{V_{15}} = 0{\cdot}9345\ \ (\because d_{45} = 0{\cdot}920).$$

(3) *A cylinder of iron, 20 inches long, floats vertically in mercury, both being at the temperature 0°C. If the common temperature rises to 100°C, how much will the cylinder sink? [Sp. gr. of iron at 0°C = 7·6 : sp. gr. of mercury at 0°C = 13·6; cubical expansion of mercury between 0°C. and 100°C. = 0·018153; linear expansion of iron between 0°C. and 100°C. = 0·001182]* (*Pat. 1942*)

Let l_0 and l_{100} be the lengths of the cylinder immersed in mercury and A_0, A_{100} be the areas of the cylinder at 0°C. and 100°C. respectively.

The density of iron at 0°C. = (7·6 × 62·5) lbs. per cu. ft. = d_0 say; and that of mercury at 0°C = (13·6 × 62·5) lbs. per. cu ft = ρ_0, say; and let their corresponding densities at 100°C. be d_{100} and ρ_{100}, then from eq. 5 Art. 21.

$d_{100} = d_0\ (1-3\times 0{\cdot}001182)$ and $\rho_{100} = \rho_0\ (1-0{\cdot}018153)$

By the law of flotation, we have $(20\times A_0)\times d_0 = (l_0\times A_0)\times\rho_0$ ... (1)

and $\{20(1\times 0{\cdot}001182)\times A_{100}\}\times d_{100} = (l_{100}\times A_{100})\times\rho_{100}$ ... (2)

From (1) we have, $l_0 = \dfrac{20\times d_0}{\rho_0} = \dfrac{20\times(7{\cdot}6\times 62{\cdot}5)}{13{\cdot}6\times 62{\cdot}5} = 11{\cdot}176''$

and from (2), $20(1\times 0{\cdot}001182)\times d_0(1-3\times 0{\cdot}001182) = l_{100}\times\rho_0(1-0{\cdot}018153)$

or $\quad 20(1+0{\cdot}001182)\times(7{\cdot}6\times 62{\cdot}5)(1-0{\cdot}003546)$

$\quad = l_{100}\times(13{\cdot}6\times 62{\cdot}5)(1-0{\cdot}018153)$; whence $l_{100} = 11{\cdot}353''$

So the extra length of the cylinder which will sink in mercury when the temperature rises to 100°C. = (11·353 − 11·176) = 6·177″.

22. Determination of the Co-efficient of Apparent Expansion of a liquid (i) by *Weight Thermometer*.—This may take the form of a specific gravity bottle or it may consist of a spherical or cylindrical

glass bulb with a short capillary stem, which is bent and drawn out to a fine point (Fig. 16). The apparatus is cleaned and weighed empty. It is then filled with the liquid, of which the co-efficient of expansion is required This is done by alternate heating and cooling, keeping the end of the stem dipped in the liquid. When quite full, it is placed for some time in a beaker of water *with the open end still well below the liquid surface* to acquire the temperature of water, say t_o°C. The bulb is then taken out, dried, and weighed again. The difference of the weights gives the weight m_o of the liquid at t_o°C. The bulb is next placed in a bath of boiling water. Owing to expansion, some liquid is expelled. It is then removed from the bath, dried, and again weighed. Let the difference of this and the first weight be m_t, which is the weight of liquid at t'°C.

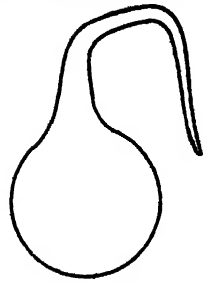


Fig. 16—
Weight thermometer.

Let V_o = volume of the thermometer at t_o°*C.*; m_o, d_o = mass and density of the liquid at t_o°*C.*; V_t = volume of the thermometer at t'°*C.*; m_t, d_t = mass and density of the liquid at t'°*C.*; γ = co-efficient of apparent expansion of the liquid between t_o°*C.* and t'°*C.*

We have, $m_o = V_o d_o$, and $m_t = V_t d_t$ $\therefore \dfrac{m_o}{m_t} = \dfrac{V_o d_o}{V_t d_t}$ ... (1)

But in determining the apparent expansion of a liquid, the expansion of the containing vessel is neglected; so here $V_o = V_t$.

$$\text{Hence} \quad \frac{m_o}{m_t} = \frac{d_o}{d_t} = \frac{d_t\{1+\gamma(t'-t_o)\}}{d_t} = 1+\gamma(t'-t\ )$$

$$or \quad \gamma = \frac{m_o - m}{m_t\,(t'-t_o)}$$

$$i.e. \quad \gamma = \frac{\text{Mass of liquid expelled}}{\begin{pmatrix}\text{Mass remaining at}\\ \text{higher temperature}\end{pmatrix} \times \begin{pmatrix}\text{Rise in}\\ \text{temperature}\end{pmatrix}}.$$

22. (a) Absolute Expansion :—The co-efficient of *absolute expansion* of the liquid can also be calculated in the following way :—

$V_t = V_o\{1+g(t'-t_o)\}$, where g is the co-efficient of cubical expansion of glass, and $d_o = d\ \{1+\gamma_a(t'-t_o)\}$ (See Art. 21.)

where γ_a is the co-efficient of absolute cubical expansion of the liquid.

$\therefore$ from Eq. 1, $\dfrac{m_o}{m_t}=\dfrac{V_o d_o}{V_t d_t}=\dfrac{V_o d_t\{1+\gamma_a(t'-t_o)\}}{V_o d_t\{1+g(t-t_o)\}}=\dfrac{1+\gamma_a(t'-t_o)}{1+g(t'-t_o)}$

or $\quad m_t+m_t\gamma_a(t'-t_o)=m_o+m_o g(t'-t_o)$,

or $\quad m_t\gamma_a=\dfrac{m_o-m_t}{(t'-t_o)}+m_o g$, or $\gamma_a=\dfrac{m_o-m_t}{m_t(t'-t_o)}+\dfrac{m_o}{m_t}g.$

If only the apparent expansion is required, g should be neglected, and the co-efficient of apparent expansion becomes

$$=\frac{m_o-m_t}{m_t(t'-t_o)}.$$

Notes.—(1) Because in the above experiment *weights* (and not) volumes) are taken for the determination of co-efficient of expansion, it should not be thought that the co-efficient of expansion is equal to the increase of one gram of liquid for 1°C. rise of temperature.

(2) The above instrument is called a *Weight Thermometer*, because by knowing the co-efficient of apparent expansion of a liquid and by finding the weight of liquid expelled at a higher temperature we can determine an unknown temperature.

Examples :—(1) *The mass of mercury overflowed from a weight thermometer is 5'4 gm. when heated from ice to steam point. The thermometer is placed in an oil bath at 20°C On heating the bath, 8 64 gms of mercury flow out Determine the temperature of the bath*

The mass of mercury overflowed for (100 − 0)°C = 5'4 gms

$\therefore$ The mass overflowed for 1°C = 5'4 ÷ 100 = 0 054 gm.

So, for the overflow of 8'64 gms of mercury, the rise of temperature of oil-bath $=\dfrac{8'64}{0'054}=160°C.$

Hence, the actual temperature of the bath = 20 + 160 = 180°C

(2) *A weight thermometer weighs 40 gms when empty, and 490 gms. when filled with mercury at 0°C. On heating it to 100°C, 6 85 gms. of mercury escape. Calculate the co efficient of linear expansion of glass, the co efficient of real expansion of mercury being 0'000182.*

Mass of mercury in the thermometer at 0°C. = 490 − 40 = 450 gms.

The mass of mercury left in thermometer at 100°C.

= 450 − 6'85 = 443 15 gms.

$\therefore$ The co-efficient of apparent expansion of mercury

$$=\frac{6'85}{443'15(100-0)}=0'000155.$$

Hence, the co-efficient of cubical expansion of glass = co-efficient of real expansion of mercury − co-efficient of apparent expansion of mercury

$$= 0{\cdot}000182 - 0{\cdot}000155 = 0{\cdot}000027.$$

$\therefore$ The co-efficient of linear expansion of glass $= 0{\cdot}000027 \div 3 = 0{\cdot}000009.$

(3) *If the co-efficient of apparent expansion of mercury in glass be $\frac{1}{6500}$, what mass of mercury will overflow from a weight thermometer which contains 400 gms. of mercury at 0°C., when the temperature is raised to 90°C. ?* (C. U. 1930)

We have, $\gamma = \dfrac{m_o - m_t}{m_t\,(t' - t_o)}$; or $\dfrac{1}{6500} = \dfrac{400 - m_t}{m_t\,(90 - 0)}$;

whence $m_t = \dfrac{26000000}{6590} = 394{\cdot}53.$

$\therefore$ The mass of mercury expelled $= m_o - m_t = 403 - 394{\cdot}53$
$= 5{\cdot}47$ gm.

(ii) **Dilatometer or Volume Thermometer method**—A dilatometer (Fig. 17) consists of a glass bulb with a graduated stem of small bore leading from it. It is used as follows :—weigh the dilatometer empty ; let this be w_1 gms. Introduce mercury in the tube to fill the bulb and a part of the stem up to the zero mark A. Weigh again, and let this weight be w_2. Put in more mercury to fill, say, up to B, the length AB being l cms. Weigh again. Let this third weight be w_3 gms. Then the weight of mercury occupying l cms. of the stem $= (w_3 - w_2)$ gms. = say, m_1 gms., and the weight of mercury in the bulb and stem up to the zero mark

Fig. 17—Dilatometer.

$$= (w_2 - w_1) \text{ gms.} = m_2 \text{ gms., say.}$$

$\therefore$ m_2 gms. of mercury would occupy $\left(\dfrac{m_2}{m_1} \times l\right)$ cms. of the stem and the volume of the bulb up to the zero mark of the stem $= \dfrac{m_2}{m_1} \times l \times a$, (if a sq. cm. = area of cross-section of the bore of the stem).

The bulb and part of the stem of the dilatometer is then put in a water bath, the temperature t_1 of which is measured and the length l_1 of the mercury height, at temperature t_1, is read accurately. Increase the temperature of the water bath up to t_2°C., and read the level of mercury again at C, the length AC being l_2 cms. Then the volume expansion of $(l_2 - l_1)$ cms. of mercury column for $(t_2 - t_1)$°C.

$= (l_2 - l_1) \times a$ c.c., and the original volume $= \left\{\left(\dfrac{m_2}{m_1} \times l \times a\right) + l_1 a\right\}$ c.c.

$\therefore$ Mean co-efficient of expansion between t_1° and t_2°

$$= \frac{\textit{Increase in volume}}{\textit{Original volume} \times \textit{Rise in temperature}}$$

$$= \frac{(l_2 - l_1) \times a}{\left(\frac{m_2}{m_1} \times l \times a + l_1 a\right) \times (t_2 - t_1)}$$

$$= \frac{(l_2 - l_1)}{\left(\frac{m_2}{m_1} \times l + l_1\right)(t_2 - t_1)}$$

Note.—The calculation will be easier if the density of the liquid is supplied (see example 2 below).

Examples.—(1) *A long glass tube of uniform capillary bore contains a thread of mercury which at 0°C. is one metre long. At 100°C. it is 16·5 mm. longer. If the average co-efficient of volume expansion of mercury is 0·000182, what is the co-efficient of expansion of glass?* (C. U. 1910)

$$\text{Co-efficient of expansion of mercury} = \frac{\text{Increase in volume}}{\text{Original volume} \times \text{rise in temp.}}$$

$$= \frac{1\cdot65 \text{ cm.} \times \text{area of cross-section}}{100 \text{ cm.} \times \text{area of cross section} \times 100} = 0\cdot000165$$

Co-efficient of cubical expansion of glass = co-efficient of absolute expansion of mercury − co-efficient of apparent expansion of mercury (see Art. 20).

$$= 0\cdot000182 - 0\cdot000165 = 0\cdot000017.$$

$$\therefore \text{ Co-efficient of linear expansion of glass} = \frac{0\cdot000017}{3} = 0\cdot0000056.$$

(2) *A glass bulb with an accurately graduated stem of uniform bore weighs 30 gms. when empty, 356 gms. when filled with mercury up to the 16th division, and 356·15 gms when filled up to the 110th division. Find the mean co-efficient of apparent expansion of the liquid which fills the bulb and stem up to the zero of the graduations at 0°C., and up to the 80th division at 10°C. (The density of mercury is 13·6).*

The capacity of the bulb and 16 divisions of the steam $= \frac{356 - 30}{13\cdot6} = \frac{326}{13\cdot6}$ c.c.,

and the internal volume of each division $= \frac{356\cdot15 - 356}{13\cdot6 \times (110 - 16)} = \frac{0\cdot15}{13\cdot6 \times 94}$ c.c.,

Hence the capacity of the bulb with the part of the stem below the zero mark $= \frac{329}{13\cdot6} - \frac{0\cdot15 \times 16}{13\cdot6 \times 94} = \frac{15320\cdot8}{13\cdot6 \times 47}$ c.c. Thus the initial volume of the liquid $= \frac{15320\cdot8}{13\cdot6 \times 47}$ c.c., and the total apparent increase of volume for 10°C.

$$= \frac{80 \times 0\cdot15}{13\cdot6 \times 94} \text{ c.c.}$$

Hence the co-efficient of apparent expansion of the liquid

$$=\left(\frac{80\times 0\cdot 15}{18\cdot 6\times 94}\middle|\frac{15820\cdot 8}{18\cdot 6\times 47}\right)+10= 0\cdot 00008915.$$

(3) The co-efficient of absolute expansion of mercury is 0·00018 ; the co-efficient of linear expansion of glass is 0·000008. Mercury is placed in a graduated tube, and occupies 100 divisions of the tube. Through how many degrees of the tube must the temperature be raised to cause the mercury to occupy 101 divisions ? (L. M.)

Let t be the number of degrees, then the length of the mercury column for t° rise of temperature $=100\ (1+0\cdot 00018t)$.

This becomes equal to 101 divisions of the tube after expansion.

$$1 \text{ division of the tube} = \frac{100(1+0\cdot 00018t)}{101}.$$

But 1 division of the tube becomes $(1+0\cdot 000008\ t)$ divisions at t°.

$$\therefore\ \frac{100(1+0\cdot 00018t)}{101}=1+0\cdot 000008\ t\ ;$$

$$\text{whence } t=\frac{1}{0\cdot 018-0\cdot 000808}=\frac{1}{0\cdot 017192}=58\cdot 2^\circ\text{C}.$$

23. Exposed Stem correction for a Thermometer.—The correction for the exposed portion of the stem of a thermometer will be best understood by the following example.

A mercurial thermometer is placed with its bulb and lower part of the stem in a liquid and indicates a temperature $t^\circ C$. The upper portion of the stem containing n divisions of mercury column is in the air at $\theta^\circ C$. Find the true temperature of the liquid.

The true temperature T° of the liquid is that which the thermometer would indicate if completely immersed in the liquid. Then n divisions of the mercury column, now at $\theta^\circ C.$, would be at $T^\circ C.$, and at that temperature would occupy $n\{1+\gamma(T-\theta\}$ divisions, where γ is the co-efficient of expansion of mercury in glass.

$\therefore$ The corrected length of the exposed portion would be greater than the actual length by $n\{1+\gamma(T-\theta)\}-n=n(T-\theta)\gamma$.

Hence, the true temperature of the liquid, $T=t+n(T-\theta)\gamma$.

Example.—*The bulb of a mercurial thermometer and the stem up to the zero mark are immersed in hot water at 100°C. while the remainder of the stem is in the air at 20°C. What will be the reading of the thermometer ?*

Using the formula given already, we have $T=100$, $n=t$, $\theta=20$, $\gamma=0{\cdot}000155$

$\therefore \quad 100=t+t\times(100-20)\times0{\cdot}000155=1{\cdot}0124\,t: \quad or \quad t=98{\cdot}77°C.$

24. Co-efficient of Absolute Expansion : (*Dulong and Petit's Method.*)—In 1816 Dulong and Petit developed a method of determining the co-efficient of absolute expansion of a liquid in which the expansion of the containing vessel has no effect on the observations from which the expansion is to be calculated. The liquid taken by him was mercury.

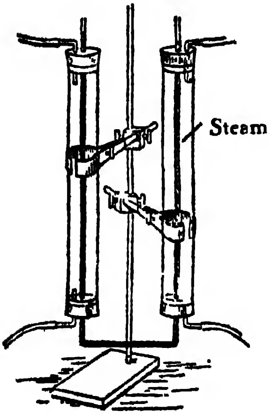


Fig. 18.—Dulong and Petit's apparatus.

The apparatus consists of a U-tube filled with the liquid (Fig. 18). One limb of the U-tube is kept cool by packing one of the jackets with melting ice, while the temperature of the other is increased and maintained by passing steam through the jacket. A piece of blotting paper constantly soaked with water is placed on the horizontal portion in order to prevent a flow of the liquid from one limb to another. Thus two different temperatures are maintained in the liquid in the two limbs.

Let h_t and h_o be the heights of the two liquid columns at t°C. and 0°C. respectively.

Let d_o be the density of the liquid of the cold column, and d_t be that of the hot column. Then, the pressure exerted on the horizontal portion of the tube by the cold column $=h_o\,d_o\,g+P$, and that by the hot column $=h_t\,d_t\,g+P$, where $P=$atmospheric pressure. But, since the two liquid columns are in equilibrium, we have, $h_o\,d_o\,g=h_t\,d_t\,g$, *or* $\dfrac{d_o}{d_t}=\dfrac{h_t}{h_o}$. But $d_o=d_t\,(1+\gamma_a\,t)$, where γ_a is the co-efficient of absolute expansion of the liquid.

$$\therefore \quad 1+\gamma_a\,t=\frac{h_t}{h_o} \qquad or \qquad \gamma_a=\frac{h_t-h_o}{h_o\,t} \qquad \ldots \quad (1)$$

Laboratory Experiment.—The above experiment can be done in a laboratory by circulating water at the room temperature through the left-hand jacket, instead of melting ice. The formula (1) should then be slightly changed as follows :

Let h_1 and h_2 be the heights of the cold and hot columns, and t_1, t_2 their temperatures. If d_1, d_2 be the densities of the cold and hot columns respectively, we have, $h_1 d_1 g = h_2 d_2 g$

$$\text{or} \qquad h_1 \frac{d_o}{1+\gamma_a t_1} = h_2 \frac{d_o}{1+\gamma_a t_2} \quad [\because \quad d_o = d_1(1+\gamma_a t_1)]$$

$$\text{or } h_2(1+\gamma_a t_1) = h_1(1+\gamma_a t_2)\,; \qquad \text{or} \quad h_2 + h_2\gamma_a t_1 = h_1 + h_1\gamma_a t_2$$

$$\text{or} \qquad \gamma_a = \frac{h_2 - h_1}{h_1 t_2 - h_2 t_1}.$$

[**Note.**—The above method is obviously independent of the expansion of glass, and so the diameters of the two limbs may be different without in any way interfering with the result.]

(*a*) *Indirect Method.*—Knowing the co-efficient of absolute expansion of mercury by Dulong and Petit's method, and the co-efficient of apparent expansion by the weight thermometer or any other method, the co-efficient of cubical expansion of the meterial of the weight thermometer is obtained from eq (1). Art. 20 (a). Again, by determining the co-efficient of apparent expansion of any liquid using the same vessel, the co-efficient of absolute expansion of the liquid can be calculated also from eq. (1), Art. 20 (a).

25. Apparent Loss in Weight of a Solid dipped in a Liquid at Different Temperatures.—A solid of volume V c.c. and known weight is weighed in the liquid at 0°C. Let the apparent loss in weight be W_o. It is then weighed again in the liquid raised to temperature t°C., and let the apparent loss in weight be W_t.

Let d_o, d_t = densities of the liquid 0°C. and t°C. respectively, γ = mean co-efficient of cubical expansion of the solid between 0°C. and t°C. ; g = acceleration due to gravity.

We have, according to Archimedes' principle, weight of the displaced liquid at 0°C. $= W_o = V \times d_o \times g$ ... ... (1)

(where V is the volume of the solid at 0°C. and so the volume of the liquid displaced at 0°C.)

When the temperature increases to t°C., the volume of the solid becomes $= V(1+\gamma t)$, which is also the volume of the liquid displaced at t°C. $\therefore$ The weight of the displaced liquid at t°C.,

$$W_t = \{V(1+\gamma t)\} d_t \times g \qquad \ldots \qquad \ldots \qquad (2)$$

From (1) and (2), $\frac{W_0}{W_t} = \frac{Vd_0 g}{V(1+\gamma t)d_t g} = \frac{d_0}{d_t(1+\gamma t)} = \frac{d_0}{d_0(1-\delta t)^{'}(1+\gamma t)}$

$$= \frac{1}{1-\delta t+\gamma t-\delta\gamma t^2} \qquad \ldots \qquad (3)$$

where δ = mean co-eff. of expansion of the liquid between 0°C and t°C.

So the loss in weight W_t at a higher temperature, is less than W_0, the loss at the lower temperature, since $\delta > \gamma$. Therefore, *the weight of of the solid in the liquid will increase with rise of temperature of the liquid.*

25 (*a*). Co-efficient of Expansion : (*Hydrostatic Method*)—

Knowing the value of γ, we can also apply this method in determining the co-efficient of expansion of the liquid.

We have, from (3) $\frac{W_0}{W_t} = \frac{1+\delta t}{1+\gamma t}$. $\qquad [\because \ d_0 = d_t(1+\delta t)]$

$$\text{whence} \qquad \delta = \frac{W_0 - W_t}{W_t\, t} + \frac{W_0}{W_t}\gamma. \qquad \ldots \qquad (4)$$

Examples.—(1) *A piece of glass weighs 47 grams in air, 31·53 grams in water at 4°C., and 31·75 grams in water at 60°C. Find the mean co-efficient of cubical expansion of water between 4°C. and 60°C., taking that of glass as 0·000024*

(C. U. 1922)

Wt. of displaced water at 4°C. = 47 − 31·53 = 15·47 gms.

∴ Volume of displaced water = 15·47 c.c., and this = volume of glass at 4°C.

Again, the volume of glass at 60°C. = 15·47 {1 + 0·000024(60 − 4)}

= 15·49 c.c. = volume of displaced water at 60°C.

Wt. of displaced water at 60°C. = 47 − 31·75 = 15·25 gms.

∴ Densily of water at 60°C. = 15·25 ÷ 15·49.

Now, if d = density of water at 4°C. ; d' = density of water at 60°C.,

γ = co-efficient of cubical expansion of water,

we have $\quad d' = d\{1-\gamma(60-4)\}$; $\quad$ *or* $\frac{15·25}{15·49} = d\{1-\gamma(60-4)\}$;

whence γ = ·000276, since $d = 1$.

[N. B. The value of the co-efficient of expansion can also be determined by using eq. (4) Art. 25(a).]

26. Anomalous Expansion of Water.—The expansion or con-

traction of water presents interesting peculiarities. If a mass of water at any temperature, say 10 C., be taken and allowed to cool, its volume will gradually diminish until it reaches the temperature 4°C, when with further cooling the volume increases instead of diminishing. This behaviour is pecular to water *The volume of water at 4°C. being the least, it has got the maximum density at that temperature* (Fig. 19).

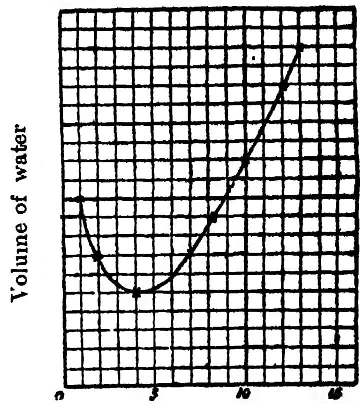


Fig. 19—
Temperature

The curve also shows that on further cooling, the volume of water increases. So enormous is the force exerted by the expansion of water that in cold countries even iron water-pipes get cracked in winter due to the freezing of water inside the pipe. Majority of substances contracts on cooling, substance like iron, antimony, and bismuth behave like water.

(Note Only *pure water* has got the maximum density at 4°C.; any impurity dissolved in water lowers the temperature of the maximum density.)

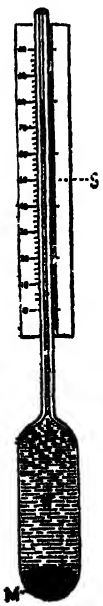


Fig. 20—Constant Volume Dilatometer.

26. (a) Constant Volume Dilatometer.—A simple experiment to measure the change in volume of water with temperature near about 4°C. can be carried out in a glass vessel whose capacity must be kept constant by some device. This can be done by taking a dilatometer [Fig. 20] with a graduated stem S of uniform narrow bore and filling it with mercury M whose volume should be about one-seventh part of the total capacity of the dilatometer. The co-efficient of expansion of mercury being about seven times that of glass, the change in volume of the dilatometer for any change of temperature will be equal to the change in mercury, and so the volume of the vessel unoccupied by mercury remains constant. In order to measure the change in volume of water, the dilatometer W is filled up with pure water up to certain point in the stem, and then placed in a water bath maintained at 0°C. The volume of

the water of the dilatometer is noted after sometime when the position of the water column in the stem becomes steady. The temperature of the bath is noted by a sensitive thermometer. The temperature is now gradually raised and the volumes of water at the corresponding temperatures are noted from the positions of 'water level in the stem from which a graph is obtained, as in Fig. 19.

Example.—*What volume of mercury must be placed in a glass dilatometer to keep the volume unoccupied by mercury constant at all temperatures. The co-efficient of cubical expansion of mercury is 0·00018, and the co-efficient of linear expansion of glass is 0·000009 per degree C.* [C. L.]

Let expansion of the vessel be V c.c., and let there be a rise of temperature of t°C. The expansion of the vessel for t°C. $= V \times 3 \times 0{\cdot}000009 \times t$.

If V_1 c.c. of mercury be used, its expansion will be $= V_1 \times 0{\cdot}00018 \times t$.

These must be equal in order to have the volume of the vessel unoccupied by mercury constant at all temperatures.

$\therefore \quad V_1 \times 0{\cdot}00018 \times t = V \times 3 \times 0{\cdot}000009 \times t$, whence $V_1 = 3/20\ V$.

Hence the volume of mercury required is such that it must be $\frac{3}{20}$th (*i.e.* approximately $\frac{1}{7}$th) of the volumn of the dilatometer.

The density and corresponding volume of 1 gram of water at various temperature :—

Temp. Centigrade	Density	Volume
0° (ice)	0·01670	1·09081
0° (water)	0·99987	1·00013
2°	0·99993	1·00003
4°	1·00000	1·00000
10°	0·99973	1·00026
20°	0·99823	1·0018
40°	0·9922	1·0073
60°	0·9832	1·0170
80°	0·9718	1·0287
100° (water)	0·9584	1·0432
100° (steam)	0·000599	1·670

The co-efficient of expansion of water varies from 0·0001 at 10°C. to 0·0006 at 80°C. Its co-efficient is 0 at 4°C., and negative from 4° to 0°C.

26 (b). Hope's Experiment.—The following experiment performed by T. C. Hope in 1805 shows that water attains its maximum density at 4°C.

Expt.—A tall cylinder is surrounded by a circular trough placed about half-way up the cylinder and the trough is filled with a freezing mixture of ice and salt [Fig. 21 (*a*)]. Two thermometers are inserted

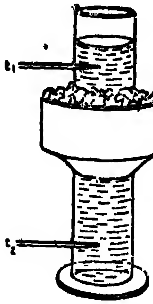


Fig. 21 (*a*)

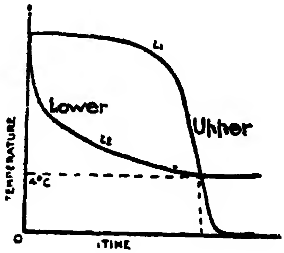


Fig. 21 (*b*)

horizontally through holes in the walls of the cylinder, one near the top and the other near the bottom of the vessel. The cylinder is filled with pure water and allowed to stand for some time. As cooling goes on, the temperature of the lower thermometer will gradually come down to 4°C., where it will remain stationary. It is because the water in the middle of the cylinder becomes denser on cooling and sinks to the bottom. This goes on until the temperature of the whole of the water below the middle portion falls to 4°C. After this, the water near the ice-jacket is gradually cooled down to 0°C., and being less dense rises up lowering the temperature of water there up to 0°C. So ultimately the upper thermometer will indicate 0°C., and the lower one 4°C., thus proving that 4°C. is the temperature of the maximum density of water. Hence it may be stated that *water at 4°C. expands whether it is heated or cooled.*

The readings of the two thermometers, entered in a graph, will be represented by Fig. 21 (*b*).

26 (c). Practical Importance of Hope's expt.—The fact that water has a maximum density at 4 C. is of great practical importance. If the density continued to increase until 0°C. was reached, ponds would freeze solid from top to bottom in severe frosts, and ultimately the whole pond would be a mass of ice, and that would destroy the animal life. But actually ice forms on the surface of a pond and the water below remains at 4°C. This saves much of the animal life.

27. Correction of Barometric Reading.—The pressure exerted by a column of zero-degree-cold pure mercury (density = 13·596 gms./c.c.)

'76 cms. in height at the sea-level in the 45° latitude (where $g = 980{\cdot}6$ cms./sec.^{2}) is called **standard pressure.** If the observed barometric height at a place is transformed so as to correspond to the above standard conditions, the height so obtained expressed in cms. is to 76 cms. as the barometric pressure at the place is to the standard pressure. But before the observed height is transformed to standard conditions, it has to be corrected, because the scale with which the height is measured may be at a different temp. from that at which it is graduated.

Temperature Correction for Scale.

Suppose the scale is graduated at 0°C. At higher temps., each division of the scale will extend in length. So the observed height, say h_t, at a temp. t°C will be smaller than its real value. Let h_o be the correct height, had the scale been maintained at 0°C. So, $h_o = h_t(1 + \alpha t)$ where α = co-eff. of linear expansion of the material of the scale.

Transformation of Corrected observed height to Standard conditions.

(a) Transformation to zero-degree-cold mercury.

The corrected height h_o is a column of mercury at t°C. To transform it to zero-degree-cold mercury with which the height will be, say H, we have

$H.d_o = h_o \,.\, d_t$, where d_o and d_t are densities of Hg. at 0°C and t°C.

i.e., $H = h_o \dfrac{d_t}{d_o} = h_o \,.\, \dfrac{d_o(1-\gamma t)}{d_o}$, where γ = co-eff. of cubical expansion of mercury

$$\therefore \quad H = h_o(1-\gamma t) = h_t(1+\alpha t)(1-\gamma t).$$
$$= h_t\{1-(\gamma-\alpha)t\} \text{ approximately.}$$

(b) Transformation to the sea-level in the 45° latitude.

The volume of g at a place depends on the latitude of the place and its elevation above the sea-level. If g = acce. due to gravity at the place of observation and g_o, that at the sea-level in the 45° latitude ($= 980{\cdot}6$ cms./sec.^{2}), and if the corrected height H measured by zero-degree-cold mercury, on transformation to the sea level in the 45° latitude becomes H_o, then

$$H_o \,.\, \rho_o \,.\, g_o = H \,.\, \rho_o \,.\, g$$

$$\text{or} \quad H_o = H.\frac{g}{g_o}$$

$$= h_t\{1-(\gamma-\alpha)t\}.\ \frac{g}{g_o}.$$

[**Note :** γ for $Hg.$ = 0·000182 per 1°C ; α for brass = 0·000018 per 1°C : α for glass = 0·000008 per 1°C.]

Hence for a barometer with brass scale,

$$True\ \ height = observed\ height \times (1 - 0{\cdot}000164t) \times \frac{g}{980{\cdot}6}\ .$$

and for a barometer with glass scale,

$$True\ height = observed\ \ height \times (1 - 0{\cdot}000174t)\frac{g}{980{\cdot}6}\ .$$

Examples.—(*1*) *The glass scale of a barometer reads exact millimetres at 0°C. The height of the barometer is noted as 763 divisions at 18°C Find the true height of the barometer at 0°C. (The co-efficient of linear expansion of glass = 0·000008 ; co-efficient of absolute expansion of mercury = 0·000181).*

From Art. 27, we have true height $H_o = H\{1 - (\gamma_a - a)t\}$

$$= 763\{1 - (0{\cdot}00018 - 0{\cdot}000008)18\} = 760{\cdot}6378 \text{ mm.}$$

(*2*) *A barometer provided with a brass scale, which is correct at 50°F., reads 754 at 40°F. what will be the true height at 32°F. ?*

The co-efficient of linear expansion of brass is 0·000018 per 1°C., so the value for 1°F. will be $(\frac{5}{9} \times 0{\cdot}000018) = 0{\cdot}00001$, and similarly the co-efficient of cubical expansion of mercury for 1°F. = 0·0001.

Let t_1 be the lower temperature at which the height should be corrected, t_2 the observed temperature, and t_3 the temperature at which the graduations are correct. (It should be noted that here the barometer is corrected at a higher temperature)

We have, $$H_{t_1} = \frac{H_{t_2}[1 + \alpha\{(t_2 - t_1) - (t_3 - t_1)\}]}{\{1 + \gamma_a(t_2 - t_1)\}}$$ (see p. 236)

$$H_{32} = \frac{H_{40}[1 + \alpha\{40 - 32) - (50 - 32)\}]}{1 + \gamma_a(40 - 32)^\circ}$$

$$= \frac{754\{1 + 0{\cdot}00001(-10)\}}{1 - (0{\cdot}0001 \times 8)} = 753{\cdot}32 \text{ mm.}$$

(*3*) *The brass scale of a barometer was correctly graduated at 15°C. At what temperature the observed reading will require no temperature correction ?*

Let t be the required temperature, then $H_o = \dfrac{H\{1 + 0{\cdot}000019(t - 15)\}}{(1 + 0{\cdot}000181t)}$

(Coeff. of linear expansion of brass = 0·000019). Here, we have $H = H_o$.

$\therefore\ \ 1 + 0{\cdot}000181t = 1 + 0{\cdot}000019(t - 15)$; or $t = -1{\cdot}8$°C.

Questions

Art. 20.

1. Distinguish between the real and apparent expansion in the case of a liquid. Establish a relation between them and the expansion of the material of the vessel. (C. U. 1916, '22, '26, '30 ; Pat. 1927, '28, '30, '41 ; cf. All. '44).

2. When hot water is thrown on the bulb of a thermometer, the mercury column first falls and then rises. Why is this ?

3. The readings of two thermometers containing different liquids agree at the freezing point and boiling point of water respectively but differ at other points of the scale. What inferences do you draw from this ?

Art. 22.

4. Describe the simple method of finding out the apparent expansion of a liquid. (C. U. 1916, '26 ; Cf. Pat. 1930).

The co-efficient of expansion of mercury is $\frac{1}{5550}$. If the bulb of a mercurial thermometer is 1 c. c. and the section of the bore of the tube 0·001 sq. cm., find the position of mercury at 100°C., if it just fills the bulb at 0°C. (Neglect the expansion of glass). (C. U. 1916).

[*Ans :* 18 cm. nearly]

5. Describe a weight thermometer, and deduce the formula you would use to determine the absolute co-efficient of expansion of a liquid with it. (Pat. 1943 ; All. '44).

6. Describe how to measure the absolute expansion of a liquid with the wt. thermometer. A wt. thermometer contains 43·218 grms. of liquid at 15°C., but only 42·922 grms. at 40°C. The co-efficient of linear expansion of glass is 0·000009. Find the absolute co-efficient of expansion of the liquid. (Punjab).

[*Ans* : Absolute co-efficient = 0·000303].

7. The density of water at 20°C. is 0·998 gm./c.c., and at 40°C. it is 0·992. Find the co-efficient of cubical expansion of water between the two temperatures. (S. C.)

[*Ans :* 0·00030/°C.]

8. Two scratches on a glass rod 10 cm. apart are found to increase their distance by 0·08 mm., when the rod is heated from 0° to 100°C. How many c.c. of too much boiling water will a measuring flask of the same glass hold up to a scratch on the neck which gave correctly one litre at 0°C ? (S.C.)

[*Ans :* 2·4 c.c.]

9. The co-efficient of linear expansion of glass is 8×10^{-6} and the co-efficient of cubical expansion of mercury is $1·8 \times 10^{-4}$/°C. What volume of mercury must be placed in a specific gravity bottle in order that the volume of the bottle not occupied by mercury shall be the same at all temperatures ? (S. C.)

[*Ans :* $\frac{2}{15}$ vol. of bottle].

10. The apparent expansion of a liquid when measured in a glass vessel is 0·001029, and it is 0·001003 when measured in a copper vessel. If the co-efficient of linear expansion of copper is 0·0000166, find that of glass.

[*Ans :* 0·0000079].

11. A weight thermometer contains 700 gms. of mercury at 100°C. What is its internal volume at that temperature? (Density of mercury = 13·6 ; co-efficient of expansion = 0·000182).

[*Ans :* 52·4 c.c.].

Art. 23.

12. Calculate the apparent co-efficient of mercury from the following data :—

A mercury thermometer wholly immersed in boiling water reads 100°C. When the stem is withdrawn so that graduations from 0° upwards are at an average temperature of 10°, the reading is 98·6°. (C. U. 1940).

[*Ans :* 0·000155].

Art. 24.

13. Describe Dulong and Petit's method of determining the co-efficient of real expansion of mercury with rise of temperature. Knowing the expansion of mercury, show briefly how you would find the expansion of another liquid between 0° and 100°C. (Pat. 1917 ; cf. '42)

14. A U-tube containing a liquid has two limbs maintained at 15°C and and 100°C. respectively. On reaching a steady state the lengths of the liquid columns are 97 cm. and 102 cm. What is the co-efficient of expansion of the liquid? Is this real or apparent expansion? Give reasons.

[*Ans :* 0·00057 approx.].

Art. 25.

15. The loss of weight of a weighted bulb, when immersed in a liquid at 0°C., is W_o. Show that the loss W at t°C. is given by $W = W_o \{1 + (\alpha - \beta)t\}$, where α and β are the co-efficients of expansion of the bulb and the liquid respectively. Sketch an apparatus to determine the apparent co-efficient of expansion of a liquid by the above formula. (All. 1922).

16. A solid body is weighed successively under a liquid at different temperatures. Explain how the heating will affect two different weights, (Pat. 1927).

17. How would you prove, without weighing that, bulk for bulk, cold water generally weighs more than when hot?

18. The density of water at 4°C. is 1, and at 80°C. it is 0·9718. Find the difference of the two weights of a cube of glass of 3 cm. edge when weighed in water at both the above temperatures. (The co-efficient of linear expansion of glass is 0·00009).

[*Ans :* 0·708 gm.]

19. A piece of glass weighs 4·525 gms. in air, 2·817 gms., when immersed in water at 20°C., and 2·881 gms. in water at 100°C. Find the mean co-efficient of expansion of water between 20° and 100°C. (co-efficient of cubical expansion of glass is 0·000255).

Art. 26.

20. Water is said to have its maximum density at 4°C. Explain what this means. In what respects is the behaviour of mercury different from that of water when both are gradually warmed from 0°C. (C. U. 1937).

Art. 26(a).

21. How would you demonstrate that water while being cooled acquires the maximum density sometime before it begins to freeze. (Pat. 1937.)

22. A uniform glass tube one metre long contains a column of mercury at an end. How long must this be in order that the length of the tube unoccupied by mercury shall remain unaltered when the whole is heated? (Absolute co-efficient of expansion of mercury 0·00018/°C., linear co-efficient of expansion of glass 0·00001/°C.).

[*Ans :* $\frac{1}{6}$ length of tube].

Art. 27

23. Show how would you correct the reading of a barometer for the expansion of the mercury and the scale. (Pat. 1926, '35 ; Cf. 1932).

24. The height of a barometer appears to be 76·4 cm. according to the brass scale which is correct at 0°C. If the temperature at the time of reading 30°C., what is the actual height of the mercury column? The co-efficient of linear expansion of brass is 0·000018. How is the quantity determined experimentally? (C. U. 1920).

[**Hints** —Correct reading = $76·4(1+0·000018 \times 30) = 76·4413$ cms.]

CHAPTER IV

Expansion of Gases

28 Expansion of Gases.—While dealing with the expansion of solids and liquids we did not take into consideration the pressure to which they were subjected, as change of pressure does not produce any appreciable change in their volume, but to make a complete statement above the condition of a gas, its *pressure*, *volume* and *temperature* must

be given. In mathematical language we say that gases have *three variables* P, V, and t, and the relations between these variables are included in the gas laws. They are as follows.

(1) The relation between pressure P and volume V, when *temperature t is constant.*

(2) The relation between volume and temperature, when *pressure P is constant.*

(3) The relation between pressure and temperature, when *volume V is constant.*

The first of these has been considered in Boyle's Law (Art. 138, Part 1) which states $V \propto 1/P$, when t is constant.

29. Expansion of Gases at Constant Pressure.—This was investigated by Gay Lussac, who stated that *all gases expand by a constant fraction* ($\frac{1}{273}$) *of their volume at 0°C. for each rise of temperature of 1°C., the pressure remaining constant.* This is known as **Charles' Law.**

Charles first found out this relationship for a gas, but he did not publish his work. In 1802 Gay-Lussac investigated the same law, who later on found Charles' manuscripts and remarked that Charles had discovered the law fifteen years earlier. So it is sometimes known also as **Gay-Lussac's Law.**

Thus, if V_0 and V_t be the volumes of a gas at 0°C. and t°C. respectively, $V_t = V_0(1 + \gamma_p t)$, *where γ_p is the constant fraction, which is called the co-efficient of expansian of the gas at constant pressure*, or simply as the *volume coefficient.*

$$\text{Thus,} \qquad \gamma_p = \frac{V_t - V_0}{V_0 t}.$$

The value of γ_p is found to be $\frac{1}{273}$ or 0 00366, and it is *approximately the same for all gases* and not different for different gases, as in the case of solids and liquids.

Thus, 1 c.c. of a gas at 0°C. becomes $(1 + \frac{1}{273})$ c.c. at 1°C. ; $(1 + \frac{5}{273})$ c.c. at 5°C : $(1 + \frac{50}{273})$ c.c. at 50°C. ; and so on.

Again, 273 c.c. of a gas at 0°C. becomes 273 $(1 + \frac{1}{273})$ c.c. *i.e.* 274 c.c. at 1°C ; 273 $(1 + \frac{100}{273})$ c c. or 373 c.c. at 100°C. ; 273 $(1 + \frac{110}{273})$ c.c. or 383 c.c. at 110°C.

[**Note.**—In determining the co-efficient of expansion of a gas, the initial volume of a gas should invariably be taken at 0°C., instead of taking it at any other temperature, which can be allowed in the case of solids and to some extent, of liquids, as the expansion of a gas for

a small change of temperature is very large in comparison with the very small expansion of a solid or that of a liquid ; or, in other words, the co-efficient of expansion of a gas is *not a very small fraction* as in the case of solids or liquids.

For the above reason we did not so much insist on specifying any lower temperature in the formula relating to the expansion of solids and liquids. Thus, *in calculating the expansion of gases, we should always mind the words* $\frac{1}{273}$ "*of their volume at 0°C.*", and we shall get wrong results if we take the original volume at any other temperature, say 10°C. or 20°C. as in the case of solids and liquids.

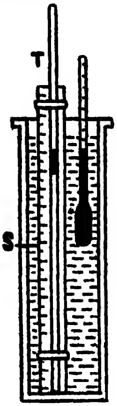


Fig. 22
Constant
Pressure Air
Thermometer.

Suppose we have 373 c.c. of a gas at 100 C., and we want to find its volume at 110°C. By directly applying the formula $V_{110} = V_{100}(1 + \frac{10}{273})$ we get,

$$V_{110} = 373(1 + \tfrac{10}{273}) = 373 + 13{\cdot}67 = 386{\cdot}67.$$

But this is not what it should be, for the correct volume of 373 c.c. at 100 C. will become 383 c.c. at 110°C., as seen before. This shows the importance of the words "of their volume at 0°C."

That the above point is not so important in the case of solids will be shown thus :

Suppose we have a rod of iron which is 100 cm. long at 0°C., then at 100 C. it will become 100 $(1 + 0{\cdot}000012 \times 100)$ or 100·120 cm. At 110 C. it will become $100(1 + 0{\cdot}000012 \times 110)$ or 100·132 cm.

Again by applying the formula directly, as in the above case, $l_{110} = l_{100}\ (1 + 0\ 000012 \times 10) = 100{\cdot}12\ (1 + 0{\cdot}00012)$ 100·1320144.

The difference in the two results, which is 0 0000144, can easily be neglected for our purposes, and this clearly shows the importance of always considering the volume at 0°C. when calculating expansion of gases].

30. Co-efficient of Expansion of a Gas at Constant Pressure.

Verification of Charles' Law—Two methods are given below for determining the co efficient of expansion of a gas at constant pressure using air in every case.

(1) **Expt.**—Take a piece of capillary glass tube T of uniform bore and about 50 cm. long (fig. 22 . Pass a stream of hot air through the tube for some time, and when the tube has been dried, seal off one end of it by a blow-pipe flame. The tube is then gently heated with the open end dipped in mercury. On allowing the tube to cool, the air contracts and draws inside a small pellet of mercury which serves as

an index. Now fix the tube on a piece of millimetre scale S and dip the whole, closed end downwards, into a long jar containing ice-cold water (water plus lumps of ice), so that the enclosed column of air is fully immersed in water. After some time take the reading of the lower end of the mercury pellet. Now heat the water until it boils, and again read the position of the pellet when it is constant. *As the tube is of uniform bore the volume of the enclosed air is proportional to the length of the air column.*

Let α be the area of cross-section of the bore, and l_1 and l_2 be the lengths occupied by the air column at 0°C. and 100°C. respectively; so $l_1\alpha$ is the volume of air at 0°C., and $l_2\alpha$ the volume at 100°C., neglecting the expansion of glass.

$$\text{The co-efficient of expansion, } \gamma_p \quad \frac{l_2\alpha - l_1\alpha}{l_1\alpha \times 100} = \frac{l_2 - l_1}{l_1 \times 100} \qquad (1)$$

The air in the tube can be replaced by any other gas, and *it will be found that the value of γ_p in every case will be the same, viz.* $\frac{1}{273}$.

(2) **Expt.**—The co-efficient of expansion is also found by using the apparatus shown in Fig. 23. The air is enclosed in the bulb A of the U-tube and kept dry by pouring strong sulphuric acid from the graduated limb B, which is open to the atmosphere. The U-tube has a short tube below as an outlet closed by a tap S, and is placed in an outer-jacket, the bottom of which is closed by a rubber cork through which the short tube passes. Water is poured into the outer-jacket until the bulb A is immersed. The water may be heated by passing steam through a bent copper tube passing through the cork. A thermometer T is suspended in the water close to the bulb and also there is a stirrer (not shown in the figure) for stirring the water.

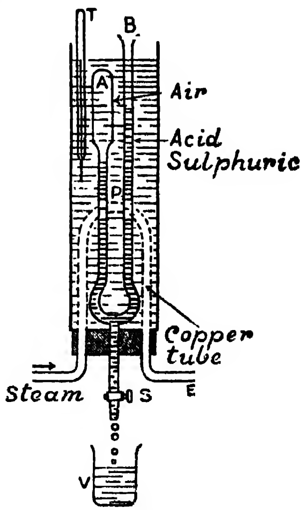


Fig. 23
Constant Pressure
Air Thermometer.

Before taking readings, sufficient time should be given to the enclosed gas to attain the temperature of the water. Now the acid is poured in at B or run out by opening S until its levels are the same in both the limbs. The air in A is then at the atmospheric pressure and its temperature is noted and volume is read by the graduations. Steam is passed through the copper tube and the water is kept constantly stirred. The tem-

perature-rise causes the air in the bulb expand and force down the liquid which rises in the other limb. The temperature is kept constant for some time by regulating the steam, during which the levels of the acid are adjusted by the tap to be the same in both the limbs, and volume and temperature are read as before. The heating is continued and readings are taken at various higher temperatures until water boils.

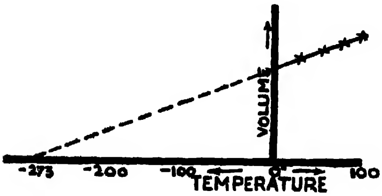


Fig. 24

If V_1 and V_2 be the volumes of air respectively at t_1 and t_2, we have, $V_1 = V_o(1 + \gamma_p t_1)$, and $V_2 = V_o(1 + \gamma_p t_2)$; or $\frac{V_2}{V_1} = \frac{1 + \gamma_p t_2}{1 + \gamma_p t_1}$ whence γ_p can be known as V_1, V_2, t_1 and t_2 are known.

30(a). γ_p *from Graph.*—If the temperature is plotted on the x-axis and the volume on the y-axis, a straight line graph is obtained (Fig 24), indicating that the expansion of the gas is uniform, or the pressure of a gas remaining constant, the volume increases regularly with the temperature. On producing the graph backwards it will cut the x-axis at about $-273°$C. ; which means that the volume of the air (theoretically) becomes zero at $-273°$C. The volumes of air, V_o at 0°C. and V_t at some other temperature, are read from the graph, from which γ_p is calculated from the relation

$$V_t = V_o(1 + \gamma_p t).$$

31. Increase of Pressure of a Gas at Constant Volume—The relation between temperature and pressure of air at constant volume can be verified by an apparatus (Fig. 25) like the Boyle's Law tube with the addition of a glass bulb in place of the straight closed tube.

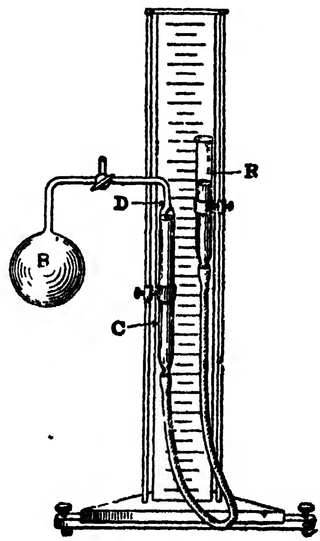


Fig. 25—Jolly's Constant Volume Air Thermometer.

Jolly's Apparatus.—The bulb B and the capillary tube D above the mercury surface contain dry air. The bulb is placed in ice-cold water (water plus lumps of ice) contained in a beaker. Now adjust the level of mercury till it reaches some point D marked

on the stem, the point being as near the bulb as possible. Read the difference in the levels of mercury in the two tubes R and C. Let it be h_1. Then the pressure of air in the bulb, say P_o, is $H+h_1$, where H is the atmospheric pressure. Now heat the water till it boils, and bring the mercury level again up to the same mark D. Read the difference in levels. Let it be h_2. Then the pressure of air in the bulb at 100°C., say P_{100}, is $H+h_2$.

Then, we have, $\gamma_v = \dfrac{P_{100} - P_o}{P_o \times 100}$ ... (2)

31(a). γ_v *from Graph.*—If the temperature is increased gradually in steps and the corresponding pressures are noted, a graph may be plotted with temperatures on the x-axis and pressures on the y-axis. On drawing the graph on a smaller scale and producing it backwards, it will be a *straight line* (Fig. 26), cutting the x-axis at about −273°C., that is, at zero pressure the temperature is theoretically −273°C.

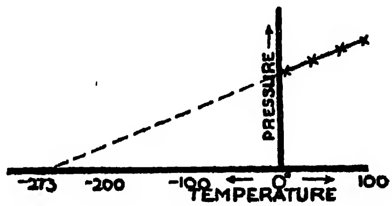


Fig. 26

The straight line indicates that the pressure increases uniformly with the temperature when the volume of the gas remains constant.

Reading from the graph, the values of P_o at 0°C, and P_t at any other temperature, γ_v can be conveniently calculated, and, in this experiment, water at the temperature of the laboratory can be used instead of ice-cold water.

The result obtained for air is $\frac{1}{273}$, and the same value is also obtained for other gases which obey Boyle's Law. So, we have another form of Charles' Law known as the **Law of Pressures**, which states that *the pressure of a gas at constant volume increases by* $\frac{1}{273}$, *of its pressure at 0°C., for each degree centigrade rise in temperature.*

32. Relation between γ_p and γ_v.—For any gas obeying the laws of Boyle and Charles it may be shown that $\gamma_p = \gamma_v$. If the temperature of any mass of the gas be increased from 0° to t° while the pressure remains constant, we have $V_t = V_o(1+\gamma_p t)$......(1). Now increasing the pressure from P_o to P_t, while the temperature remains at t°, until the volume is V_o, we have, by Boyle's Law, $P_o V_t = P_t V_o$......(2). From (1) and (2), $P_o(1+\gamma_p t) = P_t$ ......(3). If, however, the temperature of the gas had been increased from 0° to t°, while the volume remained constant, then $P_t = P_o(1+\gamma_v t)$......(4).

Hence from (3) and (4), we have, $\gamma_p = \gamma_v$, or *the volume co-efficient of a gas is equal to its pressure co-efficient.*

33. Gas Thermometer.—The use of gases as thermometric substances is a very important application of their expansion. A permanent gas, which obeys Boyle's Law, can be used as a thermometric substance, and both the instruments described above can be used as thermometers for measuring any unknown temperature. One may be called a *Constant Pressure air thermometer*, and the other a *Constant Volume air thermometer*, if air is used instead of any other gas. Taking the value of γ to be $\frac{1}{273}$, the temperature can be determined by any of the relations (1) and (2) given in Arts. 30 and 31, though in practice the constant volume air thermometer is found more convenient.

For gas thermometers, air, oxygen, hydrogen, helium, nitrogen, etc., have been used by different observers. By using helium gas in the bulb it can be used almost down to the absolute zero, and, with a non-fusible bulb, temperature even above 1000°C. can be measured.

Gas thermometers are used as standards with which mercury and other thermometers are standardised by comparison. In the National Physical Laboratory of London, a constant volume hydrogen thermometer is kept as a standard gas thermometer.

33 (a). Measurement of Temperature.—(1) To find the temperature of a given bath with a constant volume thermometer, find P_o, the pressure of the enclosed gas at 0°C. and P_t, the pressure at any unknown temperature t of the bath. Then, we have,

$$\gamma_v = \frac{P_t - P_o}{P_o t}, \text{ or } t = \frac{P_t - P_o}{P_o \gamma_v} = 273 \cdot \frac{P_t - P_o}{P_o}.$$

The value of P_o can also be determined from the graph as shown in Fig. 26.

(II) **Graphical Method.**—Plot two points corresponding to P_o and P_{100} at 0°C. and 100°C. respectively and join them by a straight line as in Fig. 26. Now find the pressure P_t, at the same volume of the enclosed gas, corresponding to the unknown temperature t of the bath, and from the graph read the value of t corresponding to P_t.

34. The Advantages and Disadvantages of a Gas Thermometer.

Advantages.—(*i*) Gases are more expansible than liquids, so a gas thermometer is much *more sensitive* than a liquid thermometer ; (*ii*) all gases expand or contract at *the same rate* and their *expansian is very regular and uniform* ; (*iii*) a gas thermometer may be used over a *much higher range* of temperature by using porcelain bulbs, instead of glass bulbs, as glass softens at above 400°C.

Disadvantages.—(*i*) It is bulky and cannot be easily moved about. and not very suitable for domestic use or for a doctor's pocket ; (*ii*) on account of the size it cannot be ussd for such work as measuring specific heats ; (*ii*) it requires the use of a barometer to follow any changes in atmospheric pressure before the measurement of temperature is complete.

Examples —1 *Find the temperature of the boiling point of a salt solution from the following readings obtained with a constant pressure air thermometer. Position of mercury at 0°C. = 7·2, and at 100°C. = 16·8 ; position when the thermometer is in boiling solution = 17·3*

Let V_1 = volume of air at the unknown temperature t°C., then $\gamma_p = \dfrac{V_t - V_o}{V_o\, t}$.

But $\gamma_p = \dfrac{V_{100} - V_o}{V_o \times 100}$; $\dfrac{V_t - V_o}{t} = \dfrac{V_{100} - V_o}{100}$

$$\therefore \quad t = \frac{V_t - V_o}{V_{100} - V_o} \times 100 = \frac{17·3 - 7·2}{16·8 - 7·2} \times 100 = 105·2°.$$

(2) *The pressure of air in the bulb of a constant volume air thermometer is 73 cm. of mercury at 0°C., 100·3 cm. at 100°C., and 77·8 cm. at room temperature. Calculate the temperature of the room.*

(As in Ex. 1), $t = \dfrac{P_t - P_o}{P_{100} - P_o} \times 100 = \dfrac{77·8 - 73}{100·3 - 73} \times 100 = 17·6$°C.

35. Absolute Zero and Absolute Scale.—

(a) If we apply Charles' Law to temperatures below 0°C., the volume of a given mass of any gas at constant pressure will diminish by $\frac{1}{273}$ of its volume at 0°C. for each degree Centigrade fall of temperature.

Thus 1 c.c.	of gas	at 0°C.	becomes		$(1 - \frac{1}{273})$	c. c. at	−1°C.
,,	,,	,,	,,	,,	$(1 - \frac{2}{273})$	,,	−2°C.
,,	,,	,,	,,	,,	$(1 - \frac{3}{273})$	,,	−3°C.
,,	,,	,,	,,	,,	$(1 - \frac{100}{273})$	,,	−100°C.
,,	,,	,,	,,	,,	$(1 - \frac{273}{273})$ or 0	,,	−273°C.

Therefore the volume of the gas will be reduced to nothing at $-273°C.$, *i.e.* the particles of the gas at this temperature are in close contact. The temperature ($-273°C.$) is thus the lowest temperature possible on the gas scale, and is known as the *Absolute Zero. The scale in which temperatures are reckoned from $-273°C.$ as zero degree, and in which other divisions are numbered starting from this temperature, is called the Absolute Scale of temperature* (see Fig. 27).

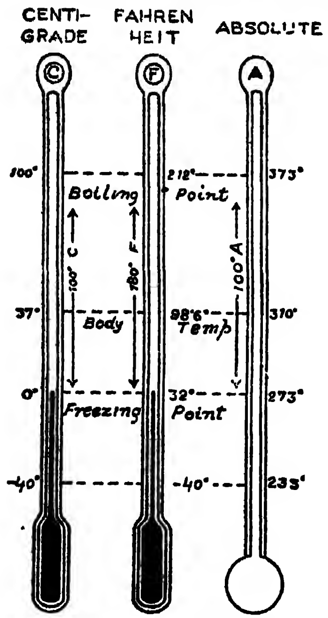


Fig. 27.—Absolute Scale.

The result stated above also follows from the relation, $V_t = V_o\left(1 + \dfrac{t}{273}\right)$.

If t is the Centigrade temperature when the volume becomes zero, we have

$$0 = V_o\left(1 + \frac{t}{273}\right) \text{ ; or } t = -273°C.$$

Thus, on the Absolute scale, $0°C. = 273°$ Absolute ; $100°C. = (100 + 273) = 373°$ Absolute ; and $t°C. = (273 + t)°$ Absolute, which is usually written $T°A$ (see Fig. 27).

Thus, *Absolute scale value = Centigrade scale value + 273.*

The above idea is also obtained from the graph in Fig. 24. It has been found there that the volume of the air theoretically becomes zero at $-273°C.$, which the scientists have called the *Absolute zero.*

The above result may be theoretically true but it is physically impossible, as all known gases liquefy and then become solid before this temperature is reached. As a matter of fact, air starts liquefying at about $-184°C.$ Hydrogen keeps to the straight line path down to $-269°C.$, By the evaporation of liquid helium a temperature as low as $-272°C.$ has been reached, but *the absolute zero has never yet been reached.* At the absolute zero temperature there would be no heat in the substance, the molecules of which would have no kinetic energy and so would stop moving.

(b). *Absolute Scale Value on the Fahrenheit Scale.—Remember* that we have so long considered centigrade scale according to which Absolute zero $= -273°C.$ But if the temperature is measured on the Fahrenheit scale, the Absolute zero becomes equal to 491·4 Fahrenheit degrees

below the freezing point (32°F.), because 273° on the Centigrade scale $= 273 \times \frac{9}{5} = 491.4$ on the Fahrenheit scale.

So Absolute zero $= 32 - 491.4 = -459.4$°F.

$\therefore$ 32°F. $= 459.4 + 32 = 491.4$°F. Absolute; and t°F. $= (459.4 + t)$°F. Absolute.

Thus, *Absolute scale value = Fahrenheit scale value + 459.4.*

Charles' Law.—Again, a Fahrenheit degree is $\frac{5}{9}$ of a centigrade degree, so the value of the co-efficient of expansion of a gas at constant pressure, which is $\frac{1}{273}$ per degree Centigrade, becomes equal to $\frac{5}{9} \times \frac{1}{273}$ or $\frac{1}{491}$. Therefore according to the **Fahrenheit scale** our **formula for Charles' Law** becomes.

$$V_t = V\left\{1 + \tfrac{1}{491}(t - 32)\right\}$$

36. Other Forms of Charles' Law.—According to Charles' Law, we get, $V = V_o\left(1 + \frac{t}{273}\right)$; Again $V' = V_o\left(1 + \frac{t'}{273}\right)$;

$\therefore$ $\frac{V}{V'} = \frac{273 + t}{273 + t'} = \frac{T}{T'}$, where T and T' denote absolute temperatures corresponding to Centigrade temperatures t and t'.

Hence $\frac{V}{T} = \frac{V'}{T'} =$ *a constant, when P is constant.*

or $V \propto T$, *when P is constant.*

In other words, *the volume of a given mass of any gas is directly proportional to the absolute temperature when the pressure remains constant.*

Similarly, from **the law of Pressures,** we get, $\frac{P}{P'} = \frac{T}{T'}$;

Hence $\frac{P}{T} =$ *a constant, when V is constant.*

or $P \propto T$, *when V is constant.*

In other words, *the pressure of a given mass of any gas is directly proportional to the absolute temperature when the volume remains constant.*

37. The Law connecting Pressure, Volume and Temperature of a Gas.—Let P, V, T denote the pressure, volume and *absolute* temperature of a given mass of gas ;

then $\quad V \propto 1/P$ when T is constant (Boyle's Law),

and $\quad V \propto T$, when P is constant (Charles' Law).

$\therefore \quad V \propto T/P$, when both T and P vary.

That is $\quad V = R\dfrac{T}{P}$, where R is a constant.

or $\quad \dfrac{PV}{T} = \text{R (a constant)} = \dfrac{P'V'}{T'}$, where P' and V' indicate the the values of pressure and volume respectively of the same amount of the gas at another *absalute* temperature T'

or $\quad PV = RT$, where R is called the *Gas constant*, whose value depends only on the mass of the gas whatever the gas is.

This is a *combined form of Boyle's and Charles' Laws*. This equation is generally called the *Gas Equation* or the *Equation of State*, for the physical state of a given mass of gas is completely determined by knowing its: pressure, volume, and temperature. Knowing any two of the three quantities, P, V, and T, the third can be calculated from the above equation, where *the value of the constant R depends on the mass of the gas under consideration.*

37 (a). The value of the gas constant R :—*One litre of air weighs* 1·293 *gm. at N. T. P. Find the value of R, considering* 1 *gm of air.*

The volume of 1·293 gm. of air at N. T. P. = 1000 c.c.

$\therefore$,, ,, 1 gm. ,, ,, ,, $= \dfrac{1000}{1 \cdot 293}$ c.c.

The normal pressure of the atmosphere $= 1 \cdot 013 \times 16^6$ dynes per sq. cm.

We have, $PV = RT$. $\therefore \quad 1 \cdot 013 \times 10^6 \times \dfrac{1000}{1 \cdot 293} = R \times 273$; So $\mathbf{R = 2 \cdot 87 \times 10^6}$ Ergs. per degree centigrade.

38. Another form. (*Change of density*).—It is often useful to consider changes of density instead of changes of volume. If D, D' represent the original and final densities of a mass M of gas, and V, V' be the corresponding volumes, then

$M = V \times D = V' \times D'$; *or* $V = M/D$, and $V' = M/D'$.

So the equation $\dfrac{PV}{T} = \dfrac{P'V'}{T'}$ becomes $\dfrac{PM}{DT} = \dfrac{P'M}{D'T'}$;

$\therefore \quad \dfrac{P}{DT} = \dfrac{P'}{D'T'} =$ a constant. *or* $\dfrac{P}{D} = \dfrac{P'}{D'}$, when $T = T'$.

Hence, *the density of a gas at constant temperature varies directly as the pressure.*

Again $DT = D'T'$, when $P = P'$.

Hence the density of a gas at a constant pressure varies inversely as the absolute temperature.

Meaning of N. T. P.—When a gas is at a temperature of 0°C. and a pressure of 760 mm. of ice-cold mercury, the gas is said to be at *normal temperature and pressure*—written as *N.T.P.*

39. Perfect Gas.—A gas for which the relation $PV = RT$ is absolutely true is called a *perfect gas*. None of the known gases is perfect, but gases such as oxygen, nitrogen, hydrogen, helium, etc. which are not easily liquefiable, behave almost like a perfect gas for ordinary ranges of temperature and pressure, while gases like sulphur dioxide or carbon dioxide, which can be easily liquefied, cannot be called a perfect gas. So *a perfect gas absolutely obeys Boyle's and Charles' Laws.*

Examples.—(1) *A quantity of dry air occupies 1000 c. c at 20°C. and under a pressure of 760 mm. of mercury. At what temperature will it occupy 1400 c c under a pressure of 750 mm of mercury?* (*C. U. 1929*)

We have, $\dfrac{PV}{T} = \dfrac{P'V'}{T'}$ or $\dfrac{760 \times 1000}{273 + 20} = \dfrac{750 \times 1400}{273 + t}$; whence $t = 131{\cdot}8°C.$

(2) *The measurement of a room is 50 ft. × 30 ft. If the temperature of the room is increased from 20°C to 25°C., calculate what percentage of the original value of air will be expelled from the room, the pressure remaining constant.*

Let V_1 = original volume of air = volume of the room = $(50 \times 30 \times 25)$ cu. ft.

V_2 = final volume of air at 25°C., $T_1 = 20 + 273 = 293°$C. Absolute.

$T_2 = 25 + 273 = 298°$C. Absolute.

We have, $\dfrac{V_2}{T_2} = \dfrac{V_1}{T_1}$; or $V_2 = \dfrac{V_1 \times T_2}{T_1} = \dfrac{(50 \times 30 \times 25) \times 298}{293} = 38139{\cdot}9$ cu.ft.

$\therefore$ Volume of air expelled $= 38139{\cdot}9 - (50 \times 30 \times 25) = 639{\cdot}2$ cu. ft.

Hence percentage of air expelled $= \dfrac{639{\cdot}9}{38139{\cdot}9} \times 100 = 1{\cdot}67$.

(3) *Find the mass of one cubic metre of dry air at 62°F. and a pressure of 75 cm. of mercury, the density of air at N. T. P. being 0·001293 gm per c. c.*

$62°F = (62 - 32) \times \frac{5}{9} = 16{\cdot}6°C. = 273 + 16{\cdot}6 = 289{\cdot}6°$C. Absolute.

The mass of a cubic metre (10^6 c.c.) of dry air at N. T. P.

$= 10^6 \times 0{\cdot}001293 = 1293$ gm.

If D be the mass of a cubic metre of air at 75 cm. of mercury and 62°F., we have

$$\frac{D \times 289{\cdot}6}{75} = \frac{1293 \times 273}{76} \text{ (see Art. 38) ; whence } D = 1203 \text{ gm.}$$

(4) *The mass of 1 litre of air at 0°C is 1·293 gm when the pressure is $1\cdot013 \times 10^6$ dynes per sq. cm. Find the value of R in the equation PV = RT*

The vol. of 1·293 gm. of air is 1000 c. c. So vol. of 1 gm is 1000/1·293c.c.

So, we have, $1\cdot013 \times 10^6 \times \dfrac{1000}{1\cdot293} = R \times 273$. [$\because$ 0°C.= 273°C. Absolute]

or $$R = \frac{1\cdot013 \times 10^6 \times 1000}{273 \times 1\cdot293} = 2\cdot87 \times 10^6 \text{ Ergs. per degree centigrade.}$$

(5) *Determine the height of the barometer when a milligram of air at 30°C occupies a volume of 20 c. c. in a tube over a trough of mercury, the mercury standing 730 mm. higher inside the tube than in the trough. (1 c. c of dry air at N. T. P. weighs 0·001293 gm).*

The wt. of 20 c. c. of air at 30°C. = 1 mgm. = 0·001 gm., and so,
wt. of 1 c. c. = 0·001/20 gm.

If P be the pressure of the enclosed air, $$\frac{P}{\frac{0\cdot001}{20} \times (273+30)}$$

$$= \frac{270}{0\cdot001293 \times 273}\text{; whence } P = 32\cdot6 \text{ mm.}$$

$\therefore$ The height of the barometer = 730 + 32·6 = 762·6 mm.

(6) *When the temperature of the air is 32°C. and barometer stands at 755 mm., the apparent mass of a piece of silver when counterpoised by brass weights in a delicate balance is found to be 25 gm. What is the actual mass? The density of silver is 10·5 and that of brass 8·4, both at 32°C.*

Let m gm. be the true mass of the silver, then its volume $m/10\cdot5$ c. c, which is also the volume of air displaced by it.

This volume reduced to N. T. P. becomes $= \dfrac{m}{10\cdot5} \times \dfrac{273}{(273+32)} \times \dfrac{755}{760}$.

But the mass of 1 c. c. of dry air at N. T. P. = 0·001293 gm.

$\therefore$ The mass of the above volume of air

$$= 0\cdot001293 \times \frac{m}{10\cdot5} \times \frac{273}{(273+32)} \times \frac{755}{760} = 0\cdot0011497 \times \frac{m}{10\cdot5}.$$

Hence the apparent mass of silver in air

$$= m - \frac{0\cdot0011497m}{10\cdot5} = m\left(1 - \frac{0\cdot0011497}{10\cdot5}\right) \text{gm.} \qquad \ldots \qquad \ldots \quad (1)$$

The volume of brass weights is 25/8·4 c.c., and the mass of air displaced by the weights = 0·0011497 × 25/8·4 gm.

$\therefore$ The apparent mass of the brass weights in air

$$= 25\left\{1 - \frac{0\cdot0011497}{8\cdot4}\right\} \text{gm.} \qquad \ldots \quad (2)$$

Since the apparent masses of silver and brass are in equilibrium, we have, from (1) and (2), $m\left\{1-\frac{0\cdot0011497}{8\cdot4}\right\}=25\left\{1-\frac{0\cdot0011497}{8\cdot4}\right\}$;

or $m=24\cdot9934$ gm.

(7) A litre of air at 0°C. and under atmospheric pressure weighs 1·2 gms Find the mass of the air required to produce at −18°C. a pressure of 3 atmospheres in a volume of 75 c. c. (*Pat. 1924*)

Let P be the atmospheric pressure. Then the pressure on the mass of the air $=3P$, and its absolute temperature $T=273-18=255°$.

Then from the formula $\frac{PV}{T}=\frac{P'V'}{T'}$; we have $\frac{P\times V}{273}=\frac{3P\times75}{255}$, where V is the volume of the mass of air at 0°C. and at atmospheric pressure P, whence $V=240\cdot88$ c.c.

But the mass of 1 litre or 1000 c. c. of air at 0°C. and atmospheric pressure is 1·2 gm. ∴ The mass of 240·88 c.c. of air $=\frac{240\cdot88\times1\cdot2}{1000}=0\cdot289$ gm.

Questions.

Art 28.

1. State concisely the relations between the volume, pressure, and temperature of a gas. Describe an experiment to prove the relation between pressure and temperature when the volume is kept constant. (C. U. 1912)

Art. 29.

2. A flask which contains 250 c. c. of air at atmospheric pressure is heated to 100°C. and then corked up. It is afterwards immersed mouth downwards in a vessel of water at 10°C, and the cork removed. What volume of water will enter the flask if the final pressure is atmospheric?

[*Ans*: 60·4 c. c.]

Art. 30.

3. Describe an experiment to find the co-efficient of expansion of a gas at constant pressure. (C. U. 1929, '35, '40 ; Pat. '32)

Art. 31.

4. Describe the constant volume air thermometer and explain how will you use it to find the melting point of wax. (All. 1927, cf. '20 ; Pat. 1926)

5. The pressure in a constant volume air thermometer is 770 mm. at 15°C. What will it be at 20°C.? (O. L.)

[*Ans*: 783 mm.]

Art. 33.

6. Explain how the thermal expansion of air can be utilised as a convenient means of measuring temperature. (All. 1918, '24)

Art. 35

7. What is meant by 'absolute temperature'? Find the value of the absolute zero on the Fahrenheit scale. (Pat. 1928 ; C. U. 1932, '38)

[*Ans :* $-459{\cdot}4$°F.]

Art. 36.

8. Why is it necessary to take account of the pressure of a gas in determining its co-efficient of cubical expansion?

200 c.c. of air at 15°C. is raised to 65°C. Find the new volume, the pressure remaining unchanged. (C. U. 1915)

[*Ans :* 234·7 c. c.]

9. A gas at 13°C. has its temperature raised so that its volume is doubled, the pressure remaining constant. What is the final temperature? (Dac. 1933)

[*Ans :* 299°C.]

10. Find the percentage increase of pressure in the tyres of a bicycle taken out of the shade (59°F.) into the sun (95°F.), disregarding the expansion of the rubber. (L. M.)

[*Ans :* 7%.]

Art. 37

11. At what temperature would the volume of a gas be doubled, if the pressure at the same time increases from that of 700 to 800 millimetres of mercury? (C. U. 1918)

[*Ans :* $t=351$°C.]

12. What volume does a gram of carbonic acid gas occupy at a temperature of 77°C. and half the standard pressure? (1 c. c. of carbonic acid weighs 0·0019 gram at 0°C. and standard pressure). (C. U. 1912 ; cf. '18, '33)

[*Ans :* 1349 c.c. nearly.]

13. Establish the relation $PV=RT$ for a gas. Given that one litre of hydrogen at N. T. P. weighs 0·0896 gm., calculate the value of R for a gramme of the gas. (C. U. 1938)

[See Art. 37 (a)]

14. State how the volume of a gas changes when its temperature and pressure both change. (Dac. 1921 ; '33)

15. Air is collected in the closed arm of a Boyle's tube and the volume found to be 32 c.c., the temperature being 17°C., and the height of the barometer 753 mm., while the mercury stands at 3·5 cm. higher in the closed arm than in the open one. What would be the volume of the air at 0°C. and 760 mm. pressure? (C. L.)

[*Ans :* 29·7 c. c.]

Art. 38.

16. The mass of 1 c. c. of hydrogen at 0°C. and 760 mm. pressure is 0·0000896 gm. per c.c. What will be its mass per c.c. at 20°C. and 760 m.m. ?

[*Ans* : 0·0000835 gm./c.c.]

17. A flask is filled with 5 gms. of a gas at 12°C. and then heated to 50°C. Owing to the escape of some of the gas, the pressure in the flask is the same at the beginning and end of the experiment. Find what weight of the gas has escaped.

[*Ans* : 0·47 gm.]

CHAPTER V

Calorimetry

40 Quantity of Heat.—If we take 10 grams of water and raise its temperature from 10°C. to 20°C., then the quantity of heat required for this purpose will raise the temperature of 1 gram of water through 100°C.. or 100 grams of water through 1°C.

From this we find that the quantity of heat required to raise the temperature of a substance through a given range depends on its *mass* and *range of temperature, i.e.* on the number of degrees through which it is heated and, we shall see later on, it also depends on the *nature* of the substance.

Calorimetry means the measurement of quantities of heat. The vessels in which the measurement of quantities of heat are carried out are called *Calorimeters.* These vessels are generally made of copper.

41. Units of Heat.—

Calorie.—It is the amount of heat required to raise the temperature of one gram of water through 1°C. This unit is called a *calorie* or *therm* or *gram-degree Centigrade Unit.* This amount is a quantity which can be added, subtracted, multiplied, and divided, just like any other quantity.

The **British Thermal Unit (B. Th. U.)** or *paund-degree Fahrenheit Unit* is *the amount of heat required to raise 1 pound of water through 1°F.*

The **Centigrade Heat Unit (C. H. U.)** is the amount of heat required to raise one pound of water through 1°C.

Thus, heat reqd. to raise 1 gm. of water through $1^\circ = 1$ heat unit
„ „ „ 2 „ „ „ „ $= 2$ „ „
„ „ „ m „ „ „ „ $= m$ „ „
„ „ „ m „ „ „ „ $t^\circ = mt$ „ „

41. (a) Relations between the Units :—

1 lb. of water = 453 6 gms. of water ; and $1^\circ F. = \frac{5}{9}{}^\circ C.$

$\therefore$ 1 B. Th. U. $= 453{\cdot}6 \times \frac{5}{9} = 252$ calories.

Thus, to convert from calories to B. Th. U. multiply the calories by 1/252 ; and to convert from B. Th. U. to calories, multiply the B. Th. U.'s by 252.

Again, a Centigrade degree is $\frac{9}{5}$ of a Fahrenneit degree, so **the pound-degree Centigrade Unit** $= \frac{9}{5}$ *or* 1·8 B Th. U , and, since, 1 pound = 453·6 gm.

$\therefore$ **The pound-degree Centigrade Unit = 453 6 calories.**

42. Principle of the Measurement of Heat.—Take two beakers of the same size. Into one of them put 50 c c of water (mass = 50 grams) at 40 C., and in the other 50 c.c. of ice-cold water. Now quickly mix the contents of the two beakers. It will be found that the final temperature of the mixture is midway between 40°C. and 0 C., *e g.* 20°C.

Again, if 100 grams of water at 60°C. is mixed with 110 grams of water at 20°C. the resulting temperature of the mixture will be 40°C.

In this experiment we assume that (*a*) the quantity of heat gained or lost by one gram of water taken at any temperature for a change of 1°C. is constant, *i e.* it is the same whether the temperature changes from, say 30° to 31°, 80 to 81, or 56° to 55° , (*b*) the exchange of heat takes place between the two quantities of water without any loss or gain of heat from any other causes.

In other words, the heat lost by 50 grams of warm water is equal to the heat gained by 50 grams of cold water, or again the heat lost by 100 gms. of water in cooling through 20°C. (from 60° to 40°) has raised the temperature of 100 gms. of water through 20° (from 20° to 40°) This is the main principle of the measurement of heat, *i.e.*

Heat lost = Heat gained.

or $50(40 - t) = 50\ (t - 0)$; *or* $2000 - 50t = 50\ t$; *or* $t = 20^\circ C.$

[**Note.**—If two masses m_1 and m_2 are added, the resultant mass $m = m_1 + m_2$, and if two quantities of heat Q_1 and Q_2 are added, the resultant quantity $Q = Q_1 + Q_2$; but *temperatures do not follow the additive law, viz.* if two bodies at temperatures θ_1 and θ_2 are mixed up, the resultant temperature θ of the mixture is not equal to $\theta_1 + \theta_2$.].

43. Specific Heat.—We have seen that by mixing 100 grams of water at 60°C. with 100 grams of water at 20°C., the resulting temperature of the mixture becomes 40°C. But if 100 grams of water at 60°C. are mixed with 100 grams of turpentine at 20°C., the resulting temperature of the mixture will be about 48°C. Thus, the heat given out by the water in cooling through 12°C. is sufficient to raise the temperature of turpentine through 22 C. If any other liquid is taken the result will be different. Again, if equal masses of different metals are heated to the same temperature, and then each of them is dropped into a beaker containing water at the room temperature, the mass of water in each beaker being the same, it will be found that waters in the beakers will rise to different temperatures. On calculating the quantity of heat in each case it will be found that different metals have given out different amounts of heat in cooling through the same range of temperature. It is also evident that these metals absorbed different amounts of heat, when heated through the same range of temperature.

Experiment.—Place a number of balls of different metals, say lead, tin, brass, copper, iron, and of the same mass, say m gms., in a vessel of boiling water. After a few minutes remove the balls and place them on a thick slab of paraffin. The balls will melt the paraffin, but not to the same amount (see Fig. 28). The balls which absorbed the most heat will, of course, sink further in the paraffin.

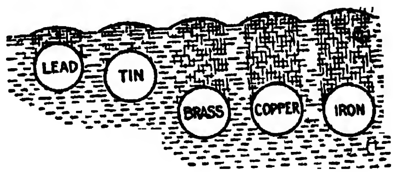


Fig. 28.

Since the mass (m) and the rise of temperature (t) are the same in each case, there is some *specific property* of the substances on which the quantity of heat taken up by each of them depended. This property is called the *specific heat* of the substance.

The heat H required to raise the temperature of m gms. of water through t°C. $= mt$ calories.

The heat required to raise m grams of mercury through the same range of temperature (t°C.) is much less than mt calories,

If H' denotes this amount of heat, we have, $H' \propto mt$; or $H' = s.mt$, where s depends upon the specific property of mercury. This is called *specific heat* which is numerically equal to the number of heat units required to raise 1 gram of a substance through 1°C.

Specific heat of a substance is given by the ratio of the quantity of heat required to raise any mass of the substance through any range of temperature to the quantity of heat required to raise an equal mass of water through the same range of temperature.

Specific heat, therefore, is a mere number involving no unit, i.e. in both the units the value of the specific heat of a substance is the same. Thus, if m be the mass of a substance and s its specific heat,

$$s = \frac{\text{Amount of heat reqd. to raise } m \text{ gms. of substance through } t^\circ C.}{\text{Amount of heat reqd. to raise } m \text{ gms. of water through } t^\circ C.}$$

Similarly, in British units

$$s = \frac{\text{Amount of heat reqd. to raise } m \text{ lbs. of substance through } t^\circ F.}{\text{Amount of heat reqd. to raise } m \text{ lbs. of water through } t^\circ F.}$$

But the amount of heat required to raise m grams of water through t°C. is mt calories, (the sp. ht. of water being unity).

Therefore, the amount of heat required to raise m grams of a substance through t°C. $= m \times s \times t$ calories.

Also the amount of heat required to raise m lbs. of a substance through t°F. $= m \times s \times t$ B. Th. U.

Thus the amount of heat required to raise the temperature of a body = **Mass × sp heat × Rise of temperature.** So we can say that **the specific heat** *of a substance is the amount of heat required to raise unit mass of a substance through one degree in temperature.*

Now the specific heat of iron is 0·11. This means that 0·11 calorie will raise the temperature of 1 gm. of iron through 1°C., or that 0·11 B. Th. U. will raise the temperature of 1 lb. of iron through 1°F., or that 0·11 pound-degree-centigrade unit will raise the temperature of 1 lb. of iron through 1°C. Similarly 1 gm. of iron in cooling through 1°C. will give out 0·11 calorie.

44. Thermal Capacity.—The thermal capacity of a body is the quantity of heat required to raise the temperature of the given body through 1°.

If m gm. be the mass of the body and s its specific heat, the thermal capacity of the body $= ms$ units of heat.

The specific heat of a substance is its thermal capacity per unit mass.

Example.—*The density of two substances are as 2 : 3, and their specific heats are 0·12 and 0·09 respectively. Compare their thermal capacities per unit volume.*

[*C. U. 1929 ; '34*]

Let the densities of the two substances be $2x$ and $3x$ respectively. Therefore, the mass per unit volume of the first substance is $2x$ gms., and that of the other is $3x$ gms. Hence the thermal capacity per unit volume of the first substance $= 2x \times 0·12$; and that of the second substance $= 3x \times 0·09$.

$$\therefore \frac{\text{Thermal capacity of the first substance}}{\text{Thermal capacity of the second substance}} = \frac{2x \times 0·12}{3x \times 0·09} = \frac{8}{9}.$$

45. Water Equivalent.—*The water equivalent of a body* is the mass of water which will be heated through 1° by the amount of heat required to raise the temperature of the body through 1°.

If m gms. be the mass of a body and s its specific heat, the amount of heat required to raise the temperature of the body through $1°C = ms$ cals. This amount of heat will raise ms grams of water through 1°C.

∴ Water equivalent of the body $= ms$ grams.

So the thermal capacity of a body is numerically equal to its water equivalent.

46 Determination of the Water Equivalent of a Calorimeter —Dry the calorimeter and weigh it with a stirrer of the same material. Fill the calorimeter to about one-third with cold water, note its temperature and weigh again, and thus get the weight of water. To this add quickly about an equal quantity of hot water after correctly noting its temperature. The temperature of this water should not be very high, otherwise the loss of heat due to radiation, etc., (which does not come into calculation) shall have to be accounted for. Now stir the mixture and note the final temperature. Weigh the calorimeter again to get the weight of water added.

Let mass of cold water $= m$ grams ; Mass of hot water $= m'$ grms ; Temperature of cold water $= t_1$°C. ; Temperature of hot water $= t_2$°C. ;

Common temperature of the mixture $= t$°C. ;

Water equivalent of the calorimeter and stirrer $= W$ gms.

Heat lost by m' gms. of water in cooling through $(t_2 - t)$°C. $= m'(t_2 - t)$ calories. Heat gained by m gms. of water in rising through $(t - t_1)$°C. $= m(t - t_1)$ calories.

Heat gained by calorimeter in rising through $(t - t_1)$°C. $= W(t - t_1)$ calories. Now, we have,

$$\textit{total heat lost} = \textit{total heat gained}$$

$$m'(t_2 - t) = W(t - t_1) + m(t - t_1)$$

$$\therefore \quad W = \frac{m'(t_2 - t)}{(t - t_1)} - m.$$

Errors and Precautions.—Heat may be lost by the hot water when being poured into the calorimeter and moreover, the hot mixture will lose some heat through radiation. Due to both the accounts, the final temp. will be too small. Again unless the temp. of the mixture is small, the loss of water by evaporation will be appreciable.

The loss of heat by radiation from the mixture may be eliminated by adopting **Rumford's method of Compensation.** In this method, the initial temp. of the water is taken as many degrees below that of the atmosphere (by addition of ice-cold water) as the final temp. of the

water after mixture will be above that of the atmosphere. So, the heat lost by radiation from the calorimeter after mixture will be exactly compemsated for by the gain of an equal quantity of heat by the calorimeter and contents before mixture.

47. Specific Heat of a Solid :—(*Method of Mixtures*).—

Regnault's Apparatus :—The complete outfit of his apparatus consists of a steam heater S (Fig. 29) and a calorimeter C thermally isolated from each other by means of the vertical partition P which can be moved up or down according to necessity. The method is to heat the given solid in the steam heater to a constant high temp. and then to drop the heated solid into the calorimeter (containing some liquid), the maximum temp. after mixture being noted. By equating the heat lost by the solid to the heat gained by the calorimeter and contents, the sp. heat of the solid is determined.

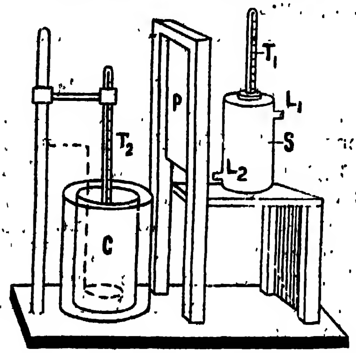


Fig. 29

The heater S (fig. 30) is a double-walled steam jacket, the inner wall being A and the outer wall B. It is open at both ends, the upper end being closed by a cork C and the lower end by a sliding wooden platform. The inner chamber AA of the heater is heated externally by a current of steam which enters from a boiler into the inlet L_1 and passes out through the exit L_2. The solid is suspended into this chamber by means of a slender thread as shewn, and the temp. of the chamber when steady (and so the temp. of the solid) is taken from the thermometer T_1 inserted vertically through a hole in the cork. The calorimeter C (fig. 29) is a copper vessel placed inside another but insulated from it by means of cork or cotton-wool pads—both being placed inside a wooden box not shewn in the figure. The thermometer T_2 held vertically gives the temp. of the contents of the calorimeter. When the solid is heated in the heater, the sliding shutter is kept down to protect the calorimeter from the heat of the heater.

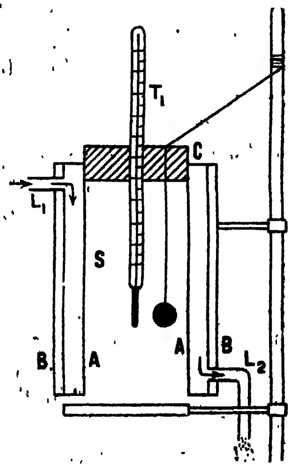


Fig. 30

Method.—When the temp. of the solid has become steady, the partition P is raised, the calorimeter C (fig. 29) is pushed into a position vertically under the heater S, the sliding wooden platform at the bottom of the heater is turned out, the suspending thread is cut whereon the solid drops into the liquid in the calorimeter. The calorimeter is then taken out, and the liquid is stirred well. The maximum temp. of the mixture is noted from the thermometer T_2.

[Fig. 30 represents a section of another very simple and efficient form of steam-heater for determining the specific heat of a solid. It consists of a fixed metal tube P bent at an obtuse angle and fixed inside a steam or water jacket. Another movable tube A is placed into the vertical part of P which is closed at the top by a cork carrying a thermometer T. In the beginning, the specimen S is dropped into this tube by removing the cork, and when the temperature becomes steady this upper tube is raised and the specimen S at once falls out of the heater into a calorimeter placed at the lower end of P.]

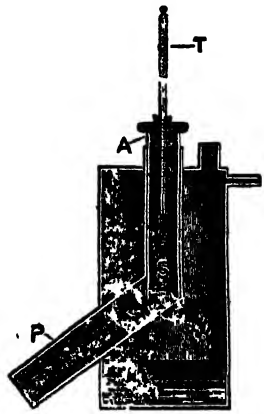


Fig. 30(a)....Steam heater

Calculation—Let the mass and sp. heat of the solid be m and s respectively and its steady temp. in the heater t_2. Suppose the mass of liquid in the calorimeter is w and s_1, its sp. heat and t_1, its initial temp. If W = water eq. of the calorimeter and stirrer and t, the maximum temp. of the mixture, we have from the *principle of heat loss equal to heat gain,*

$$m. s. (t_2 - t) = (w. s_1 + W)(t - t_1)$$

That is,
$$s = \frac{(w. s_1 + W)(t - t_1)}{m(t_2 - t)}$$

Advantages of this arrangement.—The chief points are (1) the heating of the solid to a fixed temp. without contact with moisture, (2) the rapidity of transfer of the hot solid to the calorimeter so that the heat lost in the transit is minimum, (3) the protection of the calorimeter from the heating chamber.

[**Note.**—It is to be noticed that the water equivalent of the calorimeter acts in the same way as an additional quantity of water equal to $w.s_1$ gms.]

Sources of Error :—

(1) Some heat is lost due to radiation, which should be taken into account.

This may be corrected to a certain extent by previously reducing the temperature of the liquid in the calorimeter as much below the room temperature as the final temperature is above that (Rumford's method of compensation, page 259.)

So, the loss of heat by the calorimeter during the second half of the expt. is compensated for, by an equal gain in the first half.

The outer and inner surfaces of the calorimeter are very often polished, by which the loss of heat by radiation is reduced to some extent.

For accurate work, the maximum temp. after mixture is corrected for radiation loss by the application of **Newton's law of cooling**. (Vide Art. 108, Chapter VIII).

(2) Some heat is lost in transferring the hot solid from the steam-heater to the calorimeter, so arrangement is made for dropping the hot solid directly into the calorimeter by bringing it under the steam-heater.

Some heat is also lost in heating the thermometer.

(3) The water equivalent of the calorimeter and stirrer should be taken into account in calculating the amount of heat gained.

(4) The thermometer should be very sensitive,—graduated to $\frac{1}{10}$th or $\frac{1}{5}$th of a degree Centigrade.

(5) *The change of temperature of the water in the calorimeter should be observed very accurately*, as the accuracy of the result depends more on the accuracy with which the change of temperature of the water in the calorimeter is noted, and not so much on the accuracy in weighing.

(6) The thermometer in the steam-heater should be corrected for the boiling point.

Examples :—(1) *A piece of lead at 99°C. is placed in a calorimeter containing 200 gms. of water at 15°C. The temperature after stirring is 21°C. The calorimeter weighs 40 gm. and is made of a material of specific heat 0·01. Calculate the thermal capacity of the piece of lead.*

Let C be the thermal capacity of the piece of lead.

Heat lost by the lead piece $= C\,(99-21)$ cal.

Heat gained by calorimeter and water $= 40 \times 0{\cdot}01 \times (21-15) + 200(21-15)$ cal.

Heat lost = Heat gained. Therefore,

$C(99-21)=(40\times 0{\cdot}01+200)(21-15)=200{\cdot}4\times 6$; whence $C=15{\cdot}4$ calories.

(2) *An alloy consists of 60% copper and 40% nickel. A piece of the alloy weighing 50 gm. is dropped into a calorimeter whose water equivalent is 5 gm. The calorimeter contains 55 gm. of water at 10°C. If the final temperature is 20°C, calculate the original temperature of the alloy. [Sp. ht. of copper = 0·095 ; sp. ht. of nickel = 0·11].*

The mass of copper in the alloy $=\frac{60}{100}\times 50=30$ grms., and

the mass of nickel in the alloy $=\frac{40}{100}\times 50=20$ gms.

Let t°C. be the original temperature of the alloy, then

heat lost by copper $=30\times 0{\cdot}095\times(t-20)$ cal.; heat lost by nickel $=20\times 0{\cdot}11\times(t-20)$ cal.; heat gained by water $=55\times(20-10)$ cal.

Since, heat lost = heat gained

$(t-20)\{(30\times 0{\cdot}095)+(20\times 0{\cdot}11)\}=(20-10)(55+5)$; whence $t=138{\cdot}8$°C.

(3) *Equal volumes of mercury and glass have the same capacity for heat. Calculate the specific heat of a piece of glass of specific gravity 2·5, if the specific heat of mercury is 0·0333 and its specific gravity 13·6.* (Pat. 1928)

Let the volume of the piece of glass $=V$ c.c., then its mass $=V\times 2{\cdot}5$ gm., and the mass of V c.c. of mercury $=V\times 13{\cdot}6$ gm.

Capacity for heat of V c.c. of glass $(H_1)=V\times 2{\cdot}5\times s$ (where s is the sp. ht. of glass). Capacity for heat of V c.c. of mercury $(H_2)=V\times 13{\cdot}6\times 0{\cdot}0333$.

We have, $H_1=H_2$.

$\therefore\quad V\times 2{\cdot}5\times s=V\times 13{\cdot}6\times 0{\cdot}0333\ ;\ \therefore\quad s=\dfrac{13{\cdot}6\times 0{\cdot}0333}{2{\cdot}5}=0{\cdot}181.$

48. Measurement of High Temperature by the Calorimeter Method —The method adopted in Art. 47 may be used in finding a high temperature, say that of a furnace or of a Bunsen flame.

A solid of known mass and sp. heat, preferably a good conductor of heat such as a metal, *whose melting point is below the temp. under measurement* is placed in contact with the source of high temperature. After a lapse of an interval of time when the solid has attained the constant temp. of the bath, it is taken out and immediately dropped into a calorimeter containing sufficient water to cover the solid and the rise of temp. of the water is determined with a sensitive thermometer.

If the mass of water taken	$=m$
,, ,, ,, ,, solid ,,	$=w$
Water eq. of calorimeter and stirrer	$=W$
Specific heat of the solid	$=s$
Initial and final temp. of water	$=t_1, t_2$
Unknown temp. of the bath	$=t$,

$$w.s(t-t_2)=(m.1+W)(t_2-t_1),$$

whence t can be calculated.

Example :—*In order to determine the temperature of a furnace a platinum ball weighing 80 gms. is introduced into it. When it has acquired the temperature of the furnace it is transferred quickly to a vessel of water at 15°C. The temperature rises to 20°C. If the weight of water together with the water equivalent of the calorimeter be 400 gm., what was the temperature of the furnace? (Specific heat of platinum = 0·0365).*

Let t°C. be the temperature of the furnace. The heat lost by the platinum ball in falling from t°C. to 20°C. = $80 \times 0{\cdot}0365 \times (t-20)$ cal.

and heat gained by calorimeter and water = $400(20-15)$ cal.

$\therefore$ $80 \times 0{\cdot}0365 \times (t-20) = 400(20-15)$; whence $t = 705$°C. (nearly)

49. Heating (or Calorific) Values of Fuels.—"The heating or calorific value of a sample of coal is 12000 B. Th. U. per pound"—simply means that the heat given out by the complete combustion of one pound of coal of that particular sample is 12,000 B. Th. U. The heating value of any other fuel,—solid, liquid, or gas, can be similarly expressed.

For accurate determinations of calorific values of fuels, special fuel calorimeters such as, bomb calorimeter, Juncker's gas calorimeter etc., have been devised.

50. Specific Heat of a Liquid.—

(a) Method of Mixtures—The method is the same as Regnault's method explained in Art. 47. An auxiliary solid which is not chemically acted on by the liquid and whose sp. heat is known is to be taken. Using the same notation as in Art 47, we have, by the principle of loss of heat equal to gain of heat,

$$m.\ s.\ (t_2-t)=(w.s_1+W)\ (t-t_1)$$

That is, $$S_1=\frac{m.\ s.\ (t_2-t)}{w(t-t_1)}-\frac{W}{w}.$$

[**Note.**—The specific heat of a liquid can also be determined by mixing it with a known quantity of water, but this method is not applicable when there is any *chemical action* between them, or when they do not mix well.]

(b) Method of Cooling :—

The method is based on **'Newton's Law of Cooling'** which states that *the rate of loss of heat by a body is proportional to the mean difference of temp. between the body and its surroundings.* If a hot liquid is kept in an enclosure at a lower temp., its rate of loss of heat must, therefore, depend on (i) the temp. of the liquid, (ii) temp. of the enclosure, and (iii) area of the exposed surface and nature and extent of

the surface of containing vessel. If these conditions are identical for, say, two different liquids, their rates of loss of heat will be equal irrespective of the natures of the two liquids.

Expt.—Half fill a calorimeter with the given liquid warmed to a temp., say 70°C *i.e.* sufficiently above the room temp. Place the calorimeter on non-conducting supports. Stir the liquid well and note the time taken by the liquid to fall through a certain range of temp. Repeat the above expt. by taking an equal volume of water initially at the same temp. in the same vessel so that equal surfaces are exposed to air and to the calorimeter. *Thus the two liquids are allowed to cool under similar circumstances.* Stir well. Note the time taken by the water to cool through the same range of temp.

Calculation ;—

Suppose, mass of liquid taken $= w$

„ „ water „ $= m$

„ „ calorimeter and stirrer $= W$

Time taken by liquid to cool from θ_1 to $\theta_2 = t_1$ sec.

„ „ „ water „ „ „ „ „ $= t_2$ „

Specific heat of the liquid $= s$.

$\therefore$ Rate of loss of heat of liquid and calorimeter $= \dfrac{(w.s + W)(\theta_1 - \theta_2)}{t_1}$

and rate of loss of heat of water and calorimeter $= \dfrac{(m.1 + W)(\theta_1 - \theta_2)}{t_2}$

The conditions of cooling being identical, the two rates of cooling are equal. That is,

$$\frac{(w.s + W)(\theta_1 - \theta_2)}{t_1} = \frac{(m + W)(\theta_1 - \theta_2)}{t_2}$$

or, $$s = \frac{(m + W).\,t_1}{w.t_2} - \frac{W}{w}.$$

[**Note** *that the rates of loss of heat are equal and not the rates of fall of temperature.*]

Example—*(1) 200 gms. of water at 98°C. are mixed with 200 c.c. of milk of density 1·03 at 30°C contained in a brass vessel of thermal capacity equal to that of 8 gms. of water, and the temperature of the mixture is 64°C. Find the sp. ht. of milk.* (C. U. 1925)

Wt. of milk = (200 × 1·03) gms ; Heat lost by water = 200(98 − 64) calories.

Heat gained by milk $= (200 \times 1.03) \times s(64-30)$, ($s =$ sp. ht. of milk)

Heat gained by brass vessel $= 8(64-30)$.

$\therefore$ $200 \times 34 = \{(200 \times 1.03 \times s) + 8\}\, 34$. Hence $s = 0.93$ approximately.

(2) *Three liquids of equal weights are thoroughly mixed. The specific heats of the liquids are s_1, s_2, s_3, and their temperatures are t_1°, t_2°, and t_3° respectively. Find the temperature of the mixture.*

Let T° be the temperature of the mixture of two of the liquids having specific heats s_1, s_2, and temperature t°_1, t°_2, respectively. Then, if m be the the mass of each liquid, and $T > t_1$, we have

$$m\, s_1(T-t_1) = m\, s_2(t_2-T)\ ;\ or\ T(s_1+s_2) = s_2t_2+s_1t_1\ ;\ or\ \ T = \frac{s_2t_2+s_1t_1}{s_1+s_2} \ldots(1)$$

Now mix the third liquid and let T_1 be the final temperature which is greater than T but less than t_3, then we have $ms_1(T_1-T) + m\, s_2\,(T_1-T) = m\, s_3\,(t_3-T_1)$;

$$or\ T_1\,(s_1+s_2+s_3) = s_3\,t_3 + T(s_1+s_2) = s_3t_3 + \frac{s_2t_2+s_1t_1}{(s_1+s_2)} \times (s_1+s_2) \ldots \text{from}\ \ (1)$$

$$= s_3t_3+s_2t_2+s_1t_1\ ; \quad \text{whence} \quad T = \frac{s_1t_1+s_2t_2+s_3t_3}{(s_1+s_2+s_3)}.$$

(3) *The specific gravity of a certain liquid is 0.8, that of another liquid is 0.5. It is found that the heat capacity of 3 litres of the first is the same as that of 2 litres of the second. Compare their specific heats.* (*Pat. 1926*)

Volume of the first liquid $= 3000$ c.c. ; Mass of the first liquid $= 3000 \times 0.8 = 2400$ gms. Volume of the second liquid $= 2000$ c.c. ; Mass of the second liquid $= 2000 \times 0.5 = 1000$ gms.

Heat capacity of the first liquid. $H_1 = 2400 \times s_1$ (where $s_1 =$ sp. ht. of the first liquid) ; Heat capacity of the second liquid, $H_2 = 1000 \times s_2$ (where $s_2 =$ sp. ht. of the second liquid).

We have, $H_1 = H_2$; $\because$ $2400 \times s_1 = 1000 \times s_2$; $\therefore$ $\dfrac{s_1}{s_2} = \dfrac{1000}{2400} = \dfrac{5}{12}$.

(4) *A mixture of 5 kgms. of two liquids A and B is heated to 40°C. and then mixed with 6 kgms. of water at 7.67°C. The resultant temperature is 10°C. If the specific heat of A is 0.1212 that of B is 0.0746, find the amount of A and B in the mixture.*

Let x be the amount of A, and y the amount of B, then $x+y = 5$ kgms....(1)

Heat lost by x kgms. of $A = x \times 1000 \times 0.1212 \times (40-10)$ calories.

Heat lost by y kgms. of $B = y \times 1000 \times 0.0746 \times (40-10)$ calories.

Total heat lost by the mixture $= (3.636x + 2.238y) \times 1000$ calories.

Heat gained by water $= 6 \times 1000 \times (10-7.67) = 13980$ calories.

Hence $3636x + 2238y = 13980$; But $x = 5-y$......from (1).

$\therefore$ $3636(5-y) + 2238y = 13980$; from which $y = 3.0043$ kgms.

$\therefore$ $x = 5 - 3.0043 = 1.9957$ kgms.

51. The Specific Heat of Gases.—When a gas is heated it may be allowed to expand in two ways,—(a) keeping its pressure always constant, as in constant pressure air thermometer [Art. 30], or (b) keeping its volume constant, as in constant volume air thermometer [Art. 31]. Therefore, when the mass of a gas and the amount of heat taken to raise its temperature through a certain range are known, the specific heat of the gas can be calculated at constant pressure, or at constant volnme

The **Sp. heat of a gas at constant volume** (C_v) is the amount of heat reqd. to raise the temp. of unit mass of the gas through *1°*, the volume being kept constant.

The **Sp heat of a gas at constant pressure** (C_p) is the amount of heat reqd. to raise the temp. of unit mass of a gas through *1°*, the pressure being kept constant.

C_p is greater than C_v.

Suppose 1 grm. of a gas is taken which is to be heated through 1°C. A quantity of heat will be required for the purpose when the gas is heated, but not allowed to expand, *i e* when the *volume* is kept *constant* and the pressure increases. Again, if the gas be heated and allowed to expand, *i e.* when the *pressure* is kept *constant* and volume increases, *some work is also done against external pressure* by the expanding gas, while in the first case no such work is done. Thus at constant pressure, in addition to the heat reqd. to raise the temp. through 1°C at constant volume, some additional heat must be necessary to supply the energy for the work done during expansion against the external pressure. Hence *the specific heat of a gas at constant pressure (C_p) is greater than the specific heat at constant volume (C_v).* It is found that *the ratio of the specific heat of a gas at constant pressure to that at constant volume (i e. C_p/C_v) is equal to 1·41 in the case of diatomic gases like oxygen, hydrogen, nitrogen, etc.*

N. B. For solids liquids, C_p and C_v are practically the same, as on heating expansion in volume is very small.

52. Consequence of the High Specific Heat of Water.—From a table of specific heats it will be seen that mercury has got a very low specific heat (0·033), which is one of the advantages of using mercury as a thermometric substance, because it will absorb a small amount of heat, if placed in a liquid, and so will lower the temperature of the liquid only slightly. *Water has a higher specific heat than any other liquid or any solid.* Thus a larger amount of heat is necessary to raise the temperature of a given weight of water through a certain range than is required by an equal weight of any other substance and

that is why *water is not suitable as a thermometric liquid. Moreover its sp. heat varies with temperature.*

For the above reason, the sea is heated more slowly than the land by the sun ; so during mid-day, the temperature of the coast will be greater than the temperature of the sea, but, after sun-set, the case will be just the reverse, because the sea cools more slowly than the land. For example, taking the specific heat of air to be 0·237, it is found that 1 gm. of water in losing one degree of temperature would raise the temperature of 1/0·237 gm. (*i.e.* = 4·2 gm.) of air one degree. Again, because water is 770 times heavier than air, one cubic foot of water in losing one degree of temperature would increase the temperature of 770 × 4·2 or 3234 cubic feet of air one degree. From the above consideration it is clear that islands have a more equitable climate owing to the influence of the sea, which prevents the occurrence of extremes of heat and cold, and so sea is called *a moderator of climate.*

The effect of the difference in the specific heats of sea and land is the setting up of, what is called, land and sea-breezes. During the day the sea is cooler, so the air above the sea is cooler than the air above the land, and as the hot air above the land ascends, colder air from the sea flows in to take its place producing a **sea-breeze.** After sun-set land radiates heat more quickly than the sea, and so at night colder air from the land blows to the sea producing a **land-breeze.**

Owing to its sp. heat being the highest, water is preferably used in hot water bottles, footwarmers and hot water pipes for heating purposes in cold countries. Moreover it becomes less hot than any other liquid, when kept in the sun

53. Latent heat.—It has been found that a solid substance undergoing liquefaction or fusion absorbs heat *without rise of temperature.* Similarly, a liquid during the process of solidification gives out heat *without fall of temperature. The heat absorbed or given out per unit mass* (1 gm. or 1 lb.) *of a substance during change of state* (*i.e. from the solid to the liquid* or *from the liquid to the solid state*) *without change of temperature is known as the* **Latent Heat,** which is constant for the same substance under the same condition. The word "latent" means hidden, that is, the heat which has got no external manifestation, such as rise of temperature, is called *latent heat,* but when it raises the temperature of the substance it is called *sensible heat.*

The **latent heat of fusion** of a solid is the quantity of heat required to change unit mass of the substance at its melting point from the solid to the liquid state without change of temperature. The same quantity of heat is also given out by unit mass of the substance

at its freezing point in changing from the liquid state to the solid state without any change of temperature.

Thus the amount of heat required to convert 1 gram of ice at 0°C. into water at 0°C. is called the *latent heat of fusion of ice*, or the *latent heat of water*, the value of which is 80 calories. This is also the quantity of heat given out by 1 gram of water at 0°C, in becoming 1 gram of ice at 0°C.

If the thermal unit be defined by using 1 lb. and 1°C. as units (C. H. U.) and 1 lb. be used as the unit of mass, the latent heat of fusion of ice will also be 80.

But in pound-degree-Fahrenheit units, the value must be larger in proportion to the ratio of a degree C. to a degree F., *i.e.* 9 to 5. Hence the latent heat of ice in *British Thermal Units per* lb. $=(80 \times 9)/5 = 144$.

That is, **for all latent heats, the value in calories per gram must be multiplied by $\frac{9}{5}$ to obtain its value in B. Th. U. per lb.**

"*The latent heat of fusion of ice is 80*" means that 80 calories of heat are necessary to convert one gram of ice from the solid to the liquid state *without change of temperature.*

This explains why, in cold countries, the thermometer may stand at 0°C. in winter without any ice being formed on the surface of a pond. The water must lose its latent heat before it will freeze.

The reality of latent heat may be shown by mixing 100 grams of water at 80°C, with 100 grams of water at 0°C., when the final temperature of the mixture will be 40°C. But if 100 grams of water at 80°C. be mixed with 100 grams of ice at 0°C., the final temperature will be 0°C. All the heat given out by the hot water has been used up to convert the ice at 0°C to water at 0°C.

Similarly, a liquid at its boiling point absorbs heat in order to be converted into vapour without rise of temperature. This heat is absorbed only to bring about the change of state. *The quantity of heat required to convert unit mass of a liquid at its boiling point to the vapour state without change of temperature is called the* **latent heat of vaporisation** *of that liquid at that temperature.*

The same amount of heat is also given out per unit mass of the vapour of the liquid during condensation under the same conditions.

It has been found that 536 calories of heat are necessary to change one gram of water at 100°C. into steam *without change of temperature.* The same amount of heat is also given out by one gram of steam in condensing to water at 100°C. ; or, in other words, the value of the latent heat of steam in gram-degree-Centigrade units is 536 calories.

This value in the pound-degree-Centigrade units is also 536 where the unit of heat (C. H. U.) is taken as the quantity of heat required to raise 1 lb. of water throug 1°C.

The value of the latent heat of steam in B. Th. U. (pound-degree-Fahrenheit) per lb. is $(536 \times 9)/5 = 964.8$.

Note. –The value of the latent heat of steam is rather high, and this explains why burns from steam are so severe. These burns are more painful than those from boiling water because of the heat given out by the steam in condensing.

54. **Determination of the Latent Heat of Fusion of Ice.**— Weigh a calorimeter and stirrer (w gms.). Half fill it with warm water at about 5° above the room temperature. Weigh the calorimeter with its contents again, whence the weight of water added is found (m gms.). Note with a sensitive thermometer the initial temp. (t_1°C) of the water in the calorimeter. A block of ice is broken into small fragments which are washed with clean water and dried by means of blotting paper. Get some of them and drop them into the calorimeter holding them not with finger but with the blotting paper. Stir well until all the ice is melted. Note the lowest temperature attained by the mixture (t_2°C), which should be about 5° below the room temperature. Weigh the calorimeter and contents again, whence the wt. of ice added is found (M gms.)

The gain of heat takes place in two parts, (a) an amount of heat is necessary to melt the ice at 0°C to water at 0°C, (b) a further amount of heat is required to raise the ice-cold water to t_2°C.

Heat lost by calorimeter and stirrer,

$$= (w.\ s + m)\ (t_1 - t_2) \text{ cals.}$$

where s = sp. heat of the material of the calorimeter.

Heat gained by ice in melting and by ice-cold water in rising to t_2°C, $= ML + M.1.t_2$ cals., where L = latent heat of fusion of ice.

$$\therefore \quad ML + M.\ t_2 = (ws + m)\ (t_1 - t_2), \text{ whence } L = \frac{(ws+m)\ (t_1 - t_2)}{M} - t_2.$$

[**Errors and Precaution** —(1) Fingers should not be used at the time of dropping the ice-pieces, for by so doing some ice will melt, and the melted ice, *i.e.*, water, if added along with pieces of ice, will appreciably affect the accuracy of the result. For example, if only 0.1 gm. of water (and not ice) is added, there will be an error of 0.1×80 or 8 calories of heat in the calculation, which amount was really

absent. (2) The initial temperature of water is taken 5° above the room temperature and final temperature 5° below it in order to make good the loss of heat due to radiation.

(3) The ice, during the process of melting, should be kept below the surface of water, and not allowed to float, otherwise the portion above the water surfrce will give out its heat to the outside air and not to the water in the calorimeter. For this, use a wire-gauge stirrer. Care should be taken that no water particle accompanies the thermometer while removing it.]

Examples—(1) *Find the latent heat of fusion of ice from the following data :*
Weight of the calorimeter = 60 gms. : Wt. of cal. + water = 460 gms.
Temperature of water (before ice is put in) = 88°C. ; Temperature of mixture = 5°C.
Weight of calorimeter + water + ice = 618 gms. ; Sp. heat of the calorimeter = 0·1. (C. U. 1918)

Let L be the latent heat of fusion of ice ; Mass of water $= (460-60) = 400$ gms. ; and mass of ice $= (618-460) = 158$ gms.

Heat lost by calorimeter and water $= 60 \times 0{\cdot}1 \times (88-5) + 400 \times (88-5)$ cal.

Heat required to melt 158 gms. of ice and to raise the temperature of the water formed to 5°C. $= 158L + 158\ (5-0)$ cal.

$\therefore\ 158L + 158 \times 5 = (88-5)\ (6+400)$; whence $L = 79{\cdot}8$ cals. per. gm.

(2) *A lump of iron weighing 200 gms. at 80°C. is placed in a vessel containing 1000 gms. of water at 0°C. What is the least quantity of ice which has to be added to reduce the temperature of the vessel to 0°C. ? (Sp. ht. of iron = 0·112).* (All. 1926)

Heat lost by iron is cooling to 0°C. $= 200 \times 0{\cdot}112 \times 80 = 1792$ cal.

The vessel containing 1000 gms. of water was formerly at 0°C. Now to absorb 1792 calories of heat given out by the lump of iron, the mass of ice required $= 1792/80 = 22{\cdot}4$ gms.

(3) *Find the result of mixing equal masses of ice at −10°C. and water at 60°C.* (All. 1916)

Let m gms. of ice be mixed with m gms. of water. m gms. of ice in rising to 0°C. from −10°C., will require $m \times 0{\cdot}5 \times 10 = 5m$ calories, (sp. ht. of ice $= 0{\cdot}5$). Again m gms. of ice at 0°C. in changing to water at 0°C. will require 80 m calories. But the heat supplied by m gms. of water in cooling from 60°C. to 0°C. is only 60 m calories. Out of this amount 5 m calories are required to increase the temperature of ice from −10°C. to 0°C., and the rest *i.e.* 55 m calories, can turn only $\frac{55}{80}m$ or $\frac{11}{16}m$ gms. of ice into water at 0°C. The remaining portion, *i.e.* $\frac{5}{16}m$. gms. of ice, must remain as such. Thus the result of the mixture is that $\frac{11}{16}$ parts of ice will be melted into water and $\frac{5}{16}$ parts will remain as ice at 0°C.

(4) *What would be the final temperature of the mixture when 5 gms. of ice at −10°C. are mixed up with 20 gms. of water at 30°C. The sp. ht. of ice is 0·5.* (C. U. 1926)

Let the final temperature be t°C. Heat gained by ice in going up to t°C. from −10°C. $= 5 \times 0{\cdot}5 \times \{0-(-10)\} + 5L + 5(t-0)$.

Heat lost by water $=20(30-t)$ calories. Taking $L=80$ units,

we have, $25+5\times 80+5t=20=(30-t)$. $\therefore$ $t=7°C$.

(5) *The specific gravity of ice is 0·917. 10 gms. of a metal at 100°C are immersed in a mixture of ice and water, and the volume of the mixture is found to be reduced by 125 c.mm. without change of temperature. Find the specific heat of the metal.* (*Pat* 1924)

We know that volume varies inversely as density, so when V c. c. of ice is changed into V' c. c. of water, we have

$$\frac{V}{V'}=\frac{1}{0\cdot917}\ (\because\ 0\cdot917=\text{sp. gr. of ice})=1\cdot09\ \text{c.c.}$$

$\therefore$ 1·09 c. c. of ice becomes 1 c. c. of water (wt $=1$ gm) at the same temperature, or, in other words, 1 gm. of ice in melting is reduced in volume by 0·02 c.c., and this requires 80 calories of heat

In the example, we have, the heat lost by the metal

$=10\times s\times(100-0)$ cal. $=(1000\ s)$ cal. ($s=$ sp. ht. of the metal)

The reduction in volume of the mixture $=125$ c.mm. $=\frac{125}{1000}=\frac{1}{8}$ c. c.

$\therefore$ The amount of ice melted $=\frac{1}{8}\div 0\cdot09=\frac{25}{18}$ gm.

The amount of heat required to melt $\frac{25}{18}$ gm of ice $=(\frac{25}{18}\times 80)$ calories.

By the example we have, $100\ s=\frac{25\times 80}{18}$; $\therefore$ $s=\frac{25\times 80}{18\times 1000}=0\cdot11$.

(6) *What would be the result of placing $4\frac{1}{2}$ lbs. of copper at 100°C in contact with $1\frac{1}{2}$ lbs. of ice at 0°C ? (sp. ht. of copper $=0\cdot095$ and latent heat of fusion of ice $=79$)* (*All.* 1916)

$4\frac{1}{2}$ lbs. of copper at 100°C. in cooling to 0°C. give out $4\frac{1}{2}\times 0\cdot095\times 100=42\cdot75$ pound-degree-C. heat units.

To melt one pound of ice at 0°C., 79 pound-degree-C. heat units are required.

$\therefore$ The amount of ice melted by 42·75 heat units $=42\cdot75/79=0\cdot54$ lbs.

Hence the amount of ice remaining unmelted $=\frac{3}{2}-0\cdot54=0\cdot96$ lbs.

So the result is 0·54 lbs. of water at 0°C., and 0·96 lbs. ice at 0°C.

55. High Latent Heat of Water.—The latent heat of water being high, the change from water to ice or from ice to water is a very slow process, and, during the time the change takes place, much heat is given out or absorbed. Had the latent heat of water been low, (*a*) the water of the lakes and ponds would have frozen much sooner, thus destroying the lives of aquatic animals living therein. (*b*) Ice-bergs on the mountains would have melted very rapidly with rise of temperature, thus causing disastrous floods in the neighbouring countries. The rise of temperature of a place is delayed by the presence of ice bergs near it and so the climate of the place is greatly influenced by formations of ice-bergs in the neighbourhood.

56. Ice-calorimeters.—The fact that a certain quantity of ice in melting always absorbs 80 calories of heat for each gm. of it has

been applied in the construction of ice-calorimeters for the determination of specific heats.

57 (a). Black's Ice-calorimeter.—In the simplest form of ice-calorimeters, as used by Black, a large block of ice is taken, a cavity is formed in it, and a slab of ice is taken to cover the cavity (Fig. 31). The solid (w gms.) of which the specific heat (S) is required is weighed and heated to a constant temperature (t°C) in a steam heater. On removing the slab, the water inside the cavity is soaked dry with a sponge, and the solid is quickly dropped into the cavity and covered by the slab. The solid melts some ice into water until its temperature falls to 0°C. After a few minutes, the water formed in the cavity is removed by a pipette and its mass determined (m gms.).

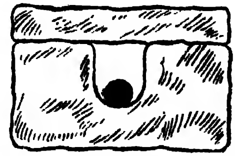


Fig. 31

Heat gained by ice in melting to water at 0°C,

$=mL$, where L is the latent heat of fusion of ice. Heat lost by the solid $=w.\ s.\ t.$

$$\therefore \quad mL = w.\ s.\ t. \quad \text{That is, } L = \frac{w.\ s.\ t.}{m}.$$

This method may also be used to determine the sp. heat S of the solid.

[**Note**—Though in this method there is no loss of heat by radiation, still it is not a very accurate method, for

(a) the water formed in the cavity cannot be completely taken out ; and

(b) during the time taken for dropping the solid inside the cavity some ice may melt by absorbing heat from the atmosphere.]

Example.—*A litre of hot water is poured into a hole in a block of ice at 0°C., which is immediately closed by a lid of ice. After a time the hole is found to contain a litre and a half of ice-cold water. What was the original temperature of the water.*

Let t°C. be the original temperature.

Mass of hot water = mass of a litre or 1000 c. c. of water = 1000 gms.

Mass of ice melted = mass of 500 c. c. of water = 500 gms.

Heat lost by water = $1000(t-0)$ cal. Heat required to melt 1 gm. of ice at 0°C. to water at 0°C. is 80 calories.

Hence heat gained by ice. = 500×80 cal. $\therefore$ $1000\ t = 500 \times 80$; *or* $t = 40$°C.

57 (b). Bunsen's Ice-calorimeter.—1 gm. of ice at 0°C. in melting to water at 0°C. decreases in volume by about 0·09 c.c. Bunsen has utilised this change of volume in the construction of a very delicate calorimeter (Fig. 32). A thin-walled test tube B is fused into a wider tube A, which is attached to a bent tube, C as shown in the figure. The other end of the bent tube is fitted with a cork D through which passes a fine capillary tube T of uniform bore having a scale S along its horizontal part. The upper part of A is filled with pure and air-free distilled water and the rest of A and the communicating tube C with mercury.

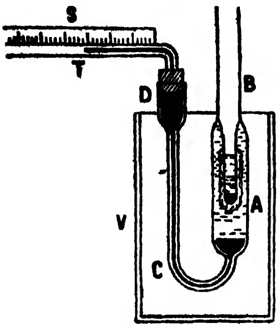


Fig 32—Bunsen's Ice calorimeter.

The apparatus is kept in a box., surrounded as completely as possible with melting ice. A mixture of some solid carbon dioxide and ether is placed in B to freeze some of the water in A forming a sheath of ice round its lower part. Now some amount of water is introduced into B, and the calorimeter is allowed to stand for a long time until the whole of it is at 0°C., when the position of the mercury meniscus in T is read on the scale S after it is steady. A small lump of metal of mass m gm. at t°C. is then put into B. This melts some of the ice surrounding B, and causes a contraction in volume and the mercury meniscus is found to move towards D. By knowing the area of cross-section a of the capillary tube, the specific heat s of the metal can be calculated as follows :—

When the metal has cooled to 0°C., the heat lost by it $= mst$ cal. But this amount is sufficient to melt mst/L gm of ice, where L is the latent heat of fusion of ice.

Now, 1·09 c.c of ice becomes 1 c.c., *i.e.* contracts in volume by 0·09 c.c., when turned into water whose mass is 1 gm.

Now L calories of heat will melt 1 gm. of ice into 1 gm. of water at 0°C., *i.e.* will cause a contraction of 0·09 c.c.

∴ For a contraction of 1 c.c., the amount of heat required $= \frac{L}{0·09}$ cal. If the mercury meniscus has moved a distance, say, d cm., the decrease in volume is $a \times d$, and for this the amount of heat necessary $= \frac{a \times d \times L}{0·09}$ cal. This amount has been supplied by the metal.

$$mst = \frac{a \times d \times L}{0.09}, \text{ or } s = \frac{a \times d \times L}{0.09 \times m \times t}.$$ If s is given, latent heat of fusion of ice can be determined by this method.

Advantages and Disadvantages—The disadvantage of this method is that it is difficult to set up the apparatus, but it is advantageous for the following reasons—(*a*) The arrangement is very sensitive. (*b*) There is no loss of heat due to radiation. (*c*) No calorimeter or thermometer is necessary. (*d*) The specific heat of a solid *available in a very small quantity* can be determined by this method.

Examples—(*1*) *Determine the specific heat of silver from the following data*—
Weight of silver dropped $= 0.92$ *gm.*; *Temperature of silver* $= 98°C$
Distance travelled by the mercury thread $= 6$ *mm.*
Area of cross section of the capillary tube $= 1$ *sq. mm*

The diminution in volume of the mercury thread $= 0.01 \times 0.6 = 0.006$ c. c.

Therefore, from above relation, we have, $s = \frac{0.006 \times 8}{0.09 \times 0.92 \times 98} = 0.0591.$

(*2*) *20 gms of water at 15°C are put into the tube of a Bunsen's ice calorimeter and it is observed that the mercury thread moves through 29 cms 12 gms. of a metal at 100°C. are then placed in the water and the mercury thread moves through 12 cms. Find the specific heat of the metal.* (*All. 1920*)

The heat given out by 20 gms. of water at 15°C in cooling to 0°C $= 20 \times 15 = 300$ cal This produces a movement of 29 cms. of the mercury thread.

Heat required for a movement of 1 cm. $= \frac{300}{29}$ cal, and for a movement of 12 cms $= \frac{12 \times 300}{29}$ cal. This amount has been supplied by the metal, which $= 12 \times 100 \times s$, where s is the sp. ht. of the metal.

$$\therefore \quad 12 \times 100 \times s = \frac{300 \times 12}{29}; \text{ or } s = \tfrac{3}{29} = 0.1 \text{ approx.}$$

(*3*) *The diameter of the capillary tube of a Bunsen's ice calorimeter is 1.4 mm. On dropping into the instrument a piece of a metal whose temperature is 100°C and mass 11.055 gms, the mercury thread is observed to move 10 cms Calculate the specific heat of the metal, given the latent heat and density of ice to be 80 and 0.9 respectively.* (*All. 1925*).

Mercury thread moves 10 cms., hence the volume of the mercury thread $= \pi \times (0.07)^2 \times 10$ c.c. $= 0.049\pi$ c.c.
Mass of 1 c.c. of ice $= 0.9$ gm., $\therefore$ The volume of 1 gm. of ice $= 1/0.9 = 1.11$ c.c.

But the volume of 1 gm. of water $= 1$ c.c. $\therefore$ The diminution in volume when 1 gm. of ice is melted, i.e. changed into water $= 1.11 - 1 = 0.11$ c.c. Hence to produce a diminution of 0.049π c.c., the mass of ice melted $= \frac{0.049\pi}{0.11}$ gm., and the heat required for this $= \frac{0.049\pi}{0.11} \times 80$ cal.

This is equal to the heat given out by the metal, which $= 11{\cdot}088 \times 100 \times s$ cal.

$$\therefore \quad 11{\cdot}088 \times 100 \times s = \frac{0{\cdot}049\pi}{0{\cdot}11} \times 80 \; ; \; \therefore \; s = \frac{0{\cdot}049 \times 22 \times 80}{11{\cdot}088 \times 100 \times 0{\cdot}11 \times 7} = 0{\cdot}1$$

58. Determination of the Latent Heat of Vaporisation.—Take a clean and dry calorimeter (Fig. 33), and weigh it together with the stirrer made of the same material (w gms.). After filling it with water upto about two-thirds weigh it again whence the mass of water (m gms.) is obtained. The steady temp. (t_1°C.) of the water is taken with a sensitive thermometer T inserted vertically. Boil some water in the boiler B, whose mouth is closed by a cork through which the bent delivery tube A passes. The free end of the delivery tube is introduced into the steam trap which is really a water-separator. It is a wide glass tube open at both ends which are closed by steam-tight corks. The delivery tube extends well into the trap. Through the cork at the bottom, two tubes pass, one a drain-off-tube C for removing the collected water and the other is an exit, being a straight tube D ending in a nozzle which dips into the water contained in the calorimeter. The screen protects the calorimeter from direct heating by the boiler.

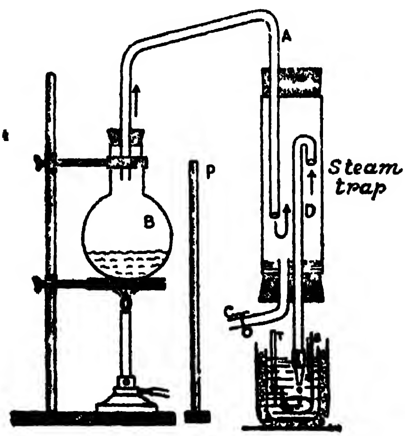


Fig. 33

Bring the calorimeter under the exit tube D such that the nozzle goes well into the water in it. After some time take away the nozzle quickly and note the highest temperature (t°C.) attained by the water. Remove the thermometer and allow the calorimeter and its contents to cool. Weigh the calorimeter with its contents again. The difference between the last two weighings gives the mass of steam condensed (M gms.).

Calculation.—Let L be the latent heat of steam and S, the sp. heat of the material of the calorimeter. Then,

heat lost by steam in being condensed to water at t°C.
$= ML + M.1.\ (100 - t)$ calories.

Heat gained by calorimeter and contents in being raised from t_1°C. to t°C.

$$= (w.s + m)\ (t - t_1).$$

Assuming heat loss equal to heat gain,

$$ML + M(100 - t) = (w.s + m)(t - t_1)$$

That is, $$L = \frac{(w.s + m)(t - t_1)}{M} - (100 - t).$$

Errors and Precautions —(1) If some part of the steam is condensed before entering into the calorimeter, the observed value of L will be low. That is why the steam delivery tube should be lagged with non-conductors like cotton-wool. In order that any condensed steam may not pass into the calorimeter, the steam trap is used. Moreover due to sudden absorption of steam by the cold water in the calorimeter, if any water from the calorimeter is sucked back it is arrested by the steam-trap and not allowed to get into the boiler *B*.

(2) To reduce the effect of radiation, the water in the calorimeter should be initially cooled a few degrees below the room temperature and steam passed till the temperature rises through the same amount above the room temperatcre (cf. Rumford's method of compensation).

(3) To protect the calorimeter from direct heating, a screen *P* is to be placed between the boiler and calorimeter.

(4) The temperature of the water in the calorimeter after mixture should not be allowed to increase by more than 15°C., otherwise much water (and so much heat) will be lost by vaporisation.

(5) If the issue of steam is too rapid, some water may be lost by splashing.

(6) The temperature of the steam should be determined in each case and cannot be taken as 100°C. without pressure correction.

Examples.—*(1) Steam at 100°C is allowed to pass into a vessel containing 10 grams of ice and 100 grams of water at 0°C., until all the ice is melted and the temperature is raised to 5°C. Neglecting water-equivalent of the vessel and the loss due to radiation, etc., calculate how much steam is condensed. (The latent heat of steam is 536 ; and latent heat of water is 80.)*

Let m be the mass of steam condensed.

Heat lost by steam $= m \times 536 + m(100 - 5)$ cal.

Heat gained by ice and water $= 10 \times 80 + 10 \times 5 + 100 \times 5$ cal.

Heat lost = Heat gained. $\therefore\ m(536 + 95) = 800 + 50 + 500$; *or* $m = \frac{1530}{631} = 2{\cdot}13$ gms.

(2) What will be the resulting temperature, if 5 gms. of ice at 0°C. are mixed with 1 gram of steam at 100°C.

Let t be the resulting temperature.

The heat required by 5 gms of ice in melting to water at 0°C. and that required by 5 gms. of water in rising through $t°C. = 5 \times 80 + 5t$ cal

The heat lost by 1 gram of steam in condensing to water at 100°C., and then cooling down to $t°C. = 1 \times 536 + 1(100 - t)$

Heat lost = Heat gained. So, $536 + (100 - t) = 5 \times 80 + 5t$, or $t = 39 \cdot 3°C.$

(3) *A calorimeter of water equivalent 4 gm contains 4 gm. of ice and 50 gms. of water at 0°C. when steam is passed into the calorimeter. Calculate how much steam must condense so that the final temperature of the mixture may be 40°C.*

Suppose x gm. be the wt. of steam condensed. The heat lost by it in coming down to $40°C. = x \times 536 + x(100 - 40)$ cal.

The heat gained by ice in first melting and then being raised to 40°C. $= 5 \times 80 + 4(40 - 0)$ cal. and the heat gained by water and calorimeter $= 4(40 - 0) + 50(40 - 0)$

∴ The total heat gained by ice, water, and calorimeter,

$$= 4 \times 80 + (50 + 4 + 4) \times 40 = 2640 \text{ cal}$$

Hence $x \times 536 + x \times 60 = 2640$; or $x \times 596 = 2640$, ∴ $x = 4 \cdot 42$ gm

(4) *How 1 kilogram of water at 50°C should be divided so that one part of it when turned into ice at 0°C, would by this change of state give out a quantity of heat that will be sufficient to vaporise the other part? (Latent heat of ice = 80 calories, latent heat of steam = 536 calories)*

Let 1 kgm. (or 1000 gm) of water be divided into two parts x and y, of which the heat given out by x gm., when turned into ice, would vaporise y gm We have $x + y = 1000$. ... ... (1)

Heat lost by x gm of water at 50°C to form ice at 0°C.

$$= (50x + 80x) = 130x \text{ calories}$$

This amount will turn y gm of water at 50°C into steam which will require $\{(100 - 50)y + 536\,y\}$ calories.

Hence $130x = 586y$; But from (1) $x = 1000 - y$, whence $x = 818 \cdot 5$ gm and $y = 181 \cdot 5$ gm.

Questions.

Art. 40.

1. Discuss the statement 'Heat is a physical quantity' (Pat 1940, '41)

Art. 46.

2. Describe an experiment to determine the water equivalent of a copper calorimeter. (C. U. 1918)

Art. 47.

3. Define specific heat. How the specific heat of a solid is determined? (C. U. 1917, '20, '28, '34, '36, '40).

4. How would you determine the specific heat of a substance by the Method of mixtures. (Dac. 1927)

5. A brass weight of 100 gms. is heated so that a particle of solder placed upon it just melts. It is then put into 100 c.c. of water at 15°C. contained in a calorimeter of water equivalent 12. If the final temperature of the water is 35°C., what is the melting point of the solder ? (sp. ht. of brass = ·088). (Pat. 1935).

[*Ans :* 289·5°C.]

6. An alloy consists of 92% silver and 8% copper. Calculate the final temperature when 50 gm. of the alloy at 100°C. are mixed with 50 gm. of oil of specific heat 0·46 at 20°C. (The sp. heats of copper and silver are 0·095 and 0·056 respectively).

[*Ans :* 29·1°C.].

7. Define unit of heat, capacity for heat, and specific heat. A piece of iron weighing 100 grams is warmed 10°C. How many grams of water could be warmed 1°C. by the same amount of heat ? The specific heat of iron is 0·10. (Punjab).

[*Ans :* 100 gm.].

Art. 48

8. Describe how you can measure the temperature of a furnace by applying calorimetric principle. (Pat. 1929).

9. A ball of platinum, whose mass is 200 gms., is removed from a furnace immersed in 153 gms. of water at 0°C. Supposing the water to gain all the heat the platinum loses and if the temperature of the water rises to 30°C., determine the temperature of the furnace. (sp. ht. of platinum = 0·031). (C. U. 1936).

[*Ans :* 770·3°C.]

Art. 49.

10. The calorific value of coke is 13,000 British Thermal Units per pound. Find the minimum amount of coke which would have to be burnt in order to heat 30 gallons of water from 60°F. to 130°F. for use in a bath. (1 gallon of water weighs 10 lb.).

[*Ans :* $\frac{21}{13}$ lb.],

Art. 50.

11. How would you determine the specific heat of a liquid ?
(C. U. 1915, '29) ; (Pat. '26, '43 ; cf. '45).

If 90 grams of mercury at 100°C. be mixed with 100 grams of water at 20°C., and if the resulting temperature be 22°C. what is the specific heat of mercury ? (C. U. 1925).

[*Ans :* 0·0285].

12. 10 gm. of common salt at 91°C. having been immersed in 125 gm. of oil of turpentine (sp. ht. 0·428) at 13°C., the temperature of the mixture is 16°C. ; supposing no loss or gain of heat from without, find the specific heat of common salt. Can you do this experiment with water instead of turpentine ? (C. U. 1936).

[*Ans :* 0·214].

13. A copper calorimeter weighing 10 gm. is filled first with water whose weight is 7·8 gm., and then with another liquid whose weight is 8·7 gm., the times taken in both cases to cool from 40°C. to 35°C. are 85 and 75 seconds respectively. Taking the specific heat of copper to be 0·095, calculate the specific heat of the liquid.

[*Ans :* 0·728].

14. Supposing you were given a thermometer reading only from 50°C. to 100°C., and some water of which the temperature was below 20°C., describe an experiment how, without using another thermometer, you could determine roughly the temperature of the water.

[**Hints.**—Take some water in another vessel whose mass is a little greater than that of the quantity given. Boil this water ; mix the two, and note the resultant temperature t°C. by the given thermometer which will be a little over 50°C. Let m be the mass of cold water, θ its temperature, and m' the mass of hot water, then we have, $m'(100-t)=(t-\theta)$. Hence calculate θ.]

Art. 51.

15. Account for the difference between the specific heat of a gas at constant volume and that at constant pressure. [All. 1931 ; cf. '44 ; '46.)

Art. 52.

16. "Water has a higher specific heat than any other liquid or solid." How will this fact affect (*a*) determination of temperature by a water thermometer over ranges for which its use is permissible, and (*b*) the climate of islands and places on the sea-coast ? (Pat. 1931).

Art. 53.

17. The latent heat of water is 80 calories. By what number will the latent heat be represented if the pound is taken as the unit of mass and the temperatures are measured on the Fahrenheit scale ? (Pat. 1931).

18. What is meant by the statement that the latent heat of steam is 536. What number will represent this latent heat, if the unit of mass is a pound and temperatures are measured on the Fahrenheit scale ? (Pat. 1941).

19. On what factor does the latent heat of a substance depend ? If the calorie be defined as the quantity of heat required to raise the temperature of one pound of water through one degree Fahrenheit, what would be the value of the latent heat of vaporisation of water in such calories, if its value in the gramme-Centigrade system in 580 ? (Pat. 1932)

[Value of latent heat in units as defined $= 580 \times \frac{9}{5} = 1044$].

20. Explain the meaning of 'latent heat.' (C. U. 1909, '13, '17 ; Pat. 1918).

Art. 54.

21. How would you determine the latent heat of fusion of ice ? (C. U. 1913, '20, '26, '28 ; Pat. 1918, '26, '28).

22. Find the result of mixing 2 lbs. of ice at 0°C. with 3 lbs. of water at 45°C. (C. U. 1931).

[**Hints**.—The amount of heat given out by 3 lbs. of water at 45°C. in cooling to 0°C. $= 3 \times 45 = 135$ pound-degree-C. heat-units ; and 80 such heat-units are necessary to melt 1 lb. of ice. So the amount of ice melted by this unit of heat $= \frac{135}{80} = 1\cdot69$ lbs. ∴ The result is $(3 + 1\cdot69)$ or 4·59 lbs. of water at 0°C. and $(2 - 1\cdot69)$ or 0·31 lb. of ice at 0°C.)].

23. Dry ice at 0°C. is dropped into a copper can of 100°C., the weight of the can being 60 grammes and the specific heat of copper 0·1. How much ice would reduce the temperature of the can to 40°C. ? (C. U. 1924).

[*Ans :* 3 grams.].

24. What would be the final temperature of the mixture when 5 gms. of ice at -10°C. are mixed up with 20 gms. of water at 30°C. The sp. ht. of ice is 0·5. (C. U. 1926).

[*Ans :* 7°C.].

25. Some ice is placed in a glass vessel held over a spirit-lamp and melts to water at 0°C. in 2 minutes ; how long will it take (*a*) before it reaches the boiling point, (*b*) before it is all boiled away, assuming there is no escape of heat.

[*Ans :* (*a*) $2\frac{1}{2}$ min. ; (*b*) $(2\frac{1}{2} + 13\frac{2}{3})$ min.]

Art. 56.

26. A ball of copper of mass 30 gms. was heated to 100°C. and placed in an ice-calorimeter. In cooling down it evolved sufficient heat to melt 3·54 gms. of ice. If the latent heat of fusion of ice is 80. what is the specific heat of copper ? (Dac. 1933)

[*Ans :* 0·094]

Art. 57.

27. Explain how the specific heat of a solid may be determined by means of the ice-calorimeter. (C. U. 1914, '15 ; Pat. 1943).

28. Describe a Bunsen's ice-calorimeter. (All. 1933).

A substance was heated to 100°C. and 0·8 gm. of it is dropped into a Bunsen's ice-calorimeter, due to which the thread of mercury in the capillary tube of 1 sq. mm. section moved through a distance of 6·9 mm. Calculate the specific heat of the substance (given that 1 gm. of water on freezing expands by 0·091 c.c.).

[*Ans :* 0·0758].

Art. 58.

29. Describe any method of determining the latent heat of steam in the laboratory. State the precautions that should be taken.

(C. U. 1931 ; All. 1918 ; Pat. 1935 ; Dac. '21).

30. A copper ball 56·32 gms. in weight and at 15°C. is exposed to a stream of dry steam at 100°C. What weight of steam will condense on the ball before the temperature of the ball is raised to 100°C. ? (sp. ht. of copper $= 0\cdot093$. Latent heat of steam $= 536$ cal.). (Dac. 1929)

[*Ans :* 0·83 gm.].

CHAPTER VI

Change of State

59. Liquefaction and Solidification.—When a solid substance changes from the solid to the liquid state, the process is known as *fusion* and when a liquid changes from the liquid to the solid state the process is called *freezing* or *solidification*

Melting point —The particular temperature at which a solid turns into the liquid is known as the *melting-point* of the solid, *which remains constant throughout the process of melting*, *i. e.* the temperature remains constant until the whole of the solid is melted although heat is being applied all the time, if the pressure on it remains constant. The temperature will rise only when the last particle of the solid has been liquefied. This fixed temperature—called the melting point—is different for different substances.

Similarly, during the process of solidification, the temperature remains constant until the whole of the liquid is solidified although heat is being withdrawn all the time The temperature will begin to fall only when the last drop of the liquid has been solidified This fixed temperature, which is different for different liquids, is called the *freezing-point*, or the *solidification temperature*, of that particular liquid. It is the same as the melting point of the substance (*i e* from the solid to the liquid state).

If the cooling process be continued very slowly, then many liquids can be cooled below their soldification temperature. This phenomenon is known as **super-cooling** or *superfusion*. This condition is not stable, for if the liquid is disturbed, or touched by a particle of the substance, solidification at once begins and the temperature rises to the solidification point.

The amount of heat given up by a substance in solidification is equal to the latent heat of fusion. In the case of water, every lb. or every gram of it must give out 80 thermal units before solidification takes place and for this reason water does not freeze at once when cooled down to 0°C. Conversely, every lb. or every gram of ice must absorb 80 units of heat at 0°C. before fusion takes place. For other substances the value of the latent heat is much smaller. So water can be called a storehouse of heat. For example, 1 cu. ft. of water weighs 62·5 lbs., which, in freezing gives up $62 \cdot 5 \times 80 = 5000$ units of heat, which, again, can raise 50 lbs. of water from the freezing point to the boiling point ($50 \times 100 = 5000$).

Viscous state.—Some substances, solid at ordinary temperatures such as iron, glass, wax, etc., have got no definite melting point. They gradually change from the solid to the liquid state passing through an intermediate plastic or viscous state. Some substances, liquid at ordinary temperatures, such as glycerine, acetic acid and also some other organic acids and oils, pass through an intermediate *viscous* state in changing from the liquid to the solid state.

Sublimation.—Some substances, such as camphor, iodine, arsenic, sulphur, etc., change directly from the solid to the gaseous state, and *vice versa.* They are called *volatile substances*, and this change of state is known as sublimation.

60. Change of Volume in Fusion and Solidification.—Most of the substances increase in volume by fusion, but few substances, such as ice, cast iron, antimony, bismuth, brass, behave like water, that is, they contract in melting and expand in solidifying. In the first case, the solid sinks in the resulting liquid while in the other, the solid floats on the corresponding liquid. A lump of cast iron floats on the liquid metal just as ice floats on water, and it is for this reason that these metals can be used for sharp castings, as on solidifying they must expand and fill up every nook and corner of the mould.

It has already been stated in Art. 116, Part I, that in freezing the volume of water increases by about 9 per cent., *i.e.* 11 c.c. of water at 0°C. becomes 12 c.c. of ice at the same temperature, and so ice floats on water with $\frac{11}{12}$ of its volume below the surface of water and $\frac{1}{12}$ above it. Thus, the volume of water formed by the melting of ice is less by $\frac{1}{12}$th of the volume of ice.

A great force is exerted by the expansion of water on freezing, which sometimes may cause great trouble as it does a good deal of damage by bursting water pipes in cold weather and by the splitting of rocks and soils, etc. On the other hand, the effect would have been still more disastrous if water contracted on freezing, as in that case ice formed would have been heavier and so would sink to the bottom of lakes or ponds, and soon the whole mass of water would be converted into a solid block of ice, and thus all aquatic animals would ultimately perish (*vide* Art. 29).

Again, ice is a poor conductor of heat and so in cold weather of the cold countries, when the surface of any lake or pond is frozen into ice, it prevents the flow of heat from the water below to the space above ice which is at a temperature lower than 0°C. So, however severe the cold may be, water cannot freeze below a certain depth. Even in regions near the North Pole, the thickness of ice formed on

the ocean reaches only about 4 or 5 metres, and this thickness changes by only a metre or two during the course of a year.

On the other hand, ice, once formed, melts only slowly by the sun's rays which must supply the latent heat required for melting If the latent heat of fusion were not necessary for the melting of ice, ice and snow would melt very rapidly, and *disastrous floods would result.*

In summer, water formed at the surface of ice being heavier sinks down and a fresh surface of ice is always exposed to the sun which helps more in melting. Thus the expansion of water on solidification. serves two purposes, it prevents accumulation of much ice in winter and also helps the melting of ice in summer.

61 Determination of the Melting-point of a Substance.—Two methods are given below for the determination of the melting point of a solid, like naphthalene, which expands on melting and contracts on solidifying.

(*a*) **Cooling curve Method.**—This method is used when an appreciable quantity of the substance is available. Put the substance in a test tube and melt it by heating in a water bath. Place a thermometer in the liquefied substance, take the tube out of the bath, dry its outside, surround it by a large vessel to protect it from air-currents, and take readings at intervals of one minute as the cooling proceeds. It will remain constant during the process of solidification after which it will fall. Take temperature readings until, sometime after, solidification is observed to be complete

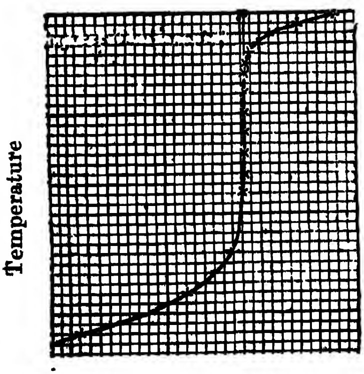


Fig. 34—Cooling Curve

Now, plotting a graph with time and temperature, a part of the curve will be seen to be parallel to the time-axis. The temperature corresponding to that part is the melting point of the substance, and Fig. 34 is the general form of the melting point for a pure single chemical substance like naphthalene. The part which is parallel to the time-axis shows no variation of temperature with time, and it corresponds to a purely liquid state, and the portion below this represents the solid state of the substance.

In the melting point curve of a substance, which is a *mixture* of different substances, such as paraffin wax, or any fat, solidification takes place over a range of temperature, and there is no definite melting point. The melting point curves for a mixture of substances have several horizontal steps corresponding to the melting points of different constituents. For substances like glass, sealing wax, etc., there is no abrupt change from the solid to the liquid state and they remain plastic over a range of temperature between the solid and the liquid state. As glass remains plastic over a wide range of temperature, so it can be worked and moulded. After taking a sharp bend, as in Fig. 34, the slope of the curve in these cases changes continuously and does not become horizontal, that is, the thermometer-readings do not remain constant for several minutes.

(*b*) **Capillary tube Method.**—This method is used when only a small amount of the substance is available. Heat a piece of glass delivery tubing in a blowpipe flame and quickly draw it out, when soft, to form a capillary tubing of about $\frac{1}{2}$ mm. diameter and with very thin walls. Take about 10 cm. of this tube *A*. Melt some napthalene in a dish and suck up about 4 cm. length of it into the capillary tube. Now seal off the lower end of the tube, and attach it by a small rubber band to the bulb of a mercury thermometer *T*, which is mounted so that the bulb and the tube dip into a beaker of water with the top of the substance just below the water surface (Fig. 35.) Now carefully heat the water stirring it all the time. After a little time, the opaque solid will change to a transparent liquid on melting; note this temperature. Now remove the burner and allow the liquid to cool stirring the water all the time, note the temperature when napthalene becomes opaque, *i.e.*, solidifies. The mean of these two temperatures gives the melting point of the substance. Repeat this experiment two or three times so as to get a very good result.

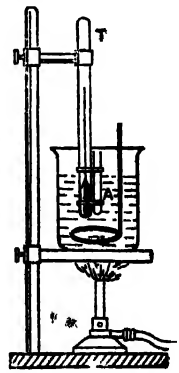


Fig. 35

Note.—Generally the temperature at which a solid melts is the same at which the liquid freezes. But for certain fats like *butter* this is not the case. For example *butter* melts at about 33°C., but it solidifies at about 20°C.

61(a). *Melting-points of Alloys.*—In the case of alloys, the melting points are usually lower than those of its constituents, and it is for

this reason that 'flux' is added to a substance with a high melting point in order to make it melt at a lower temperature.

There are other alloys like **Wood's metal**, which is an alloy of tin, lead, cadmium and bismuth, having a melting point of 60·5°C. ; and **Rose's metal**,—an alloy of tin, lead and bismuth,—having a melting point of 94·5°C. These alloys are readily fusible and so they find many applications in daily life They are used in *automatic sprinklers for buildings*, so that when a fire breaks out, a plug, made out of one of these alloys and inserted in a water pipe, melts, and thus the water rushes out from the mains. Fusible plugs are also used in closing free-proof doors automatically in the event of a fire, and *fuses* in electrical circuits are also made of these alloys.

62. Effect of Pressure on the Melting point.—The melting points of substances, such as ice, iron, etc., which *contract* on melting, are *lowered*, and the melting points of those, which *expand* on melting, are *raised* by the increase of pressure. The melting point of ice at 0°C. is lowered by about 0·0073°C for an increase of pressure of one atmosphere. Paraffin wax, which expands on melting, melts at about 54°C at a pressure of one atmosphere, and it will melt at a higher temperature if the pressure be increased.

From a simple consideration we would also expect the above facts. For, in the case of ice, any increase of pressure tends to diminish its volume and thus it helps the process of melting for which the required amount of heat energy will be less So ice will melt at a lower temperature under increased pressure. In the case of paraffin, which expands in melting, any increase of pressure will tend to diminish the volume and thus it opposes the process of melting. Hence in this case, more heat energy will be necessary to melt the substance

The depression of freezing point is used for *the determination of the molecular weight of a substance.*

Regelation—The fact, that by exerting pressure the melting point of ice can be lowered, may be shown by pressing two pieces of ice against each other and then releasing the pressure, when it will be found that the two pieces are frozen into one The pressure lowers the melting point, and so water is formed at the surface of contact. On removal of the pressure, the melting point rises ; water freezes again, and thus the two pieces are joined together, provided the temperature of the ice is not below 0°C., in which case the pressure applied by the hands will not be sufficient to reduce the melting point below the actual temperature of the ice and so the pieces of ice will not be joined together. It has been found that a pressure of about 1000 atmospheres will be necessary to melt ice when the air temperature is

$-7{\cdot}6°C$. This phenomenon of the melting of ice by pressure is known as *regelation* (L. *re*, again ; *gelare*, freeze).

This is demonstrated by the following experiments.—

(1) **Bottomley's Expt.**—A large block of ice rests at its two ends on two supports. A thin metallic wire with heavy weight on either end is placed round it. In about half an hour the wire cuts its way right through the block of ice, which is left whole. The pressure of the wire causes the ice under it to melt and the wire passes through the water formed, which being relieved of the pressure then freezes again (Fig. 36).

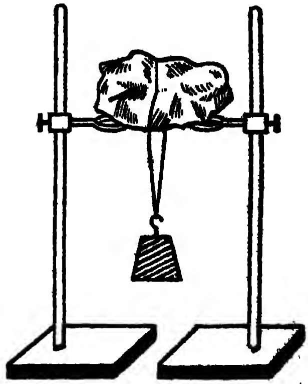


Fig. 36

It is to be noted that the ice melting beneath the wire absorbs heat on melting and the water above the wire gives out heat at the time of freezing, which is then conducted through the wire to help the ice below in melting. So the above process is helped if a metallic wire is used, for a metal is a good conductor of heat. Hence a *twine is not suitable* in this case and a copper wire will work more quickly than a steel wire.

Experiments have proved that if the block be in an ice-house where the temperature is below 0°C., the wire cannot cut through the block : the temperature of the surrounding air must be above 0°C.

(2) The lowering of the melting point of ice by increased pressure can also be shown by means of the apparatus shown in fig. 37, which is known as **Mousson's apparatus**.

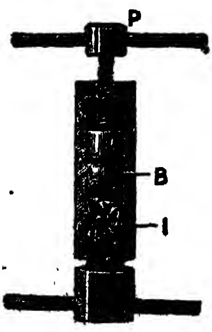


Fig. 37—Mousson's apparatus.

Expt.—The apparatus consists of an iron cylinder closed at one end with a strong screw plunger *P*. The cylinder is partly filled with water which is then frozen by keeping it inside a mixture of ice and salt. A small metal ball *B* is now placed on the top of the ice *I* in the cylinder which is then closed by the screw plunger. The whole is then surrounded by ice and the pressure is increased by driving the screw plunger in. On opening the cylinder at the bottom, the metal ball is found to come out first though the water inside is found to be still frozen. This shows that ice has melted under the increased pressure and the ball has come

down to the bottom. On relieving the pressure the water has again frozen.

Welding.—When very near the melting point, two pieces of wrought iron can be moulded together into one piece. This is called *welding*, which is an example of *regelation*. It has been found that any body, like iron or ice, which expands when cooled and contracts when heated, is cooled by pressure, instead of being heated. In the plastic condition, say at 1200°C., when iron being heated contracts, two pieces of wrought iron are brought together, and pressure is then applied due to which the temperature of the joint falls some 50°C. at the junction, and the welding is done.

63. Freezing Mixtures—Freexing mixtures are prepared by mixing generally two substances, one of which at least requires heat in passing from the solid to the liquid state. This heat is taken from the mixture and so the temperature falls. The most common freezing mixture is a mixture of ice and salt (usually 2 parts of ice to 1 of salt); every gram of ice in melting takes 80 units of heat from the mixture. If a test tube containing some water is dipped into the mixture, the water freezes. Low temperature can be produced more quickly by adding an excess of salt, but it will not effect the temperature of the freezing mixture. So, in order to freeze *ice-cream* quickly, we should add an excess of common salt.

The *freezing point of a solution is lower than that of the pure solvent*. For this reason the freezing point of the solution of common salt is lower than that of *pure* water.

Photographers' 'hypo' (sodium thio-sulphate), ammonium nitrate, etc., when dissolved in water, *lowers* the temperature to a great extent because, in dissolving, each takes the necessary amount of heat from the water.

A FEW FREEZING MIXTURES

	Parts by weight	Reduction of temperature
Powdered ice ...	2	+ 10 to −18°C
Common salt ...	1	
Crystallised calcium chloride	10	+ 10 to −50°C.
Ice ...	7	
Ammonium nitrate ...		+ 10 to −26°C.
Sodium sulphate ...		

64. Laws of Fusion.—(1) A solid under constant pressure melts at a definite temperature, called *melting-point* of the solid, and which is the same as the solidification point of the corresponding liquid.

(2) The rate at which fusion takes place is proportional to the supply of heat, but the *temperature remains constant* until the whole of the solid has melted.

(3) *Substances*, like ice, which *contract on melting*, have their *melting points lowered by increase of pressure*, while *substances*, like paraffin wax, which *expand on melting*, have their *melting points raised by increase of pressure.*

(4) Unit mass of each substance during fusion absorbs a definite amount of heat, known as *the latent heat of fusion*, which is constant for that particular subtance under the same conditions

65. Vaporisation and Condensation.—The change of a substance from the liquid to the vapour or gaseous state is known as *Vaporisation*, while the reverse process—the change from the vapour to the liquid state—is known as *Condensation* or *Liquefaction.*

The phenomena of vaporisation and condensation are almost similar to those of fusion and solidification. The following points are to be remembered about the process of vaporisation, which has got different names under the circumstances in which it takes place.—

(*a*) When the change of a substance from the liquid to the vapour state takes place slowly at all temperatures, the form of vaporisation is known as *evaporation.*

(*b*) When the change takes place more rapidly at a definite temperature the form of vaporisation is known as *ebullition* or *boiling.*

In this case also, every lb. or every gram of a substance requires a quantity of heat at its boiling point in order to be converted from the liquid to the vapour state, the quantity of heat being known as the latent heat of vaporisation of the substance. The same amount of heat will also be necessary to be abstracted from unit mass of the substance at its boiling point in order to be converted from the vapour to the liquid state. This amount of heat in the case of water at 100°C is 536 units.

66 Phenomena during Change of State.—The following phenomena are observed for a substance when it changes its state from solid to liquid or from liquid to vapour :—

(*a*) Latent heat is absorbed , (*b*) Temperature does not change, until the whole of the substance has changed state , (*c*) The volume of the substance changes.

67. Evaporation and Ebullition (or Boiling)—If a shallow dish containing water be left in a room the water will soon disappear. Similarly, volatile liquids like alcohol, ether, turpentine, petrol, etc., when

exposed to the air, will disappear much more rapidly This gradual change to the gaseous state must be taking place from the surface of the liquid and it goes on at all temperatures

So evaporation is the gradual slow change of a substance from the liquid to the gaseous state which takes place at the surface of the liquid at all temperatures

Factors governing evaporation.

(*a*) *The temperature of the liquid* The higher the temperature the faster is the formation of vapour

(*c*) *The nature of the liquid* A quantity of ether will disappear faster than the same quantity of water under the same conditions, *i e*, a liquid having a low boiling point will be evaporated quickly

(*c*) *The renewal of air over the liquid surface* The rate of evaporation increases by renewing air over the liquid surface That is why, wet linen dries up more quickly on a windy day than on a calm day

(*d*) *The pressure of the air* The less the pressure of air on the liquid the greater is the rate of evaporation So the rate of evaporation is maximum in vacuum Evaporation in vacuum is used in chemical works for preparing extracts from solutions

(*e*) *The area of the exposed surface* The greater the area of the surface of a liquid exposed to the air, the greater is the evaporation So hot liquid is taken in flat dish to get it cooled quickly

Boiling.—If a liquid is heated, the rate of evaporation increases, and if the heating be continued, a stage will be reached when rapid evaporation takes place throughout the mass of the liquid, and there is no further rise of temperature This stage is known as *ebullition* or *boiling* and the constant temperature is called the *boiling point* of the liquid which is different for different liquids So *ebullition* or *boiling* is the *rapid* change of a substance from the liquid to the gaseous state, which takes place *throughout the mass of the liquid at a definite temperature*, depending on the pressure under which the change takes place

Distinction between evaporation and boiling—The difference between evaporation and ebullion is that the former takes place at the *surface* of the liquid at *all temperatures* whereas the latter takes place *throughout the mass* of the liquid at a *particular temperature*, and evaporation is a *slow* process while ebullition is a *rapid* one

68. Cold caused by Evaporation.—*Evaporation produces cooling* When evaporation takes place from a liquid, the temperature of the

liquid falls, because the latent heat necessary for vaporisation is supplied by the liquid itself and so there is loss of heat. This is the reason of the cooling effect of the wind on the moist skin, or the wind coming through *Khas-Khas* screens in summer months. The cooling effect will be rapid if a few drops of ether or alcohol are placed on the skin instead of water. One gram of water, say, at 15°C. would require much more than 536 calories to change it into vapour at that temperature. The bulb of a thermometer wrapped with a piece of muslin will show a fall in temperature when a few drops of ether are poured over the muslin.

A porous pot keeps water cooler than a non-porous pot.—

In hot countries, water is put into earthen vessels which are porous. The water which oozes out of the pores are evaporated and thus the water inside is kept cool. Water in this case will be much cooler than the water kept in a glass or metallic vessel of the same size because, in the first case, the evaporation takes place all over the vessel, while, in the other case, it takes place only from the surface of water at the mouth.

The watering of the streets in summer not only lays down dust but produces a cooling effect by evaporation.

When drinking hot milk, or tea, it is generally poured in a shallow saucer before drinking, in order to expose a larger surface of the liquid so that evaporation may take place more rapidly. In summer, dogs are seen to hang out their tongues in order to expose a surface to air for evaporation so that they may enjoy the cooling effect caused by it.

The reason of using a fan in summer is to increase the rate of evaporation of the perspiration coming out of the pores of our skin. Generally, the vapour formed out of the perspiration remains over the skin due to which the rate of evaporation becomes slow, but, when a fan is used, the wind produced by the fan removes the layer of vapour and this renewal of the air in contact with skin increases the rate of evaporation. This causes greater absorption of heat from the skin due to which cold is produced.

Experiments.—The absorption of heat, and the consequent production of cooling, by an evaporating liquid may be shown by the following experimennts, where it will be seen that it is possible even to freeze a liquid by the loss of heat caused by its own evaporation.

(1) A few drops of water are placed on a block of wood and a thin copper calorimeter containing some ether is placed on the water. The ether is now made to evaporate rapidly by blowing air through it by foot bellows. The ether in rapidly evoporating takes heat from

the water, under the beaker, which will ultimately freeze, and thus the beaker becomes attached to the wood by a layer of ice formed between them.

(2) **Wallaston's Cryophorus.**—This illustrates the above principle. It consists of a bent glass tube having a bulb at each end containing a little water and water vapour only, but no air. All the water is transferred to bulb P and the bulb A is surrounded by a freezing mixture (Fig. 38). The vapour in A condenses; the pressure inside falls and more water evaporates from P, the water in which is gradually cooled and ultimately may be frozen into ice.

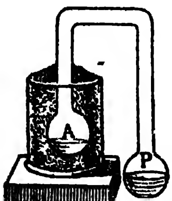


Fig. 38.—Cryophorus

(3) A shallow metal dish containing a little water and another dish containing strong sulphuric acid are placed under the receiver of an air-pump. On exhausting, the pressure inside falls, the water of the dish rapidly evaporates, and the vapour formed is absorbed by the sulphuric acid, and thus always keeping the pressure inside low. So the water continues to evaporate rapidly, whereby the temperature of the water falls and ultimately a thin layer of ice forms on the surface of the water. This is known as *Leslie's Experiment.*

69. **Refrigerators and Ice Machines.**—The cooling produced by an evaporating liquid is utilised in mechanical refrigerators and ice machines. The liquids commonly used are ammonia, carbon-dioxide, sulphur dioxide, freon etc. A refrigerator is used for industrial purposes in *cold storage* warehouses, in the house-hold (especially in hot countries), in air-conditioning plants. in hotels, and in theatres etc.

Ice Machines.—The essential parts of the machine (Fig. 39) are the following :—

(1) Coil A, called the *evaporator*, is surrounded by a strong solution of brine.

(2) Coil B, contained in a tank, called the *condenser*, is surrounded by running water.

(3) A *pump*, which can work both as an exhaust and a compression pump.

(4) A *regulating valve V.*

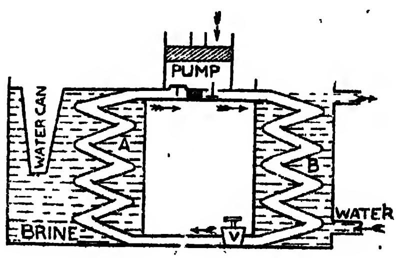


Fig. 39.—Refrigerating machine.

Action —The whole working takes place in three steps.—

(*a*) The refrigerating gas is compressed by a pump driven by an electric motor or steam engine. The compressed gas enters the coil B opening the exit valve.

(*b*) The compression produces heat (as in a bicycle pump), and the gas is cooled by the running cold water round the pipes B containing the compressed gas.

(*c*) The gas is liquefied under pressure at the temperature of the water and the liquefied gas is allowed to pass slowly through the regulating valve V to the coils in box A, where the pressure is kept low by working the pump as an exhaust pump. The liquid now evaporates very rapidly absorbing latent heat from the surrounding brine solution The cold brine, the temperature of which falls several degrees below 0°C., is then caused to circulate round the water in the can, which is to be frozen, or, to do cold-store work, through the rooms which are to be kept at a low temperature.

During the suction stroke, the admission valve between the coil A and the cylinder of the pump, which can open into the cylinder only, is lifted owing to the fall of pressure in the cylinder and the liquid rapidly evaporates, producing cold in the brine solution. The next compression stroke, by which the evaporated gas is again compressed and liquefied, starts the cycle again.

In order to cool the air of theatres and hotels of hot countries, the air is directly forced over the cooling coils in the evaporator and the brine solution is dispensed with.

70. Saturated and Unsaturated Vapour.—Whenever a liquid evaporates at any temperature it exerts a definite pressure like a gas, which is called the *vapour pressure* of the liquid at that temperature.

Take two barometer glass tubes, A and B, each about a metre long and 4 or 5 mm. in diameter. Fill each completely with dry mercury and, after closing the open end with the thumb, invert both of them in a trough V of mercury (Fig. 40). Clamp the tubes vertically side by side. The mercury in both of them will be found to stand at the same level, the height of which above the surface of mercury in the trough indicates the atmospheric pressure at the room temperature. Now, if a few drops of water or ether are introduced into the Toricellian vacuum of one of the barometer tubes by means of a bent pipette P, the liquid rises through the mercury on the top of the tube and evaporates immediately. The pressure of the vapour formed there depresses the mercury column slightly. If further small quantities are

introduced little by little, the quantity of vapour on the top of the tube will be increased and there will be further depression of the mercury column. On continuing this process, a stage will be reached when there will be no more evaporation and no more depression of the mercury column. The liquid introduced will be collected on the surface of mercury. At this stage, the space [above C in Fig. 40 (1)] is said to be **saturated** with the vapour, or is said to be full of *saturated vapour*. Before this stage, the space was *unsaturated*, or was full of *unsaturated vapour*. As there was no further depression of the mercury column after the vapour became saturated, it was evident that the *saturated vapour exerted its maximum pressure at that temperature*, and the difference BC of mercury levels in the tubes B and A (which serves as a barometer) gives the pressure of the water vapour in B [Fig. 40 (1)].

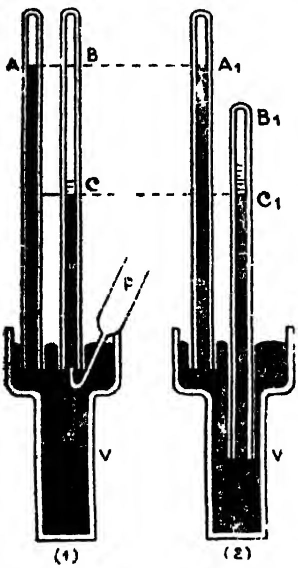


Fig. 40

Thus, *a vapour is said to be saturated when in contact with its own liquid confined in a closed space, it exerts the maximum pressure at that temperature*, and the vapour is said to **unsaturated** when it is not exerting the maximum pressure possible at the temperature.

Repeating the above experiment with different liquids, it will be found that *saturated vapours of different liquids exert different pressures at the same temperature.*

71. Unsaturated Vapour and Boyle's Law—In the above experiment the height B_1, corresponding to height B in Fig. 40 (1), of the mercury column [Fig. 40 (2)], read before introducing any liquid in the tube, will measure the atmospheric pressure. This height will diminish after introducing only two or three drops of the liquid, because the mercury head is depressed due to vapour formed. The difference between these two heights of mercury columns gives the pressure of the unsaturated vapour occupying the space over the mercury head. On now pushing down the tube into the trough [Fig. 40 (2)] until the volume of the vapour is halved, and taking the difference between the new height C_1 of the mercury column and the barometric height, it will be found that the new (pressure which is the difference $A_1 C_1$ of mercury levels) is twice the previous pressure. Similarly, making the volume of the vapour one-third, or one-fourth, of the initial volume,

the pressure will be found to be three times, or four times, the original pressure. Thus *the unsaturated vapour obeys Boyle's Law.*

72. Effect of change of Volume on Saturated Vapour.—If the tube containing the saturated vapour in the presence of its liquid be pushed into the mercury, the volume will diminish, and some vapour will be condensed, but it will be seen that the height of the mercury column remains constant, as C_1 in Fig. 40(2), proving thereby that the *pressure exerted by the saturated vapour at a given temperature is independent of the volume occupied by the vapour at that temperature.*

Again, if the tube is gradually raised the volume of the vapour will increase and more liquid will evaporate to saturate the space above mercury, but the height of the mercury column remains unchanged, that is, the pressure of the vapour remains unchanged. Thus, the pressure of saturated vapour is independent of its volume, *i.e. a saturated vapour does not obey Boyle's Law.*

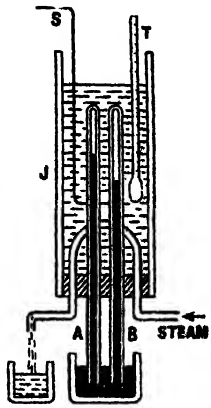


Fig. 41

72 (a). Effect of temp on Saturated Vapour.—At constant volume, if temp. is increased, more liquid evaporates and the pressure increases. The pressure is at any stage equal to the saturation vapour pressure at the higher temp. If temp. is lowered, some of the vapour is condensed and thereby the pressure falls to the saturation vapour pressure at the lower temp., but the change of pressure in either case does not obey Charle's law.

If, at the time of increasing the temperature, no more liquid is supplied, the vapour becomes *unsaturated*, and then the pressure will be inversely proportional to the volume of the vapour. Thus, *unsaturated vapour* behaves approximately as an ordinary gas, and *obeys approximately Boyle's and Charles' Laws.*

73. Measurement of Saturation Vapour pressure of water at diff. temps. (Regnault's expt.).—Regnault determined the saturation vapour pressures of water over a wide range of temperatures and has incorporated them in a chart, popularly called the **Regnault's table** of vapour pressures. He used the following arrangement of apparatus for the temperatures between 0°C. and 50°C. and the arrangement in fig. 45 for temps. between 50°C. and 230°C.

Set up the two barometer tubes A and B (fig. 41) vertically with their upper ends in a water bath J provided with a stirrer S. The temp. of the bath is regulated as desired whether below or above the

room temp., by the addition of ice or passing steam through a heater tube immersed in the bath ; the inequality in temp. being prevented by constant stirring. The liquid whose vapour pressure is to be determined is introduced into the R. H. S. barometer tube B by means of a bent pipette until a thin layer of the liquid collects on the Hg-level *i.e.* until the space is saturated at the desired temp. When the desired temp. is attained, it is determined by a sensitive thermometer T and the difference of levels in the two barometer tube is read, which gives the vapour pressure. The expt. is subject to the errors due to

(a) the pressure of a thin layer of the liquid in the experimental tube.

(b) difference in surface tension of Hg. in the two tubes, in one the Hg being in contact with water while it being in contact with its own vapour in the other.

74. **Conclusion**.—From the above experiments we establish the following facts :—

(1) When sufficient liquid is present to saturate a closed space with its vapour, the vapour exerts its *maximum pressure*, which is constant for the given liquid at that *particular temperature, and increases with the increase of temperature.*

If sufficient liquid be not present, the space is said to be unsaturated with that vapour.

(2) Rise of temperature of a saturated vapour in the presence of the liquid increases the vapour pressure, the pressure at any temperature being maximum *at that temperature.*

If no liquid is present when temperature rises, the vapour becomes unsaturated.

(3) Keeping the temperature constant, if the volume of a saturated vapour, in the presence of its own liquid, be increased, more vapour will be formed. and, if diminished, some will be condensed [Fig. 40(2)], but *the pressure will remain constant.*

If no liquid is present when volume increases, the vapour becomes unsaturated, and changes of pressure and volume take place according to Boyle's Law.

(4) Saturated vapour does not obey any of the gas laws, while unsaturated vapour behaves like a gas and obeys the gas laws.

The above phenomenon can be compared to the solution of a soluble solid, say sugar, in water. When the solution contains the maximum amount of sugar possible at that temperature, it is called a *saturated* solution like the 'solution' of the maximum amount of water vapour in air. If the sugar solution is cooled, some sugar crystallises out, so if air saturated with water vapour is cooled, part of the water vapour condenses. Again, by increasing the temperature of the sugar solution, more sugar can be dissolved, and, similarly, the warmer the air the more water vapour will it hold in suspension.

75 Pressure exerted by a Mixture of Gas and Vapour : Dalton's Laws.—If a known quantity of gas or vapour is introduced into the Torricellian vacuum of a barometer tube, it will exert some pressure which will be known by the difference between the heights of the mercury columns, as explained before (Fig. 39). If a known quantity of another gas or vapour is introduced into another similar tube, the pressure exerted by it can also be measured in the same way. If, now a similar mass of the same gas or vapour (as in the third tube) is introduced into the second tube, the total pressure exerted by both the gases or vapours will be seen to be equal to the sum of the individual pressures exerted by each of them The laws, regulating the pressure of vapours or gases, known as **Dalton's Laws of Vapour pressure** are given as follows :—

1. *The pressure exerted by a vapour, which saturates a given space, is the same for the same temperature, whether the space contains a gas or is a vacuum.*

2. *The total pressure exerted by a mixture of gases or vapours, which have no chemical action upon one another, is the sum of the individual or partial pressure exerted by each constituent, if this occupied the whole space alone at the existing temperature.*

76. Gas and Vapour.—There is no hard-and-fast line of difference between the two terms, gas and vapour. But generally, the term **gas** is used to describe the state of a substance when it is *above its* **Critical temperature** (—*i.e. the temperature below which the substance can be liquefied by pressure, and above which it cannot be liquefied under any pressure whatsoever*), while the term **vapour** describes the gaseous state of a substance *below its critical temperature.*

The pressure which will liquefy a gas at its critical temperature is called its *critical pressure.*

The critical temperature of carbon dioxide is 31°C., and its critical pressure is 73 atmospheres. Thus above 31°C. carbon dioxide is a true gas, while below this temperature it is a vapour.

Examples.—(*1*) *A quantity of oxygen is collected over water in a graduated tube. The height of the column of water left in the tube is 68 mm., and its temperature is 30°C; assuming the space occupied by oxygen to be saturated with aqueous vapour, find the pressure of the oxygen, the maximum pressure of vapour at 40°C. being 31·55 mm., and the barometric height 758 mm.*

The pressure of 68 mm. of water is equivalent to $\frac{68}{13·6}=5$ mm. of mercury.

The atmospheric pressure balances the pressure of oxygen, the pressure of the vapour, and the column of water in the tube; hence, if x mm. be the pressure of oxygen, we have $x+5+31·55=758$; $x=721·55$ mm.

(*2*) *A quantity of dry air at 25°C. occupies a length of 156 mm. in a tube over mercury, the mercury standing 612 mm. higher inside the tube than outside. A small quantity of water is then passed up into the tube and the mercury column falls to 599·4 mm. Find the pressure of aqueous vapour at 25°C., the laboratory barometer standing at 759 mm.*

The original pressure of dry air $=759-612=147$ mm. When the air is saturated with the vapour, it occupies a length of $156+(612-599·4)=168·6$ mm.

$\therefore$ The final pressure of dry air alone $=\frac{147\times156}{168·6}=136$ mm. ... (Boyle's law)

Hence, if x mm. be the pressure of the vapour, we have

$$x+136+599·4=759\ ;\quad \therefore\quad x=23·6 \text{ mm.}$$

(*3*) *1000 c.c. of a gas are collected over water at 20°C and 760 mm pressure, the space being saturated with aqueous vapour. Find the volume of dry gas at N.T.P., the maximum vapour pressure at 20°C. being 17·4 mm.* (*All. 1920*)

760 mm. is the combined pressure of dry gas and aqueous vapour at 20°C.

$\therefore$ According to Dalton's laws, pressure of dry gas alone at 20°C. $=760-17·4=742·6$ mm. Volume of dry gas $=1000$ c.c.

Hence, we have, if V be the volume of dry gas at N. T. P.,

$$\frac{742·6\times1000}{273+20}=\frac{760\times V}{273}\quad\text{or}\quad V=910·4 \text{ c.c.}$$

(*4*) *A mass of air is saturated with water vapour at a temperature of 100°C.; on raising the temperature to 200°C. without change of volume, the pressure is raised to two atmospheres. Find the pressure at 0°C. of this volume of the dry air alone.* (*Pat. 1926*)

Let P = pressure of the dry air at 100°C.

∴ The total pressure of the moist air at 100°C = $(P+760)$ mm. (∵ The pressure of water vapour at 100°C. = 760 mm.)

Then from the formula, $\frac{P}{T}=\frac{P'}{T'}$, where T and T' are absolute temperatures corresponding to pressures P and P' we have

$$\frac{P+760}{373}=\frac{2\times760}{273+200} \quad or \quad P=438{\cdot}64 \text{ mm.}$$

Again, if P_o be the pressure at 0°C., we have

$$\frac{P_o}{273}=\frac{438{\cdot}64}{273+100}; \quad \therefore \quad P_o=321{\cdot}04 \text{ mm.}$$

77. **Boiling.**—When vapour is given off by a liquid rapidly and violently throughout the whole mass of the liquid, the liquid is said to boil, the temperature remaining constant under constant pressure throughout the process of boiling. This constant temperature is known as the *boiling point* of the liquid.

When water is heated in a glass vessel. bubbles will appear inside the vessel which will rise to the surface with increase of temperature. These are air-bubbles dissolved in water. After a time, bubbles of steam formed at the bottom, while rising above towards the colder layers, collapse due to condensation. This produces a peculiar 'singing' sound. On further rise of temperature, the steam bubbles rise vigorously to the surface and boiling begins.

If *pure* water, which has been previously boiled to drive away dissolved air, be heated in a clean vessel, bubbles will not be formed for some time and the temperature will rise above the boiling point (*vide* super-cooling in Art. 59). Then suddenly large bubbles will be formed which will burst forth with explosive violence and there is a tendency for the whole liquid to be thrown out. The temperature of the liquid now comes down to its normal boiling point. This phenomenon is called **boiling by Bumping**.

Bumping may be prevented by introducing some rough material, say a few fragments of glass or porcelain, into the liquid, as the presence of air in the crevices will facilitate boiling.

Condition for Boiling.—*A liquid boils at a temperature at which the pressure of its vapour is equal to the pressure to which the surface of the liquid is exposed.*

Experiments.—(1) A barometer tube is filled with mercury and inverted over a trough of mercury (fig. 42). The tube is completely surrounded with a jacket through which steam can be passed. Introduce some water into the tube by means of a bent pipette, and gradually pass steam into the jacket. As the temperature rises, more and more water vapour is formed at the top of the mercury column, which depresses the mercury column until, if there be sufficient liquid present, the mercury inside the tube is at the same level as that in the trough. This means that the pressure of the water vapour at the temperature of the steam, *i.e.*, the boiling point, is the same as the outside pressure, which is the atmospheric pressure, or in other words, *water (or any other liquid) boils at a temperature when its vapour pressure is equal to the pressure on the surface of the liquid.*

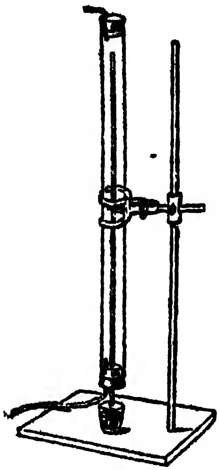


Fig. 42

Consulting the table of the pressure of water vapour it will be seen that the maximum pressure of water vapour at 100°C. is 760 mm., so water boils at 100°C. when the external pressure is 760 mm., similarly water boils at 90°C. when the external pressure is 525·5 mm

Thus the **boiling point** of a liquid can be defined as *the temperature at which the maximum vapour pressure of the liquid is equal to the external pressure to which the surface of the liquid is exposed.*

(2) Take a bent tube *AB* closed at *B* as shown in Fig. 43. The small arm contains some well-boiled water below which is mercury *M* which also rises in the longer arm. The level of mercury in the longer arm is below that in the other Now introduce the tube into a flask containing some water such that the tube is above the surface of water in the flask. Boil the water and allow the steam, which surrounds the lower part of the tube, to escape through an exit tube. In a short time it will be found that the mercury assumes the same level in the two arms, showing that the maximum vapour pressure at the boiling point is equal to the atmospheric pressure.

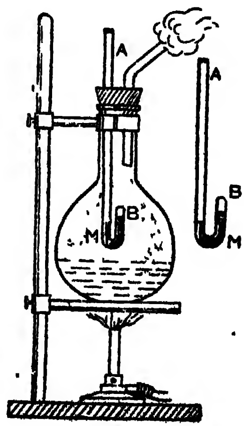


Fig. 43.

78. Boiling Point depends on Pressure—From the experiments already described it follows that *the boiling point of a liquid will change if the pressure to which the surface of the liquid is exposed changes* Thus water will boil at a temperature higher than 100 C if the atmospheric pressure is higher than 760 mm, and similarly, it will boil at a lower temperature if the pressure is lowered So, on the top of a high mountain, water will boil at a temperature lower than 100°C

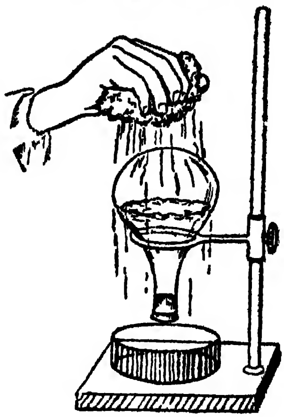


Fig 44

(1) **Boiling under Reduced Pressure**—This is demonstrated by the following experiments—

Franklin's Expt—(*a*) Boil some water in a strong glass flask until all the air is expelled Now remove the burner and invert it after it is tightly corked (Fig 44) The space above the surface of water contains saturated water vapour When boiling ceases, pour some cold water on the flask This condenses some vapour inside the flask and thus reduces the pressure over the surface of water but water boils again This shows that boiling is possible at a temp below 100°C by reducing the pressure on the liquid

(*b*) The same result can be produced by placing a beaker containing some boiling water in the receiver of an air pump On pumping out some air, as soon as boiling ceases, the water will again be found to be boiling

(2) **Variation of Boiling Point with Pressure** A liquid can be boiled at different temperatures by changing the pressure of air above the surface of the liquid The arrangement is shown in Fig 45 The liquid is placed in a boiler A which is connected with a large air reservoir *B* through a Liebig's condenser *C* The reservoir *B* is connected with mercury manometer *M* and an air pump The liquid is heated until it boils under a given pressure and the boiling point is read by means of a sensitive thermometer *t*, the bulb of which is placed in the vapour and not in the liquid The reason for this is that liquids sometimes may boil irregularly when the temperature rises several degrees above

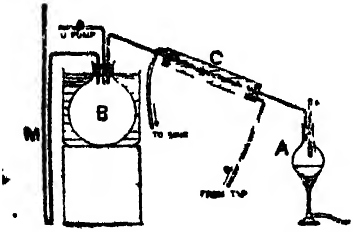


Fig 45

the true boiling point. The condenser condenses the vapour and restores it back to the boiler *A*. The reservoir *B* containing air is surrounded by water to keep its temperature constant. The pressure in *B* is adjusted to a definite value by connecting it with a compression or exhaustion pump as is required for increasing or reducing the pressure. Take the reading of the thermometer when it becomes stationary after boiling commences, and record the manometer reading at the same time. When the liquid boils, the pressure of its vapour is equal to the superincumbent pressure which is indicated by the manometer M. By altering the pressure to a new value, the boiling temperature is altered and the saturation vapour pressure of the liquid for that temperature is obtained. By this means Regnault was able to determine saturation pressure of water vapour up to 230°C,, the pressure at that temperature being 27½ atmospheres, and he used this method for determining vapour pressure of water above 50°C.

On the top of a mountain the pressure is less than that at sea-level, so the boiling point of water there is less than 100°C. For example, water boils at 93·6°C. at Darjeeling, which is about 7200 ft. above the sea-level ; and at Quito (in S. America), the highest city in the world (9520 ft above the sea-level), the normal height of the barometer is 52·5 cm. and water boils at 90°C At the top of Mont Blanc (15, 781 ft.) water boils at 85°C.

It has been found that the boiling point of water decreases by 1°C. for every 960 ft. increase in elevation above sea-level, or, in other words, *for a reduction of pressure of 26·8 mm. the boiling point falls by 1°C.*

Papin's Digester.—The cooking power of boiling water depends upon the temperature at which it boils, hence on the top of a very high mountain it is impossible to cook food in an open vessel. But, by increasing the pressure, water can be made to boil at any higher temperature. So, for cooking food on the top of a very high mountain, a special closed vessel, provided with a safety valve, is used, the pressure within which can be raised to about 760 mm This special contrivance is named *Papin's Digester.* Ordinarily by closing a saucepan with a cover the difficulty of cooking, tea-making, etc., can be solved to some extent.

Boiling under increased pressure is useful for the manufacture of artificial silk ; for the preparation of pulp used in paper-making by boiling wood with caustic soda ; etc.

Boiling under diminished pressure also has its uses. For instance, in the preparation of condensed milk, much of the water of the milk is

driven off at a low temperature in order to keep the food value of the milk unaltered. Sugar is also refined by a similar process

79. Boiling Points of Solutions—What has been said so far regarding boiling points is confined to pure liquids only, such as water, ether, etc The law, namely, *a liquid boils at a temperature at which its vapour pressure is equal to the pressure on its surface*, is also obeyed by the boiling points of solutions but the vapour pressure of a solution at a particular temperature is always less than that of a pure solvent at the same temperature, so the temperature of the solution has got to be raised above the boiling point of the solvent before it will boil So (a) *the boiling point of a solution is always higher than that of the pure solvent, and* (b) *the amount by which the boiling point is increased is proportional to the concentration of the solution*

So besides the effect of pressure, *the boiling point of a liquid is also effected by the presence of substances dissolved in it* For example, the boiling point of sea water is about 104°C, while that of pure water is 100°C and it has already been said that the increase of the boiling point depends upon the weight of the substance dissolved So, *the purity of a liquid can be tested by its boiling point*

80 Laws of Ebullition—(1) *Every liquid has got a definite boiling point at a particular pressure, by increasing or decreasing the pressure the boiling point is raised or lowered*

(2) *A liquid boils when the maximum pressure of its vapour is equal to the atmospheric pressure*

(3) *The temperature at which a liquid boils remains stationary until the whole of the liquid is evaporated*

(4) *The temperature during boiling is constant so long as the pressure is constant A definite quantity of heat, known as the latent heat of vaporisation, is absorbed by one gram of the liquid in changing from the liquid to the vapour state at the same temperature*

81 Ebullition and Fusion Compared.—

(*a*) The temperature remains stationary throughout each process, when the corresponding latent heat is absorbed

(*b*) As there is super-cooling of a liquid under some conditions, so there may be super-heating that is, the liquid may be heated above its boiling point without boiling

(*c*) Both freezing and boiling points of a liquid are changed with pressure, though in the first case it is very small

(*d*) For both the processes there is generally an increase in

(e) In the case of a solution, the freezing point of a solution is *lower*, but the boiling point is *higher*, than that of the pure solvent.

82 Change of Volume of Water with Change of State — When water is changed from the solid to the liquid state its volume decreases upto 4°C, after which it gradually increases upto 100°C., and, when it is changed into steam at 100°C. and at atmospheric pressure, its volume is increased more than 1670 times, that is, a cubic inch of water produces about a cubic foot of steam. The curve (Fig. 46) shows diagrammatically (not according to scale) the changes in volume

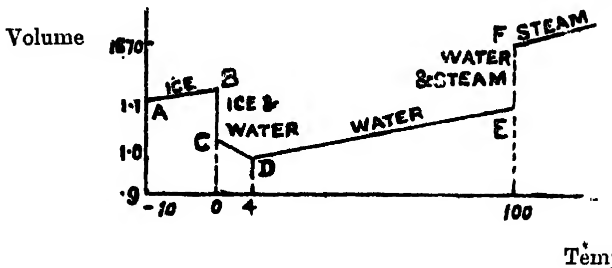


Fig. 46

when 1 gram of ice at -10°C. is heated to steam. The portion AB of the curve represents the expansion of ice as its temperature increases from -10 to 0°C. The portion BC represents the state of melting of ice when the volume diminishes, the temperature remaining constant at 0°C. The portion CD shows the diminution in volume of water as its temperature rises from 0°C. to 4 C., when it attains the minimum volume, after which the volume of water increases as its temperature rises from 4°C. to 100°C. at which the water begins to boil. This is represented by the portion DE. The portion EF shows the state when water boils and changes into steam, the temperature remaining constant at 100°C., but the volume of the steam formed is enormously increased being about 1670 times the volume of water taken. The portion beyond F shows the increase in volume of the steam with the rise of temperature.

The large expansion, when water is turned into steam, *accounts in some degree for the large volume* (536 cals. in the case of water) *of the latent heat of vaporisation.* The required latent heat alters the condition of the molecules in changing the liquid state to the vapour state and also has to do external work in displacing the atmosphere. The work required for this is great as the expansion is great, and so the amount of heat is equivalent to this work.

83. Determination of Height by Boiling Point : *(Hypsometry).*—The height of a place can be determined by knowing the atmospheric pressure at the top and at the bottom of the place. The pressure of the vapour of a liquid at its boiling point is equal to the superincumbent pressure. So, by determining the boiling point by means of a hypsometer at the top and bottom of any high place, the atmospheric pressure at both these places can be known by referring to Regnault's table of maximum pressure of water vapour. The process of determining height by this method is called *hyspometry.* Ths meaning of the word "hypsometer" is *height-measurer.* Roughly it has been estimated that there is a difference of 1 inch in the barometric pressure for every 900 ft. change in altitude. The height can also be calculated in the following way :—

The difference of pressures will be equal to the weight of a column of air of height equal to the height of the place, and of area equal to 1 sq. cm. The pressure and temperature of the air may be taken to be the mean of the two pressures and temperatures at the top and the bottom without much error.

Let P_1 = atmospheric pressure at the top corresponding to the boiling point observed there.

P_2 = atmospheric pressure at the bottom ; H = unknown height.

$t_1°$ and $t_2°$ = temperatures of the air at the top and at the bottom respectively.

Mean temperature $t = \dfrac{t_1 + t_2}{2}$, Mean pressure $P = \dfrac{P_1 + P_2}{2}$.

If V_0 be the volume at N. T. P. corresponding to H c.c. of air at pressure P and temperature t, we have

$$\frac{PH}{273+t} = \frac{76 \times V_0}{273} \quad \therefore \quad V = \frac{PH}{273+t} \times \frac{273}{76}.$$

The weight of this air $= \dfrac{PH}{273+t} \times \dfrac{273}{76} \times 0{\cdot}001293 \times 981$,

since the density of air at N. T, P. = 0·001293 gm. per c.c.

Again, the weight of the air column $= (P_2 - P_1) \times 13{\cdot}6 \times 981$,

since 1 c.c. of mercury weighs 13·6 gms.

$$\therefore \frac{PH}{273+t} \times \frac{273}{76} \times 0{\cdot}001293 \times 981 = (P_2 - P_1) \times 13{\cdot}6 \times 981 ;$$

Hence $H = \dfrac{(P_2 - P_1) \times 13{\cdot}6 \times (273+t) \times 76}{P \times 273 \times 0{\cdot}001293}$ cm.

The result obtained is, however, approximately correct as the assumptions are only approximately true.

Questions.

Art. 60.

1. Why would the height of water in a vessel containing ice cold water and a lump of ice floating on it be unaffected when the ice melts ? (C U 1938)

2. A piece of ice is placed in a beaker Water is then poured into the beaker until it is on the point of overflowing Will the water overflow when the ice melts ? What differences will be observed in the experiment, (1) if hot water be used, (2) if water at 4°C be used ? (Pat 1922)

[**Hints.**—(1) In this case the volume of ice diminishes as it melts as well as the volume of hot water diminishes as its temperature falls, so water will not over flow (2) In this case, when the piece of ice melts the water formed out of it fills up the space previously occupied by the portion of ice under water, but at this reduces the temperature of water in the beaker, which has got the least volume at 4°C, the volume of water will increase a little and the water will overflow]

Art. 62.

3. Explain the phenomenon of regelation and describe experiments to illustrate it (C U 1940)

It is found that a copper wire with a heavy weight at each end will cut its way through a block of ice, but a piece of twine will not Explain these results.

4 What is the effect of pressure upon the melting point ? In what way, if at all, does it differ in the case of paraffin and of ice ? Explain the phenomenon of regelation (Pat 1919, C U 40)

Art. 67.

5. Explain the meaning of evaporation and ebullition Describe suitable experiments to illustrate their meaning (C U 1914)

Art. 68.

6. What is the cause of the cooling effect produced in a room when grass (*Khas Khas*) screen moistened with water is placed in front of the door ? (C U 1911)

7. A glass bottle and a jug of porous earthenware are both filled with water and exposed to air side by side What difference do we notice between the temperature of the water in the two vessels after a few hours ? Explain why this happens If there is very little or no difference of temperature, what conclusion may we draw as to the state of the atmosphere and why ? (C. U. 1915 ; Cf. Pat. 1928)

[**Hints.**—In the glass vessel evaporation takes place only over the small surface exposed to the outside air through the neck of the bottle, while in the other case evaporation takes place through the pores of the whole vessel, hence there is greater fall of temperature

There is no evaporation as the atmosphere is saturated with water vapour. Hence there is no difference of temperature]

Art. 69

8 Explain the construction and action of some kind of practical freezing machine that does not require the freezing mixture (Pat 1931)

Art 70

9 Distinguish carefully between saturated and unsaturated vapours (C U 1929 41 45, Cf Pat 1921, 41)

10 How would you find out whether a space is saturated or not ? (C U 1929, 32, Pat 1931, Dac 1931)

11 What is meant by maximum vapour pressure of water vapour? Describe an experiment to determine it from the laboratory temperature up to 100°C (See also Art 72) (C U 1921 Cf 16 17, 24 34 Cf Dac 1931)

Art 71

12 Two barometers stand side by side A few drops of water are introduced into the vacuum of one and a little air into the other What would be the effects on the errors of the barometer readings thus produced for (*a*) a change in the atmospheric pressure (*b*) a change in the temperature (C U 1909)

Arts 71 & 72

13 What is the saturated vapour pressure ? Under what conditions is a vapour able to exert such pressure ? What happens when unsaturated vapour is compressed till further compression is impossible ?

If water boils at 99°C, when the pressure is 733 mm, what is the saturated pressure at 101°C ? Explain briefly (Pat 1929)

Ans [760 + (760 − 733) = 787 mm]

14 Distinguish between saturated and unsaturated vapour and discuss their behaviour as regards changes contemplated by Boyle's and Charles laws. (Pat 1931, '42)

Art 73

15 Explain how the maximum tension of aqueous vapour is determined at temperatures below and above the normal boiling point (C U 1932)

[For determining the maximum tension of aqueous vapour from 0°C to 50°C, see Art 73 (Fig 41), and from 50°C upwards see Art 78, Fig 45]

16. Water is sprinkled in a room containing a barometer State how will the barometer be affected under the following conditions —

(*a*) The doors and windows are closed and the room is gradually heated.

(*b*) The room is heated but with doors and windows open. (Pat. 19[illegible])

Art. 74.

17. Into a cylinder exhausted of air and provided with a piston, there is introduced just enough water to saturate the space at 20°C. Describe what happens under the following conditions :—

(*a*) The volume of the space is increased by pulling up the piston.

(*b*) The volume is diminished by pushing the piston down.

(*c*) The volume remaining as at first, the temperature is increased to 50°C.

(*d*) The temperature falls to 10°C. (C. U. 1910, '23, '24)

Art. 75.

18. 50 c.c. of a gas are collected in an inverted tube over water. The height of the barometer is 77 cm., the temperature of the room is 17°C. and the water level inside the tube is 7·6 cm. above that outside. What is the volume of the dry gas at 0°C. and at 76 cm. pressure. The maximum pressure of aqueous vapour at 17°C. is 14·4 mm.

[*Ans :* 46·5 c.c.]

19. Enunciate Dalton's laws of partial pressure. (All. 1920 ; Pat. 1926, '40)

20. A mass of air is saturated with water vapour at 100°C. On raising the temperature to 200°C. without change in volume, the mixture exerts a pressure of 2 atmospheres. What was the pressure due to air alone in the initial condition ?

[*Ans :* 438·6 mm.] (Pat. 1938)

Art. 76

21. Distinguish carefully between a gas and a vapour.
(Pat. 1926, '44 ; C. U. 1927)

Art. 77.

22. Describe an experiment to show that the vapour pressure of a liquid exposed to air at its boiling point is equal to the atmospheric pressure.
(C. U. 1915 ; Pat. 1915, '24, '26, '31)

23. Distinguish between boiling and evaporation. What conditions determine whether a liquid will boil or evaporate.

(See Art. 67) (C. U. 1914, '25, '41 ; Pat. 1928, '41, '44 ; Cf. Dac. '31)

[**Hints.**—A liquid evaporates as long as the vapour pressure at the temperature of the liquid is less than the atmospheric pressure, and it boils when these two pressures are equal.]

24. Explain how a knowledge of the boiling point of water would enable you to determine the barometric pressure.

Into the Toricellian vacuum of a barometer, water is introduced drop by drop till some water is left over. From the depression of the mercury column it is possible to determine the temperature of the room. How ? (C.U. 1918, '20)

Hints.—A liquid boils when its vapour pressure is equal to the superincumbent pressure. Knowing the boiling point we can find out the vapour

pressure from Regnault's table which will be the same as the barometric pressure.

From the depression of the mercury column the maximum vapour pressure at room temperature is known. Now, by consulting Regnault's table, the temperature corresponding to this vapour pressure is known, which is the same as the temperature of the room].

Art. 78.

25. Why does it take a longer time to cook food on the top of high mountains ? At Darjeeling the barometric height is found to be almost 23″ only. At what temperature will you expect water to boil there ? (Pat. 1919)

[**Hints.**—There is a change of 0·04°C. in the boiling-point for a change of 1 mm. (or 0·04 inch) in pressure].

[*Ans :* 93°].

26. Define boiling-point of a liquid. Describe suitable experiments to show that water can be made to boil at temperatures greater or less than 100°C.. (See Art. 77.) (C. U. 1930, '41 ; Dac. 1932)

Art. 82.

27. Heat is continuously applied to a mass of ice at −10°C. until it becomes steam at 100°C. If the temperature is taken at intervals of time and a graph is plotted of the temperature against the time, what would be the shape of the curve obtained ? Give reasons for this. (Pat. 1935 ; C.U. '22)

Art. 83.

28. Explain how you are able to determine (approximately) the height of a mountain by finding the boiling-points of water at its top and bottom. (C. U. 1914, '24, '25, '28, '45 ; Cf. Pat. 1928, '31, '48)

CHAPTER VIII

Hygrometry

84. Hygrometry.—Aqueous vapour, more or less, is always present in the atmosphere as evaporation is constantly going on from the surfaces of water such as seas, rivers, lakes, etc. *Hygrometry* is that part of Physics which deals with the measurement of the amount of aqueous vapour present in a given volume of air. The formation of cloud, dew, mist, fog, etc., proves that water vapour is present in the atmosphere.

On a warm, damp day the outside of a tumbler of cold water soon becomes covered with dew due to the condensation of water vapour from the air.

It has also been observed that on a cold night water vapour condenses on the inside of the glass panes of a sitting room window. The room receives much water vapour from the breathing of the persons in it, but this vapour cannot saturate the warm air of the room. The glass of the window being thin is cooled to a lower temperature by the cold air outside. The air in contact with the glass is cooled to a lower temperature when it becomes saturated with water vapour which is condensed on the glass.

85. Dew-point and Relative Humidity.—Ordinarily the quantity of water vapour present in the atmosphere is not sufficient to produce saturation, and so the pressure exerted by the vapour is less than the saturation pressure at the existing temperature ; but the same quantity of vapour may be sufficient to produce saturation at a lower temperature. So, if the air be gradually and progressively cooled down, the pressure of the aqueous vapour remains constant, but a temperature is soon reached at which the quantity of vapour actually present is sufficient to saturate the air, and so the pressure of this aqueous vapour now becomes the saturation pressure for this lower temperature. This temperature is called the *dew-point*, for below this if the temperature is lowered, some of the vapour is condensed as *dew*. So, the *Dew-point may be defined as a temperature at which the amount of water vapour actually present in the air saturates it.*

Hence the (saturation) vapour pressure of water at the dew-point is a measure of the amount of water vapour present in the air ; which again is proportional to the pressure exerted by the water vapour in the air under the initial conditions. Therefore, if we know the dew-point we can know the pressure exerted by the actual amount of water vapour in the air.

Hence *the* **dew-point** *may also be defined as the temperature for which the saturation vapour pressure corresponds to the pressure of water vapour actually present in a given volume of air under given conditions*

Relative Humidity.—For meteorological work, the *degree of saturation* of the atmosphere is more important than the actual amount of water vapour in the air. This is known as the *Relative Humidity* or the *Hygrometric State of the Air*. This may be defined as—

$$\frac{\text{The mass of water vapour actually present in any volume of air at } t^\circ\text{C.}}{\text{The mass of water vapour necessary to saturate the same vol. at } t^\circ\text{C.}} \quad \ldots(1)$$

But so long as the water vapour is unsaturated and so obeys

Boyle's Law, the mass of water vapour existing in a given volume of air is proportional to the pressure exerted by the vapour, which, again, is equal to the saturation vapour pressure at the dew-point. That the mass of water in a given volume of air is nearly proportional to the pressure it exerts will be clear by consulting the table given on p. 289, where m represents the mass of water vapour necessary to saturate 1 cubic metre of air at the temperatures shown, and p, the saturation pressure of water vapour at that temperature.

Hence, *Relative Humidity*

$$= \frac{\text{Pressure of water vapour actually present in the air at } t^{\circ}\text{C.}}{\text{Pressure of water vapour necessary to saturate the air at } t^{\circ}\text{C.}} \quad \ldots \quad (2)$$

$$= \frac{\text{Saturation vapour pressure at the dew-point}}{\text{Saturation vapour pressure at the temperature } (t^{\circ}\text{C.}) \text{ of the air}} \quad \ldots \quad (3)$$

Relative Humidity is generally expressed as the percentage saturation of the air, and it is calculated by applying either (1) or (3) above as.—

Relative Humidity

$$= \frac{\text{Mass of water vapour actually present in any vol. of air at } t^{\circ}\text{C.} \times 100}{\text{Mass of water vapour necessary to saturate the same volume at } t^{\circ}\text{C.}} \text{ per cent.}$$

$$= \frac{\text{Saturation vapour pressure at dew-point} \times 100}{\text{Saturation vapour pressure at air temperature } (t^{\circ}\text{C.})} \text{ per cent.}$$

Absolute Humidity is defined as the mass of water vapour actually present in a given volume of air. This is generally expressed as the mass in grams per cubic metre of air.

Example.—*On a certain day the dew-point was found to be 12°C., when the temperature of the air was 16°C. Calculate the relative humidity of the air.*

By consulting the table of vapour pressure it will be seen that

aqueous vapour pressure at 12°C. = 10·51 mm., and at 16°C. = 13·62 mm.

$$\therefore \quad \text{Relative humidity} = \frac{10{\cdot}51}{13{\cdot}62} = 0{\cdot}77 \text{ or } 77 \text{ per cent.}$$

86. Dryness and Dampness.—Our *sensations of dryness* or *dampness* do not depend only *on the actual quantity of water vapour*, but on the *quantity of vapour necessary to saturate* the air at that temperature, and it is on the *ratio of the above two quantities*, *i.e.* *on the relative humidity*, that our sensations of dryness or dampness chiefly depend. It is found that on a cold misty day in winter, when the air seems to

be quite 'damp', the actual amount of water vapour in a given volume of air is frequently less than that on a hot day in summer, when we say that the air is 'dry' The dampness or dryness of the air is *judged by the rate* at which evaporation goes on, and which depends upon *how far the air is from the saturation state, i e* how much more vapour it can take up, and does not depend chiefly upon how much water vapour the air already contains

Such things as wet clothes will be dried more quickly when the relative humidity of the atmosphere is low, because in such cases the atmosphere can readily take up more water vapour Again, the evaporation of moisture from such things as wet clothes will be more rapid if the air in contact with them is constantly being renewed

The ventilation of buildings is necessary for two reasons to remove the carbon dioxide exhaled by us and also to remove the water vapour evaporated from our lungs and bodies

Our bodies are constantly emitting water vapour, which is very important from the standpoint of health We know how difficult it is to work in a stuffy room This is because the air in the room contains a lot of water vapour, that is, the air is nearly saturated with moisture due to which normal evaporation from our skin cannot go on, and this produces a feeling of uneasiness

This is particularly the case when the temperature of the atmosphere is high, as the feeling of easiness depends upon evaporation from the body so that its temperature may not rise above the normal value Hence the weather in India near about Bengal during the wet season is more oppressive than that in other parts where the temperatures are 10° to 20°F higher and the atmosphere is drier

If the relative humidity of air is about 100 per cent, we perspire and the weather feels sultry and oppressive

Relative humidity is determined regularly at meteorological stations, because it affords information as to the likelihood of rain We can expect rain when air contains a considerable amount of water vapour. This *damp air is lighter than dry air*, because water vapour is lighter than air *The density of water vapour relative to dry air is 5/8.*

The record of the relative humidity is useful to the Public Health Department as certain diseases thrive in damp atmosphere It is also important for certain industries, for example, cotton weaving and

spinning can be conducted satisfactorily only when the air is comparatively damp. For this reason the damp climate of Lancashire has been found suitable for the development of cotton industry.

87. Hygrometers.—Hygrometers (Gk. *hygros*, wet + *metron*, a measure) are instruments used for the determination of the hygrometric state of air, at any place and time.

The instruments by which this is done can be divided into three classes :—

(1) *Dew-point Hygrometers* :— (a) *Daniell's Hygrometer* : (b) *Regnault's Hygrometer.*

(2) *Wet and Dry Bulb Hygrometer,*

(3) *Chemical Hygrometer.*

88. (1) Dew-point Hygrometers :—

(*a*) **Daniell's Hygrometer.**—It (Fig 47) consists of two bulbs *A* and *B* bent downwards connected by means of a wide tube. One of the bulbs *A* contains ether, and the other bulb *B* with the tube connected to it is full of ether vapour, the air having been expelled before the apparatus was sealed up. There is a delicate thermometer *t* inside the bulb *A* containing ether. The bulb is silvered, or gilt within, while the other is covered with muslin. Another thermometer *T* attached to the stem *C* indicates the temperature of the air.

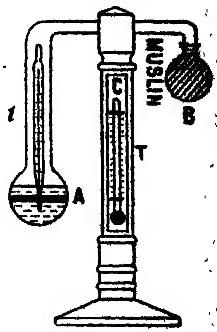


Fig. 47—Daniell's Hygrometer.

To determine the dew-point, some ether is poured on the muslin which on evaporating cools the bulb and condenses a portion of the ether vapour inside. The pressure inside being thus reduced, ether from the other bulb *A* evaporates, and so it becomes colder. The temperature is reduced until dew-point is reached. The temperature of the thermometer inside the bulb is noted as soon as the first film of dew appears on the silvered surface. The cooling process is discontinued by allowing the muslin to dry up and again the temperature is noted when the film just disappears. The mean of these two temperatures is the dew-point.

Sources of Error.—This form of hygrometer is rather defective for the following reasons:—(*i*) Ether evaporating outside *B* contaminates the air and this affects the hygrometric state of the air. (*ii*) It is rather difficult to observe the exact moment of appearance or disappearance of dew as there is no comparison standard (see Regnault's hygrometer). (*iii*) Inside the bulb *A*, ether evaporates mostly at the

surface of the liquid, which is thus cooled more rapidly than the interior and thus the actual dew-point is not observed. (*iv*) Because glass is a bad conductor, temperature outside A is not the same as that inside.

Precaution :—With any hygrometer, observation ought to be taken either (*a*) by a telescope, or (*b*) by placing a piece of glass between the observer and the apparatus, so that the result may not be affected by the heat from the body or breath.

(*b*) **Regnault's Hygrometer.**—This is a better form of hygrometer. This consists of a test tube having a side tube (Fig. 48), the lower part E of the test tube being made of silver. The mouth of the tube is closed by a cork through which passes a delicate thermometer T. A glass tube A also passes down through the cork nearly to the bottom of the tube.

To work the instrument, some ether is placed in the test tube. The side tube is connected to a vertical brass tube, which again is connected to the rubber tube C with an aspirator. The vertical brass tube is supported in a clamp. A second glass tube similar to the first, fitted with a thermometer t inside it, is also attached to the same support. By opening the aspirator, which is full of water in the beginning, a current of air is drawn through the tube E, which causes rapid evaporation and sufficient fall of temperature to condense water vapour on the outside of the silver tube E. The temperature is noted and the aspirator is shut off when dew is first observed. The temperature is again noted as soon as the dew disappears. The *mean* of these two temperatures is the *dew-point*. The aspirator must not be placed too close to the hygrometer, for the water released from the aspirator may then alter the humidity of the space around. The other tube is not an essential part of the instrument, and serves only as a standard of comparison of brightness of the two silver surfaces E and F. The thermometer t inside the other tube gives the temperature of the air. The relative humidity is then given by,

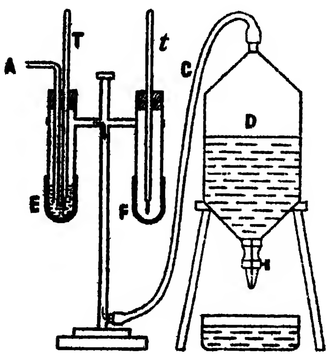


Fig. 48.—Regnault's Hygrometer.

$$\text{Rel. Humidity} = \frac{\text{Saturation vapour press. at the dew-point}}{\text{Saturation vap. press. at the tem. } (t^\circ C) \text{ of the air}}$$

Advantages of Regnault's Hygrometer.—

(*i*) By regulating the flow of water of the aspirator the rate of evaporation of ether in the tube can be better controlled than in the Daniell's Hygrometer.

(*ii*) Silver being a very good conductor of heat, the temperature of the ether, indicated by the thermometer, is practically the same as that of the silver surface which is in direct contact with the ether and the atmosphere.

(*iii*) The presence of the dummy tube facilitates the observation of the appearance and disappearance of dew on comparing the brightness of the two silver surfaces.

(*iv*) The continuous agitation of ether by the bubling of air through it keeps the temperature uniform throughout its mass.

(*v*) Observations being taken from a distance by a telescope, the result is not affected by breath or heat from the body.

(2) Wet and Dry Bulb Hygrometer (*Mason's Hygrometer or Psychrometer*).—The humidity of the atmosphere can also be judged by observing the rate of evaporation. When the atmosphere is dry, evaporation goes on more rapidly than when it is nearly saturated.

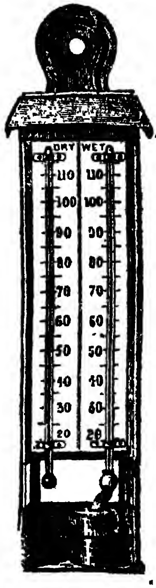


Fig. 49.—Dry and Wet bulb thermometers.

Wet and dry bulb hygrometer is a reliable apparatus which *does not depend on a determination of the dew-point.* It consists of two mercury thermometers, placed side by side, the bulb of one of which is covered with muslin, which is always kept moist by dipping its free end into water contained in a small vessel (Fig. 49). The continuous evaporation from the wet bulb keeps its temperature always lower than the other thermometer which is quite dry. The difference between the two temperatures indicates the humidity condition of the air. The less saturated—*i.e.* the drier the air, the quicker is the evaporation and the more rapid is the cooling; so the difference between the readings of the two thermometers will be great and hence the *dew-point is low.* When the difference is small, it indicates that evaporation from that wet bulb is very slow, and this is due to the presence of considerable water vapour in the air, hence the *dew-point is high.* If the air is already saturated, no evaporation will take place, and the two thermometers will give the same reading.

The dew-point and relative humidity can be found by means of a table as given below :

t°C.	0	1	2	3	4	5	6
10	9·2	8·1	7·0	6·0	5·0	4·0	3·1
11	9·8	8·7	7·6	6·5	5·5	4·5	3·5
12	10·5	9·3	8·2	7·1	6·0	5·0	4·0
13	11·2	10·0	8·9	7·6	6·5	5·5	4·5
14	11·9	10·7	9·4	8·3	7·1	6·1	5·0
15	12·7	11·4	10·1	9·0	7·8	6·6	5·5
16	13·5	12·2	10·9	9·7	8·4	7·3	6·0
17	14·4	13·0	11·7	10·4	9·1	8·0	6·7
18	15·4	13·9	12·5	11·2	9·9	8·6	7·5
19	16·3	14·9	13·4	12·0	10·7	9·4	8·1
20	17·4	15·9	14·3	12·9	11·5	10·2	8·8

The first column (t°C.) gives the temperature of the dry bulb thermometer, and the second column the corresponding vapour pressure of water in millimetres. The numbers 1, 2, 3, etc., at the top of the other columns indicate the difference of temperatures in degrees Centigrade between the dry and wet bulb thermometer. This will be clear from the example given below.

By Formula.—The relative humidity and dew-point can also be calculated by determining the pressure f (in mm.) of aqueous vapour from the following formula $f = F - 0{\cdot}00077\,(t - t') \times H$, where t is the reading of the dry bulb and t' that of the wet bulb thermometer on the Centigrade scale, F, the saturation pressure of aqueous vapour at t°C., and H the atmospheric pressure (in mm.)

The dew-point can also be determined from the **Glaisher's formula.**

If t_0 = dew-point, then $t - t_0 = F(t - t')$, where F is the Glaisher's factor.

Example.—*The reading of the dry bulb thermometer is 18°C., and that of the wet bulb 16°C. Find the relative humidity of the air, and the dew point.*

The difference in dry and wet bulb temperatures $= 18 - 16 = 2$°C.

In the first column we find 18°C. and on the same level in the second column we find 15·4. Then 15·4 mm. is the vapour pressure at 18°C. Now at the same level in column headed "2"—the difference of the two temperatures, we find 12·5. Then 12·5 mm. is the vapour pressure at the dew-point.

Hence the relative humidity $= \dfrac{12{\cdot}5}{15{\cdot}4} = 0{\cdot}80$ or 80 per cent.

The dew-point is the temperature at which 12·5 is the saturation vapour pressure. From the second column we find that 12·7 mm. is the vapour present at 15°C. and 11·9 mm. at 14°C. So the dew-point is a little below 12·7°C. We observe from the table that there is a change of 0·8 mm. in the vapour pressure for a change of 1°C. in temperature from 14° to 15°; so for an increase of (12·5 − 11·9) mm. in vapour pressure, the change in temperature = (0·6)/0·8 = 0·75°C.

∴ The actual dew-point is (14 + 0·75) = 14·75°C.

(3) Chemical (or Absorption) Hygrometer.—The mass of water vapour present in a given volume of air can be measured directly in the following manner :—

The apparatus consists of an aspirator A (fig. 50) filled up with water and provided at the bottom with an outlet tap. It is connected with a bottle *B*, called the trap bottle containing conc. H_2SO_4 which is connected successively to the drying U-tubes *C* and *D* filled with dry phosphorous pentoxide or anhydrous calcium chloride. The thermometer placed near the open end of the tube *D* registers the temp. of the air. In an expt., the tubes *C* and *D* are detached and weighed (W_1). Water is then allowed to run out from the aspirator by opening the exit tap whereon a slow current of air is drawn into the aspirator. When a considerable amount of water has passed out, the tap is closed and the water level in the aspirator marked. The tubes *C* and *D* are taken out and weighed again (W_2). *The difference in the two wts. gives the quantity of moisture absorbed from the air* at a certain temperature. In the second part of the expt., a tube charged with pumice stone soaked in water is then connected to the tube *D*. The aspirator is again filled up with water upto the same level as in the beginning of the first expt., the tap opened and water allowed to come out till its level again falls as before. In this case the same volume of air saturated with water vapour is sucked in. The wt. of the tubes *C* and *D* is again taken (W_3). *The difference between the second and the third weights gives the mass of an equal volume of saturated air at the same temperature.* Then relative humidity,

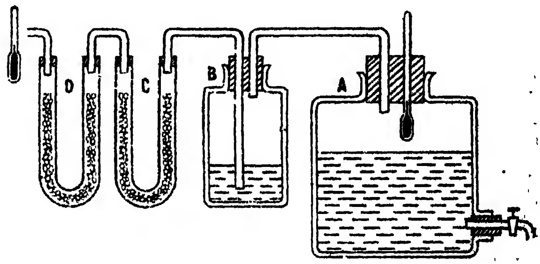


Fig. 50

$$= \frac{W_2 - W_1}{W_3 - W_2}$$

Examples—*A cubic metre of air at 30°C of which the relative humidity is 0·8 is cooled to 5°C. Find the quantity of vapour which will be condensed into water. The maximum pressure of aqueous vapour at 30°C = 31·6 mm and at 5°C. = 6·5 mm.*

$$\text{Relative humidity} = \frac{\text{Mass } m \text{ of vapour present in 1 cu. m of air at } 30^\circ\text{C}}{\text{mass of vapour necessary to saturate 1 cu. m at } 30^\circ\text{C}}.$$

$$0{\cdot}8 = \frac{m}{\frac{5}{8} \times 1{\cdot}293 \times \frac{31{\cdot}6}{760} \times \frac{273}{303}}, \text{ whence } m = 24{\cdot}21 \text{ gm}$$

3. *If 200 gms of water are collected to evaporate in a room containing 50 cubic metres of dry air at 30°C and 760 mm what will be the relative humidity of the air in the room?*

If f be the pressure of the vapour formed, we have (see Ex. 1)

$$200 = 50 \times \frac{5}{8} \times 1{\cdot}293 \times \frac{f}{760} \times \frac{273}{273+30}, \qquad f = 4{\cdot}17 \text{ mm. The maximum}$$

pressure of aqueous vapour at 30°C is 31·6 mm. $\quad \text{R. H.} = \frac{4{\cdot}17}{31{\cdot}6} = 0{\cdot}13$

4. *The temperature of the air in a closed space is observed to be 15°C and the dew point 8°C. If the temperature falls to 10°C how will the dew point be affected? (Press. of aq. vapour in mms. of mercury at 7°C = 7·49 at 8°C = 8·02)*
(*Pat.* 1925 31 '40, 41)

If the volume of the space be reduced then when the space is saturated with vapour some vapour will be condensed but the pressure will remain constant, and if the space be not saturated, then there will be no condensation on reducing the volume and pressure will be increased instead of remaining constant.

As it is a closed space (*i.e.* volume is constant) the pressure is proportional to the absolute temperature.

$$\frac{\text{Press. at } 10^\circ\text{C}}{\text{Press. at } 15^\circ\text{C}} = \frac{(10+273)}{(15+273)} = \frac{283}{288}$$

But the pressure at 15°C = maximum pressure at 8°C (the dew point) = 8·02 mm.

$$\therefore \qquad \text{Press. at } 10^\circ\text{C} = 8{\cdot}02 \times \frac{283}{288} = 7{\cdot}88 \text{ mm (approx.)}$$

Now, a temperature is to be found for which 7·88 mm. will be the maximum pressure. That temperature will be the dew point corresponding to 10°C. From the data given it is seen that for a change of 1°C in temperature there is a change of (8·02 − 7·49) = 0·53 mm in pressure.

Therefore, for (8·02 − 7·88) or 0·14 mm change in pressure, the change in temperature = ¼° approximately. So, at 10°C the dew point is lowered by ¼°.

89 Mass of Aqueous Vapour. (*Mass of Moist Air*).—It is often required to find the mass of water vapour present in a given volume of moist air. Assuming that the vapour obeys the gaseous laws and knowing that the density of water vapour compared with that of air is 5/8 (or 0·62) at the same temperature and pressure, the mass of a given volume of moist air can be calculated as follows :—

Suppose we want to find the mass of 1 litre of moist air at t°C. when the height of the barometer is P mm. and the vapour pressure obtained from dew-point observation is f mm. According to Dalton's Laws the pressure of the air alone is $(P-f)$ mm. The volume of 1 litre reduced to N. T. P. becomes,

$$V = 1 \times \frac{273}{273+t} \times \frac{(P-f)}{760} \text{ litre}$$

The mass of 1 litre of air at N. T. P. is 1·293 gm.

$\therefore$ Mass (m_1) of V litre of air at N. T. P. (which is the same as 1 litre at t°C and $(P-f)$ mm.) $= 1·293 \times \frac{273}{273+t} \times \frac{P-f}{760}$ gm.

$\therefore$ Again, the pressure of water vapour is f mm., hence its mass

$$(m_2) = 0·62 \times 1·293 \times \frac{273}{273+t} \times \frac{f}{760} \text{ gm} \qquad \dots \quad (1)$$

$\therefore$ The mass of 1 litre of moist air $= m_1 + m_2$

$$= 1·293 \times \frac{273}{273+t} \times \frac{(P-f)}{760} + 0·62 \times 1·293 \times \frac{273}{273+t} \times \frac{f}{760} \text{ gm.}$$

$$= 1·293 \times \frac{273}{273+t}\left(\frac{P-f+0·62f}{760}\right) \text{ gm.} = 1·293 \times \frac{273}{273+t} \times \frac{P-0·38f}{760} \text{ gm.} \qquad \dots \quad (2)$$

Example.—*Find the mass of a litre of moist air at 32°C. and 758·2 mm., the dew-point being 13°C. The maximum pressure of aqueous vapour at 13°C. is 12·7 mm.*

The whole gaseous mass may be divided into two portions,—one litre of dry air at 32°C. and (758·2 − 12·7) mm. or 745·5 mm., and one litre of water vapour at 32°C. and 12·7 mm. (Dalton's Law). 1 litre of dry air at 32°C., and 745·5 mm. reduced to N.T.P. becomes $= 1 \times \frac{273}{273+32} \times \frac{745·5}{760}$ litre.

The mass of this air $= 1.293 \times \frac{273}{305} \times \frac{745.5}{760} = 1.1352$ gm. since 1 litre of dry air weighs 1.293 gm.

The mass of aqueous vapour $= \frac{5}{8} \times 1.293 \times \frac{273}{305} \times \frac{12.7}{760} = 0.0121$ gm.

$\therefore$ The mass of 1 litre of moist air $= 1.1352 + 0.0121 = 1.1473$ gm.

90. Condensation of Aqueous Vapour.—It is known that the condensation of aqueous vapour gives rise to the formation of dew, fog, cloud, rain, hail, snow, and hoar frost

Cloud.—Cloud is formed when large masses of warm air rise high up in the air and become cooled either by coming in contact with cold air above or by the lowering of pressure in the upper regions due to expansion. Clouds are nothing but minute drops of water formed by the water vapour being condensed on floating dust particles.

Mist. When the temperature of the air falls below the dew point, condensation takes place throughout the large mass of air on suspended particles of hygroscopic dust or other hygroscopic nuclei, such as sea-salt or molecules of sulphur dioxide, giving rise to *mist*

Dew.—During the day the air in contact with the objects, which are heated by direct radiation, contains a large amount of water vapour which remains unsaturated due to high temperature. During the night cooling takes place and objects, which radiate their heat very rapidly, cool below the temperature of the surrounding air and in consequence, the air in contact with them becomes saturated with the vapour it contains. With further cooling a portion of the vapour is deposited as dew on the surfaces of the cold bodies. Plants are good radiators of heat, so dew is deposited copiously on leaves and grasses. The earth is a good radiator, but water is not

The conditions favouring the formation of dew are (i) *a clear sky* for free radiation. (ii) *absence of wind* in order that air in contact with any object may remain there to be cooled below the dew point (iii) *a good radiating surface* which should not be in good thermal contact with other bodies

Fog is a dense mist or cloud, which is often dispersed by the sun. The hygroscopic dust particles in the air serve as nuclei for the formation of fog. In large towns, having many chimneys, the dense fogs are due to the condensation of moisture on soot and other dust particles.

Rain.—When several tiny globules of water coalesce they form a large drop and fall as rain.

Rain-gauge.—It is an instrument to measure the amount of rainfall in a locality. The instrument in common use is known as '*Symon's Rain-gauge*', which consists of a funnel provided with a circular brass rim having a diameter of five inches. It is fitted to a collecting vessel, which is generally a bottle *B*, placed within a metal cylinder (Fig. 51). The funnel *F* is kept one foot above the ground. The rain passing through the funnel collects into the bottle and the quantity collected in a certain period is measured by a glass cylinder graduated to hundredths of an inch. The rainfall of a place is expressed in inches per annum. An inch of rainfall means that the amount of water collected would fill to the depth of one inch a cylinder with its base equal to the rim of the funnel.

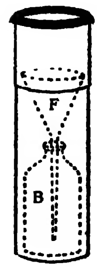


Fig. 51
Rain gauze.

Here in India we have the greatest amount of rainfall in Cherapunji. The average value of annual rainfall in Bengal is 75 inches, in Behar and Orissa 52 inches, in Bombay 45 inches, and in Cherapunji 100 inches.

Snow.—When condensation of water-vapour takes place at a temperature below 0°C. it is directly frozen as crystals of snow.

Questions

Art. 84.

1. Why does a glass tumbler 'cloud over' on the outside when 'ice-cold' water is poured into it? (C. U. 1930 ; Dec. '29)

Art. 85.

2. Explain the formation of dew. Show that the pressure of unsaturated vapour in a room is equal to the saturation pressure at dew-point.

Define Relative Humidity. On what factors does it depend? Obtain an expression for its determination. (Pat. 1932 ; All. '45)

Art. 86

3. (a) Define "relative humidity". Does our opinion of dampness or dryness depend upon the absolute quantity of water vapour present? Explain your answer fully. Describe Daniell's Hygrometer and explain how it is used to determine relative humidity. (See Art. 88). (All. 1922)

(b) Wet clothes are usually seen to dry sooner in the cold weather than in the rainy season, though the temperature in the latter case is higher." Explain. (All. 1945)

Art. 88

4. Define humidity and describe two different methods of determining it in the laboratory. What result would you expect to get on a very wet day in the rainy season? (Pat. 1931, '27 ; cf. C. U. '37)

5. Describe an experiment to determine the dew-point. (C. U. 1944

Calculate the dew-point when the air is $\frac{3}{5}$ saturated with water vapour, the the temperature being 15°C., given that for the pressures of 7, 9, 11, 13 mm. of mercury, the corresponding boiling points of water are 6°, 10°, 13°, and 15°C. respectively. (Pat. 1935, '37)

[*Ans* : 9·4°C.]

6. Describe a dew-point hygrometer and explain its action. What use is made of this instrument in weather forecasting ? (C. U. 1933)

[**Hints.**—If the difference between the room temperature and the dew-point is very small, it indicates a moist weather, and if there is much difference between the two temperatures, it indicates a dry, *i.e.* good weather.]

7. Calculate the temperature at which dew will be deposited when the hygrometric state of the air at 20°C. is 53 per cent.

[*Ans* : 10°C.]

8. Define dew-point and describe an accurate method of finding it. Why is dew more copious on some bodies than on others ?

[See Art. 90) (All. 1917, '26 ; Pat. '45 ; cf. C. U. '37)

Art. 89.

9. Calculate the mass of 7·5 litres of moist air at 27°C., given that the dew-point is 15°C. and the barometric height 762·75 mm. Calculate also the humidity of the air. Vapour pressure of water at 27°C. and 15°C. = 25·5 mm. and 12·75 mm. respectively. (Bombay Univ.)

[*Ans* : 8·923 gms. ; 0·5]

10. Knowing the dew-point how would you determine the amount of aqueous vapour present in the atmosphere ? (All. 1919)

11. What is meant by Relative humidity ? Explain how the determination of the dew-point enables you to calculate the relative humidity of a particular place. (All. 1946)

Art. 90.

12. Cloudless nights are better for the formation of dew than cloudy ones. Explain. (Dac. 1929)

CHAPTER VIII

Transmission of Heat

91. Modes of Transmission.—There are three different modes of tranmission of heat :—

(1) **Conduction.**—In conduction heat passes along a substance from the hotter to the colder parts, or from a hotter body to a colder one in contact, without any transference of material particles.

Solids are heated by conduction.

(2) **Convection.**—In convection heat is transferred from one part of the body to another by the bodily movement of the hot particles.

Liquids and gases are heated by convection.

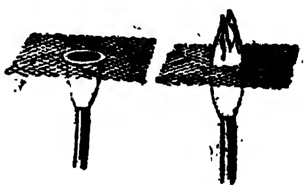
(3) **Radiation.**—In radiation heat is conveyed from one body to another, entirely separated from it, without heating the intervening medium which may be material or vacuous.

The heat of the sun is received on the earth's surface by radiation.

92. **Conduction**—When one end of a body is heated, the molecules there vibrate vigorously, and this increased agitation (i.e. the increased heat energy) is passed on by collision from particle to particle. Some substances conduct heat better than others. Metals are generally good conductors, while substances like glass, cloth, paper, wood, felt, etc., are all bad conductors of heat. Air and other gases are bad conductors of heat.

Good and bad Conductors. (Experiments).—(1) Prepare a small vessel of *thin* paper. Place a piece of copper wire-gauze on tripod stand and then place the paper vessel on it. Now carefully put some water into the vessel and heat the water gently from below the wire-gauze. After sometime the water will begin to boil. As the paper is *very thin*, the heat is conducted rapidly through the paper to the water and so the heat is not sufficient for the paper to be charred. The temperature of water does not rise above 100°C.

(2) Lower a piece of wire-gauze upon the flame of Bunsen burner. The flame burns *below* the gauze and does not pass through the meshes of the gauze (Fig. 52 *a*.). Now put out the gas, and holding the gauze about two inches *above* the top of the burner, turn the gas on. Light the gas above the flame. It burns, but the flame does not come down the gauze (fig. 52 *b*). No combustible substance will burn, even in presence of air, unless it is raised to a certain temperature known as the '*temperature of ignition*' of that particular substance. The reason why the flame does not pass through the meshes of the gauze is that the metal wires conduct away the heat so rapidly that the temperature of the gas near the flame does not rise high enough to be ignited.



(*a*) (*b*)

Fig. 52

(*a*) **Davy's Safety lamp** (Fig. 53) used in mines is an example in which good conductivity of a metal has been utilised. It consists of an oil lamp, the flame of which is surrounded by a cylindrical wire-gauze of close mesh. Even if the lamp is surrounded by an explosive gas, the heat is conducted away so rapidly that it prevents any flame from passing from the inside to the outside, and, when brought in an atmosphere charged with an explosive gas, danger is indicated by the *character of the flame.*

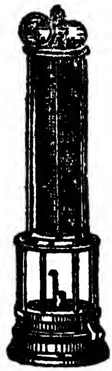


Fig. 53—Davy's Safety lamp.

(*b*) *Other illustrations.*—The advantange of the conductivity of glass is taken when opening a glass stopper stuck tight in the neck of a bottle. If the neck of the bottle is gently and carefully heated, the neck expands before the stopper which thus becomes loose.

Our feeling of warmth or cold on touching different bodies depends to a great extent on conductivity. Thus, if we touch iron and flannel, both being placed in the *same* room, the temperature of which is *below* that of the hand, iron appears to the colder, because it rapidly conducts the heat from the hand, and flannel being a bad conductor conducts very little heat. If the temperature of iron and flannel be *above* that of the hand, as when kept in warm air (or rooms), iron appears to be warmer, because it rapidly gives out more heat to the hand than the flannel.

This is the reason why a piece of metal appears hotter to the touch than a piece of wood when both have been lying long in the sun ; and for the same reason a marble floor appears colder than an ordinary cemented floor.

(*c*) *Use of Bad Conductors.*—In summer ice is packed in felt or saw-dust, which being bad conductors do not conduct heat to the ice from outside. We use woollen dress in winter because it *conducts very slowly* the heat of our bodies to the outside air, and thus the feeling of warmth is maintained. Again, the handles of kettles and tea-pots are very often made of wood, or of vulcanite, in order that the heat from the hot water or tea may not pass through them as much as through a metal handle. Besides this it should be noted that the very low conductivity of cotton, wool, felt, and other fabrics of open texture is largely due to the *low conductivity* of air enclosed in the fabric. For this reason wool is preferred to cotton for preparing warm clothings as the texture of wool is more loose and so it contains more air. It should be noted that bad conductors not only keep *in* heat, but they also keep *out* heat.

93. Comparison of Conductivities.—The conductivity of different substances can be compared by the following experiments :—

Experiments.—(1) Take a cylinder one half of which is brass and and the other half wood. Wrap a piece of thin paper tightly round the cylinder, and hold the middle portion on a Bunsen flame (Fig. 54) It will be seen that the paper over the wooden portion is scorched long before any effect is produced on the other half.


Fig. 54

The brass, being a good conductor, conducts away the heat so rapidly that the paper is not scorched ; while wood, being a bad conductor, is not able to do this.

(2) **Ingen-Housz's Expt.**—A number of metal or other rods, of the same length and diameter are introduced into the holes in the front of a metal trough. All the rods are already covered with a coating of wax, and the metal trough is then filled with boiling water (Fig. 55). Heat is carried along each rod, and, at the proper temperature, wax melts. After sometime a a steady state (Art. 95) is reached, when there is no further sign of melting. It will be observed that the wax has melted up to different distances along different rods showing that conducting power for different substances is different.

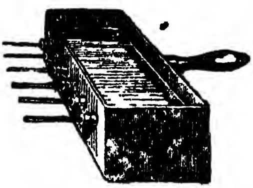


Fig. 55

It can be proved that after the steady state is reached *the thermal conductivities of the different rods are proportional to the squares of the lengths of the wax melted in the rods.*

Thus, if l_1, l_2, l_3, etc., are the lengths of the rods, and if k_1, k_2, k_3, etc., are their thermal conductivities (Art. 94), we have, $k_1 : k_2 : k_3 \ldots\ldots = l_1^2 : l_2^2 : l_3^2 \ldots\ldots$

94. Thermal Conductivity.—

If Q = total quantity of heat transmitted through a plate,
then it is found, $Q \propto A$, the area of the plate,
$\propto (\theta_1 - \theta_2)$, the difference of temperature between opposite faces,
$\propto t$, the time in seconds in which the quantity Q passes,
$\propto 1/d$, d being the thickness of the plate,

$$\therefore\ Q \propto \frac{A(\theta_1 - \theta_2)t}{d}\text{ ; that is, } Q = \frac{K.A.(\theta_1 - \theta_2)t}{d}.$$

where K is a constant characteristic of the material of the plate. It is called the *thermal conductivity*, or the *co-efficient of conductivity* of the material.

If in the above equation we take $A=1$ sq. cm.; $d=1$ cm.; $(\theta_1-\theta_2)=1°$C.; $t=1$ sec., then we have $K=Q$ calories.

That is, **thermal conductivity** *is the amount of heat which passes in one second through the opposite faces of a unit cube (i.e. 1 cm. cube) of a material, the difference of temperature between the opposite faces being 1°C.*

Note.—The quantity $(\theta_2-\theta_1)/d$, or, in other words, the fall in temperature per unit length is called the *temperature gradient.*

95. Thermal Conductivity and Rate of Rise of Temperature.—When a metal rod is heated at one end, its temperature goes on increasing because each portion of the rod receives more heat than it transmits. This state of the rod is known as the *variable state.* After sometime each portion of the rod may receive somewhat more heat than it conducts through the neighbouring portion, but this excess is lost by convection or radiation, and so the temperature of each part of the rod remains constant. This state is called the *stationary* or *steady state* of the rod.

In the *variable state*, the rate of increase of temperature depends not only on the thermal conductivity of the substance but also on its *specific heat*, which is the quantity of heat required to raise unit mass of the substance through unit temperature. The quantity of heat reaching any portion of the rod will depend on the thermal conductivity, but the rise of temperature produced by that amount of heat will depend on the specific heat of the material. If the specific heat is low, the temperature of any portion of the rod rises quickly until the *stationary state* is reached, even if the conductivity of the substance is not good, because in this case, only a small amount of the heat that comes along is necessary to raise the temperature. But, on the other hand, if the specific heat is high, the temperature of any portion of the rod will rise very slowly (to the stationary temperature) even if the conductivity of the substance is also high. Considering a unit cube (*i.e.* volume $=1$ c.c.) of the material of the rod,

If $d=$ density of the material, *i.e.* mass of unit volume,

$s=$ specific heat of the material, $t°C.=$ rise of temperature per second, $Q=$ quantity of heat reaching the volume per second,

we have, $d.\,s.\,t.=Q$; or $t=Q/d.\,s$;

That is, *the rise of temperature during the variable state produced in a unit volume of the rod is directly proportional to the quantity of*

heat reaching the volume, and so, to the thermal conductivity, and inversely proportional to the product of the density and specific heat, that is, the thermal capacity of unit volume.

$$\therefore \quad t = \frac{K}{d.s.} = \frac{\text{Thermal conductivity}}{\text{Thermal capacity of unit volume}}.$$

The ratio $K/d.s.$ has been termed by Lord Kelvin as *diffusivity (or thermometric conductivity)* of the substance.

Taking the case of iron and bismuth, we have the thermal capacity of unit volume of iron $(7{\cdot}8 \times 0{\cdot}11 = 0{\cdot}858)$ much greater than that of bismuth $(9{\cdot}8 \times 0{\cdot}03 = 0{\cdot}294)$, and so the rate of melting of the wax (see Fig. 55) at the beginning is much lower for the iron. But the thermal conductivity of iron being 7 times greater than that of bismuth, a longer length of wax is melted along the iron.

Thus, remember that both the *thermal conductivity* and *specific heat* play important part during the *variable state*; but when the *stationary state* is reached, no more heat is absorbed, and then the flow of heat depends on *the thermal conductivity* only. Therefore in comparing the thermal conductivities of different substances, *we should wait until the stationary state is reached.*

Before the steady state is reached in the Ingen-Housz's experiment described in Art. 93, heat diffuses through the different rods at different rates depending on the diffusivity of the substances and so the rate of melting wax gives a measure of diffusivity along the rod.

Example 1 —*If conductivity of sandstone is 0·0027 C. G. S. units and if the underground temperature in a sandstone district increases 1°C. for 27 metres descent, calculate the heat lost per hour by a square kilometre of the earth's surface in that district.* **(*Pat. 1926.*)**

We know that, $Q = \dfrac{K.\ A.\ (\theta_1 - \theta_2).\ t}{d}$

Here $K = 0{\cdot}0027$; $A = 1$ sq. kilometre $= 10^6$ sq. metres $= 10^{10}$ sq. cm. ;

$(\theta_1 - \theta_2) = 1°C.$; $d = 27$ metres $= 2700$ cm. ; $t = 3600$ sec.

$$\therefore \quad Q = \frac{0{\cdot}0027 \times 10^{10} \times 1 \times 3600}{2700} = 3{\cdot}6 \times 10^7 \text{ calories.}$$

N. B. The area given in sq. metre must be reduced to sq. cm. as the value of thermal conductivity is given in cm.

2. *An iron boiler 1·25 cm. thick contains water at atmospheric pressure. The heated surface is 2·5 sq. metres in area and the temperature of the under side is 120°C.*

If the thermal conductivity of iron is 0·2 and the latent heat of evaporation of water 536, find the mass of water evaporated per hour. (*Pat. 1930, '41.*)

Here $K=0{\cdot}2$; $A=2{\cdot}5\times100\times100$ sq. cm.

$\theta_1=120°$C. ; $\theta_2=100°$C.. ($\because$ the boiling point of water at atmospheric pressure is 100°C.) ; $t=60\times60$ sec. ; $d=1{\cdot}25$ cm.

$$\therefore \quad Q=\frac{0{\cdot}2\times2{\cdot}5\times10^4\times(120-100)\times3600}{1{\cdot}25}=288\times10^6 \text{ calories.}$$

The latent heat of evaporation of water is 536, *i.e.* 536 calories of heat are required to evaporate 1 gm. of water. Therefore the number of grams of water evaporated by 288×10^6 calories of heat $=\dfrac{288\times10^6}{536}=537{,}313$ gms.

3. The absolute conductivity of silver is 1·53 ; its specific heat is 0·056, and its density is 10·5. Find (i) the thickness of a silver plate 1 sq. cm in area that would be raised in temperature through 1°C. by the quantity of heat transmitted in 1 second through another plate of silver of the same area and 1 cm. thick with a difference of temperature of 1°C. between its opposite faces ; (ii) the rise of temperature produced in a plate of silver 1 sq. cm. in area and 1 cm. thick by the same quantity of heat.

(*i*) Let x cm. be the thickness of the first plate, then its mass $=x\times1\times10{\cdot}5=10{\cdot}5x$ gm. Therefore the quantity of heat required to raise the temperature of this mass through 1°C. $=10{\cdot}5x\times0{\cdot}056$ cal.

But the heat which flows through the second plate in one second $=1{\cdot}53$ cal. Hence $10{\cdot}5x\times0{\cdot}056=1{\cdot}53$. $\therefore$ $x=2{\cdot}6$ cm.

(*ii*) Let θ°C. be the rise of temperature produced in the plate of silver 1 sq. cm. in area and 1 cm. thick by 1·53 cal. of heat, then

$$1{\cdot}53=m\,s\,\theta=10{\cdot}5\times0{\cdot}056\times\theta. \quad \therefore \quad \theta=2{\cdot}6°\text{C.}$$

96. Conductivity of Liquids and Gases.—

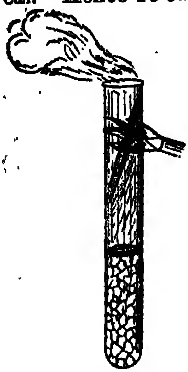


Fig. 56.

Expt —Wrap copper wire round a piece of ice until it will sink in water. Place this in a test tube and pour water in the test tube (Fig. 56). Now heat the upper part of water with a flame. In this way water can be boiled at the upper part without melting the ice.

Liquids are generally bad conductors of heat, but *mercury*, which is a good conductor of heat, is an exception.

The **Conductivity of gases** is less than that of liquids. Gases are heated by convection.

97. Convection.—When liquids and gases are heated, the heat

is carried away by the *actual movement* of particles. These movements arise from differences in temperature between different parts of the same substance. When the temperature at some part of a liquid or a gas increases, it causes diminution in density, and the hotter portion being lighter rises, its place being taken by colder and heavier portion from the sides. Thus, convection currents are set up which can easily be visible by heating some water in a flask in which some colouring matter is kept in the bottom of the flask (Fig. 57).

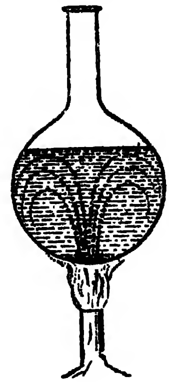


Fig. 57

98. Convection Currents in Liquids—Convection currents may be illustrated by the apparatus shown in Fig. 58.

Expt—A flask B (Fig. 58) and a reservoir A open at the top are connected by two glass tubes AB and CD. AB runs from the top of the flask to the top of the reservoir and CD runs from the bottom of the flask to the bottom of the reservoir. The whole apparatus is filled with water. The water heated in B ascends the tube AB and the colder water in the upper vessel runs down the tube CD to fill the place. Thus a circulation is set up and finally all the water reaches the boiling point. The motion becomes visible on dropping some dye into A, when the colour can be seen travelling down along the tube CD.

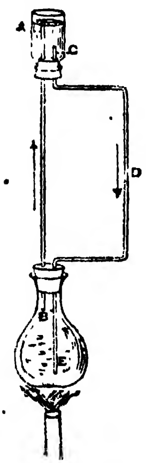


Fig. 58

The above experiment illustrates the principle applied in *hot-water-heating system* for buildings. In this case, a pipe rises from the upper part of the boiler to a reservoir at the top of the building and the downward pipe passes through a number of metal coils placed in various rooms and ultimately enters the boiler again. The water, in circulating through the pipe, is cooled and heat is given out to the rooms.

This method of heating illustrates all *the three processes of transmission of heat—i.e.* conduction, convection and radiation. It is by *conduction* that heat passes from the furnace to the water through the boiler; it is carried to the interior of the pipes by *convection*, and the whole system is a good example of a continuous water *convection* current. Heat is carried to the exterior of the pipes by conduction and it escapes into each room from the pipes and coils by *radiation*.

99 Convection of Gases.—The ascent of smoke up a chimney is a familiar example of convection In the same way convection currents are produced in the chimneys of oil-lamps. Hot air above the fire rises up the chimney, its place being taken by cold air drawn from the room. Thus a fire helps to ventilate a room. Winds are caused by convection currents in the atmosphere.

Warmth of Clothing—The warmth of clothing depends to a large extent upon convection. A loosely woven thick cloth consists of wool fibres separated by air spaces. The heat of the body trying to escape to the outside must do so either by the zig-zag paths among the fibres or it must go through the shorter and more difficult path partly through the non-conducting fibres, and partly across the air spaces by setting up convection currents. Thus a loosely woven cloth is really warmer in cold air, which is at *rest*, than another cloth having the same amount of material but closely woven. The air should be at *rest*, otherwise heat of our bodies will be lost by convection. For this reason closely woven cloth is necessary for people exposed to strong winds, that is, for aviators and motorists who can use leather cloth. So our 'warm' clothes are not really warmer than other objects in any room.

Ventilation.—The ventilation of a room depends on merely establishing convection currents between the outside air and the air in the room.

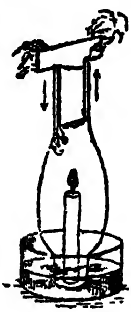


Fig. 59

The following experiment will illustrate it.—

Expt —Place a lighted candle on a saucer and pour water around it (Fig 59). Put a lamp chimney over the candle. The flame after a while goes out as no fresh air can get in from below, and through the sides of the chimney.

Repeat the experiment and introduce a piece of T-shaped metal or card-board down the chimney. The candle continues to burn. This is because the T-piece had divided up the chimney into two halves, one for up-draught to get rid of hot gases, and the other for down-draught to take in fresh air. The existence of these two currents can be shown by holding a piece of smouldering paper near the top of the chimney.

Two things are necessary for proper ventilation of a room—an outlet for the warm and impure air near the top of the room, and an inlet for the cold pure air near the bottom of the room.

Chimneys—The draught in a chimney of an ordinary lamp or over a furnace is due to convection. The heated air and smoke go up the chimney, while fresh cold air enters at the bottom and thus a convection current is set up. The *draught* is due to the *difference* in weight between the cold air outside and the hot air inside the chimney. The taller the chimney the greater will be this difference in weight and so greater will be the draught. So the factory chimneys are *taller*. But tall chimneys will be of no advantage unless there is enough fire at the bottom to keep the gas hot all the way up. In order that the descending currents may be prevented, narrow chimneys are better than wide ones.

Gas-filled Electric Lamps.—The heat of the filament of a gas-filled electric lamp, which contains a small quantity of some inert gas, such as argon or nitrogen, is carried away to the upper part of the bulb by means of convection current set up by the heated filament. As the heat from the filament is carried away, the filament can be raised to a higher temperature without any risk of melting than if surrounded by a vacuum. Besides this, the convection currents have another advantage ; they carry to the top of the bulb the tiny metal particles of the gradually disintegrating filament which cause the blackening of the lamp. Thus the blackening, which would otherwise take place over the entire inside surface, being prevented, these lamps last longer than that of the vacuum type (see Ch. VIII, Part VII).

100. Air Conditioning.—It is known that by the oxidation of the food we take in, generally more heat is generated than the body can use, and so there must be some way of removing the surplus in order to prevent a rise in temperature above normal ; otherwise we become uncomfortable. The comfort of the body depends upon three factors or conditions—*temperature, humidity*, and *ventilation.*

The normal *temperature* of the body being 98·6°F., the surrounding air must be at a lower temperature than this in order that it may remove some part of the body heat. In cold countries most homes are provided with various devices for warming the air in cold weather. It is also possible to cool the air in summer, which is generally done in some stores and theatres by means of refrigerators (see Art. 69). The relative *humidity* of the air should be near about 60 per cent. in order that we may feel comfortable This is controlled by addition of moisture as by a Humidifier or by condensing the moisture present as in a De-humidifier. Lastly, there must be proper and ample *ventilation* by suitable arrangement.

101. Natural Phenomena.—

Winds.—Winds are due to convection currents set up in the atmosphere due to unequal heating.

Land and Sea Breezes.—Convection currents account for land and sea breezes.

Sea breeze :—

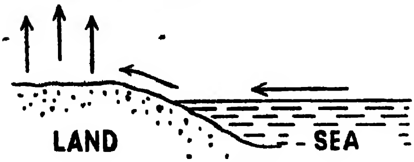


Fig. 59 (a)

During the day time land becomes more heated than the sea, firstly because of its greater absorbing power and secondly due to its lower sp. heat. In the evening, therefore, air above the land being more heated rises up and colder air over the sea blows towards the land by convection, causing sea breeze (fig. 59. a).

Land breeze :—

Since good absorbers are good radiators, during the night the land loses more heat than the sea. Sp. heat of land being lower again, the temp. of the land in the early hours of the morning will be lower than that of the sea. So convection currents of air will flow from the land towards the sea (fig. 59. b), causing what is called the land breeze.

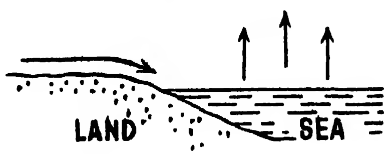


Fig. 59 (b)

Trade Winds.—Heated air over the tropics rises up and cold air from the north and the south moves towards the equator, but, owing to the rotation of the earth from west to east, the wind gets a relative velocity in the north-eastern direction in the northern hemisphere and south-eastern direction in the southern hemisphere. The first is known as *north-east trade wind* and the other as *south-east trade wind.*

102 Distinction between Conduction, Convection and Radiation.—(1) In conduction and convection heat is propagated in a material medium , while in radiation the assistance of no material medium, either solid, liquid or gas, is essential, and heat energy passes through a vacuum or a material medium *without affecting the temperature* of the intervening medium ; but conduction and convection raise the temperature of the medium. In conduction there is no transference of material particles while in convection the heated particles bodily move.

Heat energy is transferred to us from the sun through thousands of miles of so-called vacuous space where there is no material medium. Heat is received by radiation not from the sun only. We also receive heat by radiation from a fire or any other hot body. If you hold your hand *below* an electric lamp your hand will get warmer. This is not due to conduction, for air is a bad conductor of heat, and it is also not due to convection, for a convection current always has the tendency to move upwards. So it is due to radiation.

(2) A body emits radiation in *all directions* and in *straight lines ;* while in the other processes, it is not so. For this reason, a screen, placed between the source of heat and any body, cuts off the radiation.

(3) Transference of heat by radiation takes place almost *instantaneously*, while the other processes are comparatively very *slow.*

Radiant energy travels with the velocity of light, *i.e.* 186,000 miles per second.

(a) Nature of Radiation. (*Ether Waves*).—When we stand before a fire we feel hot. It is obvious that we are not getting heat from the fire by conduction ; also the convection currents carry heat upwards and bring cool air from around to the fire. So the heat we feel is not due to convection. Again, we know that we get something from the sun and the fire that can give rise to sensation of heat and sometimes of light. This something is called *Radiation.* Radiant energy reaches us from the sun, a distance of about 92,000,000 miles, in about $8\frac{1}{4}$ minutes only. The atmosphere, which surrounds the earth, does not extend upwards indefinitely. How, then, the radiant energy is communicated to us from the sun ? To explain this, scientists have assumed the existence of a medium, called the *ether*, which is a very delicate medium and which is present everywhere, even in the interstices of the molecules of even the hardest solids, just as air is present everywhere between the leaves and branches of a tree.

Just as by disturbing the surface of water in a pond waves can be created, which spread outwards from the point of disturbance, so transverse waves are created in the ether by the rapid vibration of the molecules of a hot body, and these waves pass outwards in all directions with the velocity of light (186,000 miles per sec.) When these waves are stopped by a body, the molecules of the body are made to vibrate producing heat in the body. There are some substances which allow these waves to be transmitted through them. These are called **dia-thermanous** substances ; while the other substances which do not allow the waves to pass through them, are known

as adia-thermanous or a-thermanous. A vaccum is perfectly dia-thermanous. Dry air, rock-salt, carbon bisulphide are also good dia-thermanous substances. Wood, slate, metals, etc., are dia-thermanous. The latter class gets heated by absorbing radiant energy. It is to be noted that *radiant heat* or radiation, strictly speaking, *is not heat, but is the energy which being absorbed by certain bodies manifests itself as heat ;* and *during transit it is only the energy of wave which moves in the intervening ether.*

(b) Radiant energy.—Any form of energy transmitted by means of ether waves is called *radiant energy.* These ether waves differ amongst themselves in **frequency** *(i.e.* in the number of vibrations per second) and consequently in the **wave-length,** just as there are little ripples and big waves on the surface of the sea.

Waves of different lengths produce different effects. Very long ether waves carry *electricity*, and they are used for the transmission of wireless messages. Waves shorter than these give us *radiant heat* and still shorter waves affect our eyes, which we have called **light** The waves, which are still shorter, or rather too short to affect the eyes, can produce *chemical action* on photographic plates. These are called **Ultra-violet rays**. Still shortor waves are known as **X-rays**, and waves still shorter than the X-rays are **Gamma rays** which are given out by radio-active substances. The shortest ether waves about which full information is not yet known are called **Cosmic rays** and are supposed to be coming from stars, etc., *i.e*, from sources outside the earth and its atmosphere.

A hot body at a low temperature is not visible in a dark room as it emits only heat radiation. But at a sufficiently high temperature it becomes visible, when it emits also comparatively smaller waves, which can excite in our eyes the sensation of light in addition to that of heat ; so, at a high temperature, it emits both kinds of radiation (heat and light). As water waves are produced by the vibration of water particles, so the ether waves are produced by the vibration of ether particles. Vibrations of ether particles of certain degrees of rapidity produce mainly heating effects on bodies on which they fall ; while certain others of higher degrees of rapidity can produce in our eyes the sensation of light. Longer waves are produced by slow vibration and shorter waves by rapid vibrations of ether particles. For example, vibrations between $3{\cdot}75 \times 10^{14}$ (*red*) and $7{\cdot}5 \times 10^{14}$ (*violet*) times per second producing ether waves of approximate lengths between $\cdot 80 \times 10^{-6}$ cm. (*red*) to 40×10^{-6} cm. (*violet*) can produce the sensation of vision. This is the range of **Luminous radiations** ; while the frequencies of **Actinic radiations**, which can produce chemical changes

are higher than $7{\cdot}5 \times 10^{14}$ times per second *i.e.* beyond the violet end of the visible light. *So heat and light are both forms of radiant energy, and the difference between them is a difference in degree rather than in kind.* The waves producing thermal effect, and which do not affect our sense of vision, vary in lengths between 80×10^{-6} cm. to 0·03 cm. These are called **Infra-red** waves. The waves which are smaller than 40×10^{-6} and produce actinic effects, *i.e.*, produce chemical changes on plants and certain salts of silver due to which photography becomes possible, are called **Ultra-violet** waves. These vary in lengths between 40×10^{-6} to $1{\cdot}4 \times 10^{-6}$ cm. Waves smaller than these are popularly known as *X-rays*, the wave lengths of which vary between the limits 1×10^{-5} to 6×10^{-10} cm. There are also waves shorter than X-rays which are known as *Gamma rays* and beyond this region there are waves of still shorter length known as *Cosmic rays*.

On the other side beyond the Infra-red region there are very big ether waves which do not affect any of our bodily senses. Very long ether waves whose lengths vary from about 10 metres to several miles are known as "*wireless*" waves. The length of the wireless waves can also be as small as 0·01 cm.

103. Instruments for Dectecting and Measuring Thermal Radiation.—

(1) Ether Thermoscope.—The ether thermoscope (Fig. 60) contains some quantity of coloured ether and ether vapour, the whole of the air from within having been expelled before sealing the instrument. One of the bulbs is coated with lamp-black which is a perfect absorber of thermal radiation. When thermal radiation falls on the black bulb, its temperature and consequently that of the contained vapour, rises. This increases the pressure of the vapour on the ether inside the bulb. Hence the level of the ether in the black bulb is pushed down and that in the other bulb rises.

Fig. 60 Ether thermoscope.

(2) Differential Air Thermoscope.—This was first used by Leslie. It consists of a glass tube bent twice at right angles, terminating in two equal bulbs containing air. The tube contains coloured sulphuric acid up to a certain height, and the quantity of air in the bulbs is so adjusted that the liquid stands at the same level in the two tubes when both the bulbs are at the same temperature. For a slight difference of temperature of the air in the bulbs, there is a small difference in level of the liquid due to the expansion of air in the warmer bulb, which depresses the liquid column nearest to it and raises that in the other.

(3) **Thermopile.**—This is a very sensitive electrical instrument (described in Art. 53, Part VII) which is used by modern experimenters.

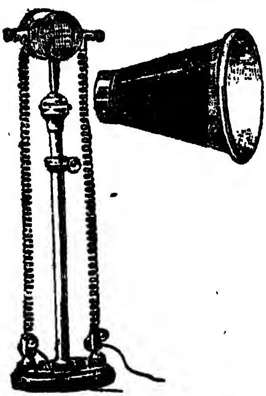


Fig. 61—Thermopile

104 Radiant Heat and Light compared.—**(A)** *Similarity.*—

(1) *Radiant heat and light travel in a vacuum as well as in air in all directions with the same velocity.*

At the time of an eclipse of the sun, when the moon comes directly between the sun and the earth, it is seen that heat and light from the sun are cut off at the same instant, showing that heat and light energy travel everywhere in all directions and with same velocity (186,000 miles per second).

Again the heat radiated from the hot wire enclosed in an exhausted electric bulb can be felt by an ether thermoscope placed outside the bulb.

(2) *Radiant heat and light travel in straight lines.*

Two wooden screens are taken having a small hole in the middle of each. They are arranged parallel to each other, and a red hot ("red-hot" at about 525°C.) metal ball is placed opposite to the hole of one of the screens. If now the lamp-black-coated bulb of an ether thermoscope (Fig. 60), or a thermopile (Fig. 61), be placed far away from the other screen and opposite to the hole in it, it will be observed that, when the two holes are in the same straight line with the ball, the thermoscope, or the thermopile, is greatly affected, while the effect is very little when the two holes are not in the same straight line. This proves that radiant energy travels in straight lines.

(3) *Heat rays can be reflected in the same way as light obeying the same laws as in the case of light.*

(*a*) *Reflection at a Plane Surface.*—Two tin plate tubes are supported horizontally in front of a polished tin-plate so as to be equally inclined to the plate. Now placing a hot metal ball near the end of one tube and a thermopile, or the black bulb of an ether thermoscope, near the end of the other, the instrument is affected. The effect on the instrument will be much less when the tubes are placed unequally inclined to the plate. It will be found that the effect is a maximum when the tubes make equal angles on the opposite sides of the normal to the reflecting plate (See Fig. 14, Part III).

(*b*) *Reflection at a Concave Spherical Surface.*—If two large concave metallic reflectors (see Fig. 15, Part III) are placed coaxially facing each other at a little distance apart, then the blackened bulb of the thermoscope placed at the focus of one of them will be seen to be greatly affected by a red hot ball placed at the focus of the other reflector. The difference in effect may be noticed by displacing a little, the reflector near the thermoscope.

(4) *Heat rays can be refracted in the same way as light, and obey the laws of refraction of light.*

The rays from the sun, *i.e.* both the heat and light rays, can be concentrated at a point by means of a convex lens, and a piece of paper placed at the point may be easily ignited by the heat rays.

A better effect will be obtained by using a convex lens made of rock-salt, instead of glass, as rock-salt, being dia-thermanous to heat rays, absorbs only a small percentage (about 7 per cent.) of them, while glass absorbs a considerable amount of heat rays.

(5) *The amount of heat received per second per unit area of a given surface, i.e. intensity of radiation, by the absorption of thermal radiation emitted by a source of heat at a constant temperature, is inversely proportional to the square of the distance between the source and the absorbing surface. This is known as the* **Inverse Square Law.**

(The Inverse Square Law holds for heat radiation in the same way as it does for light radiation.)

Proof.—A rectangular tin-plate box (Fig. 62), one face of which is coated with lamp-black, is filled with hot water and kept at a constant temperature. A thermopile is taken, one face of which is covered by a brass cap and the other face, meant for absorbing thermal radiations, is provided with a conical reflector in order to shield it from air currents. The thermopile is connected with a galvanometer, (*i.e.* an instrument to detect the presence of an electric current), and

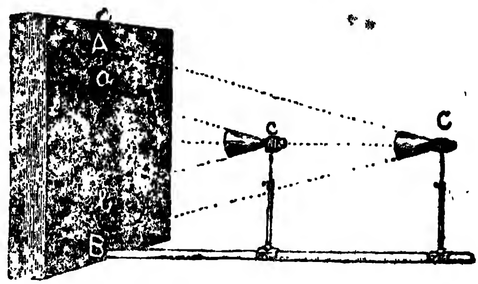


Fig. 62

placed at a certain distance from the lamp-black-coated surface. The galvanometer deflection is noted. Let the circle *ab* be the base, on the tin-plate box, of the imaginary cone having it apex *c* on the thermopile.

Radiation from points outside the circle ab (on the box) is not absorbed by the blackened inside surface of the conical reflector (Fig. 62) and so cannot reach the actual thermopile. The thermopile is now moved to another distance at C, which is *twice* the former distance so that the base of the cone is enlarged to AB. It is observed that there is *no change in the deflection* of the galvanometer as long as the whole of the circle AB lies on the black surface, showing that the quantity of heat received per unit area of the thermopile in each case is the same. As the thermopile is moved away, *the intensity of radiation*, that is, the *amount of heat energy received per unit area of the surface of the thermopile*, decreases, but the area of the source of radiation increases (from the area of ab to that of AB). The total amount of heat energy received depends on the product of these two factors, which remain constant.

Since the temperature of the box is kept constant throughout the experiment, the amount of heat emitted will be proportional to the areas of the circles ab and AB, that is, $\pi r_1^2 : \pi r_2^2$: or $r_1^2 : r_2^2$; or $1^2 : 2^2$ or as $1 : 4$, where r_1 is the radius of the ab and r_2 that of AB. Hence we might expect the galvanometer deflection at C to be 4 times that at c. But by experiment we find that deflection is the same. Therefore it follows that the intensity of heat radiation at C, which is at twice the distance, must be $\frac{1}{4}$ of that at c. This verifies the law of inverse squares.

(B) **Difference between Radiant Heat and Light.**—Both heat and light waves are due to ether vibrations. They differ only in the *frequency of vibration*, and so in their *wave-lengths*.

165. Emissive or Radiating Powers of Different Bodies compared.—

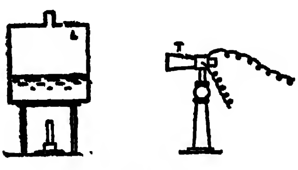


Fig. 63

The *Emissive (or Radiating) power* of a source is defined as *the amount of heat radiation received on unit area placed normal to the rays at a distance of 1 cm. from the source.*

The same body, even at the same temperature, will radiate different quantity of heat depending on the nature of its surface. To show this, take as a source of radiant heat, a cubical tin L containing boiling water. One face of the cube is coated with lampblack, another with black enamel, a third with white paper, and the fourth with polished tin. Such a cube is usually termed a *Leslie's cube* (Fig 63). Take a thermopile T connected with a galvanometer and keep the face of the

thermopile equidistant from each face of the cube in turn and for the same time. The deflection of the galvanometer will be proportional to the emissive or radiating power of each surface, the faces being supposed to be at the same temperature. It will be noticed that deflection of the galvanometer is greatest for the lamp-black surface, the black enamel surface comes next in order, then comes the white paper and it is least with the polished tin surface. This shows that *lampblack is the best radiator, and a polished metallic surface is the worst radiator.*

At 100°C. *white lead* is as good a radiator as lampblack, but above this temperature *lampblack* is by far the best radiator.

106. Absorption of Radiation.—Different surfaces at the same temperature emit different amounts of radiation, but when thermal radiation is incident on the surface of a body, the temperature rises according to the proportion absorbed by it. The proportion absorbed varies with the nature of the surface.

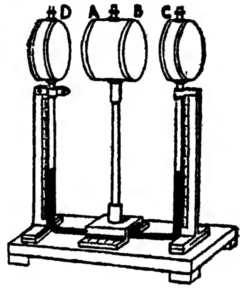


Fig. 64

The absorbing power of a surface is the ratio of the amount of radiation absorbed by the surface in a given time to the total amount of radiation incident upon it.

Emissive and absorbing power of a surface.—

Ritchie's Expt—The apparatus consists of two cylindrical metal vessels C and D filled with air and connected by a glass tube bent twice at right angles in which some coloured liquid has been placed. A large cylindrical vessel AB is supported between C and D. The surfaces A and C are coated with lampblack while the other surfaces D and B are polished. When AB is filled up with boiling water, the level of the coloured liquid is found to remain the same, which shows that C and D are the same temperature. The face A emits more than the face B, but the black face C absorbs more than the polished face D. As the level of the liquid remains the same, it shows that one vessel gains as much heat energy as the other, *i.e., the emissive power is equal to the absorbing power.*

As lampblack is the best absorber, and the polished metallic surfaces the worst absorbers, we conclude that **good absorbers are good radiators**. A body which absorbs all the radiation incident on it is called a *perfectly black body.*

107. Selective Absorption of Heat Radiation.—Different bodies, even when at the same temperature, will radiate as also absorb heat differently, and generally bodies which can reflect heat radiation very

well, are bad absorbers of heat. For example, *bad reflectors like lampblack, ashes, etc., are good absorbers of heat.* It takes less time to boil water in an old kettle covered with soot than in a new one which is polished. The soot absorbs heat better than the polished metal and so water boils quickly in an old kettle. In winter, ice and snow kept beneath the ashes melt sooner than the ice and snow which are uncovered, because ashes are good absorbers of heat *Good reflectors such as polished metals are bad absorbers and also bad radiators of heat.*

(a) **Some Practical Applications.**—In our everyday life we require for some purposes good reflectors of thermal radiation, while for other purposes good absorbers are necessary. A few examples are given below :—vessels such as teapots, calorimeters, etc., which are meant for retaining their heat are made with polished exteriors because black bodies radiate less than white ones. For cooking purposes vessels should preferably be black with rough exteriors. Black clothing is preferred in winter as it absorbs almost the whole of the heat rays falling on it and thus becomes warm, while white clothing is more suitable in summer as it absorbs very little of the sun's heat rays. The advantage of the white-painted walls and roofs of a building is that they keep the building warmer in winter and cooler in summer than if they were painted with a dark colour. In order to cool down hot liquids quickly it is better to use a black stone vessel and not a metal cup with polished surface. Dry air absorbs very little heat radiation. It transmits nearly the whole amount of heat radiation falling on it, *i.e.* it is a *dia-thermanous substance*, while *moist air absorbs* heat radiation to a great extent. Thus the moisture of the air helps us in two ways ; it prevents the earth from becoming too much heated during the day time by absorbing sun's rays and also from becoming too much cooled at night by absorbing the radiation escaping from the heated surface of the earth. We know that **a clear night is colder than a cloudy night** *as clouds are practically opaque to the long heat rays radiated from the surface of the earth.*

Water only transmits 10 per cent. of heat radiation and alum transmits less. But when alum is mixed up with water, the transmitting power of the latter is increased.

Gases are bad radiators of heat, so fire-bricks, which are good radiators, are used in the construction of furnaces in which the hot gases are made to play on the fire-bricks, which are heated by contact and then radiate the heat freely.

(b) Dewar flask (Thermo-flask or Vacuum flask) :—

It is an example where the loss or gain of heat by a system through

conduction, convection or radiation has been reduced to a minimum. It is used for keeping a hot liquid hot and a cold liquid cold for a good length of time.

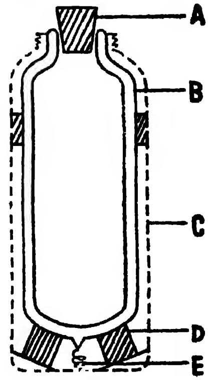


Fig. 65

It consists of a double-walled glass flask *B* (fig. 65) placed on a spring *E* within a metal or wooden casing *C*, its mouth being closed by a cork stopper *A*. The space between the flask and the outer casing *C* is preferably packed with a non-conductor like felt. The space between the two walls of the flask is exhausted of air by pumping out the air through the nozzle at the bottom which is finally sealed off. The outer surface of the inner wall and the inner surface of the outer one are silvered. The vacuum belt around the liquid kept in the flask prevents any loss or gain of heat through conduction and convection while radiation is reduced to a minimum for the silvering of the surfaces. The non-conducting packing of felt reduces any sharing of heat by conduction through the walls. Conduction, convection and radiation, the three possible modes of exchange of heat being guarded, the liquid remains almost in a state of thermal isolation and thus maintains its own temp. for pretty long time.

(c) Green-House.—It is an example of selective absorption of glass. The amount of heat transmitted through a substance depends upon the source of heat ; for example, glass transmits about 50 per cent of heat when the heat rays come from a source which is at a high temperature *e.g.* the sun, or a hot fire. *Glass is adia-thermanous* to heat rays when the source is below 100°C. This is why heat accumulates in a green-house, the glass windows of which allow rays of heat from the sun to pass through them. These rays heat the objects inside, but when the bodies inside, which are evidently at a temperature below 100°C., radiate their heat, glass windows do not allow it to pass out. Glass thus serves as a *trap* to the sun-beams.

A **Glass fire-screen** is also an example of the above principle. It will absorb most of the thermal radiations falling on it, while only a small part is transmitted along with the luminous portion. One, therefore, will see the fire while much of the heat is cut off. Ordinary *glass* not only *absorbs* the long *infra-red waves* but also the short *ultra-violet waves*. It *transmits* only the visible *light waves*. A special kind of glass has, however, been made which can transmit infra-red radiation. So these are used for camera lenses for long distance photography.

Quartz glass and "*Vita glass*" transmit ultra-violet portion of the radiation and they are often used for window-panes in hospitals.

(d) Temperature of the Moon's surface.—Like glass, water is dia-thermanous to radiation from a hot source, but adia-thermanous to that from cold bodies. This fact has been applied to measure the temperature of the surface of the moon It is known that moon reflects the sun's radiation and also emits its own. These two different types of radiation have been separated by passing the radiation into water when the sun's radiation will be transmitted, while that from the moon will be absorbed, by calculating the amount of which the temperature of the moon's surface can be determined.

108. The Law of Cooling.—

Expt.—(Verification of Newton's Law of cooling.) Take some hot water in a calorimeter and note the temperature of the water at an interval of one minute for a period of about 20 minutes carefully stirring the water all the time. Now, note the fall of temperature for any interval of time, 2 minutes, and also the mean difference of temperature between the water and the air of the room during the same two-minute interval. Calculate the ratio of the fall of temperature during this interval to the mean difference of temperature, and repeat the process taking the fall of temperature at various points all over the period of 20 minutes. It will be found that these ratios are practically equal.

As the mass of the liquid is constant, this experiment shows that the rate of cooling of the water is proportional to the mean difference in temperature between the water and its surroundings. This will be true for any other liquids, and this fact was first expressed by Newton, being known as **Newton's Law of Cooling,** which states,—

The rate of loss of heat, i.e., the rate of cooling is proportional to the mean difference of temperature between the body and its surroundings.

Again taking two or three calorimeters and recording the time and temperature as before, it will be seen that *the amount of heat lost per second depends on the extent and natnre of the radiating surface.* This shows that the rate of cooling does not at all depend on the nature of the liquid.

[Vide Art. 50(*b*) for the determination of specific heat of a liquid by the method based on Newton's law of cooling].

109. Prevost's Theory of Exchanges—Experiments on radiant heat have led to the Theory of Exchanges, first stated by Prevost of Geneva which states that a body radiates heat at all temperatures, the amount of which depends upon the *temperature of the body* and the

nature of its surfaces, and not upon its surrounding objects. A red hot ball will radiate the same amount of heat whether placed in a furnace or in an ice-house. Again, an ice ball will radiate equally whether kept in an ice-house or in front of a fire. Again bodies receive heat from the surrounding objects, and the balance of heat of the body is a differential effect. It depends upon the difference between the amount emitted and the amount absorbed. If the amount emitted equals the amount absorbed, the temperature of the body remains constant. When a man stands before a fire, he gains more heat than he loses by radiation, his temp. being less than that of the fire. So he feels hot. It is the differential effect of an exchange.

Questions

Art. 91.

1. Distinguish between conduction and convection of heat. Illustrate the difference by examples. (C. U. 1918, '28, '38, : Dac. 1938)

2. Point out the various ways in which a hot body may lose its heat. What methods would you adopt to reduce the rate at which heat is lost in each of these ways ? (C. U. 1924 ; Pat. 1928)

3. (a) Distinguish between conduction, convection and radiation of heat. Describe experiments to illustrate the distinctions. (Pat. 1940 ; Cf. Dac. 1931, '33)

(b) Of what importance are these in calorimetric determinations, and what arrangements would you make to eliminate their effects ? (Pat. 1932 ; Cf. '28)

4. What are the different methods for the transmission of heat from point to point ? Clearly explain their difference with suitable examples. (C.U. 1931, '41)

Art 92.

5. If you touch a piece of iron and a piece of wood lying exposed to the heat of the sun, which feels hotter and why ? (Dac. 1930 ; Pat. '43)

6. On a cold day a piece of wood and a piece of iron, when touched with fingers, appear to be different at different temperatures, though a thermometer placed successively against each gives the same reading. How do you account for this, and how would you verify your explanation by experiment ? (Pat. 1928)

Art. 93.

7. Define thermal conductivity. Explain the statement that the thermal conductivity of glass is 0·002 C. G. S. units. (All. 1944)

How will you show experimentally that different substances have different conductivities ? (*See* Art. 94) (Pat. 1937 ; '43)

8. State briefly how you would compare experimentally the conductivities of a rod of copper and one of lead. (C. U. 1938)

Art. 94

9. The opposite faces of a cubical block of iron of cross section 4 sq. cm. are kept in contact with steam and melting ice. Determine the quantity of ice melted at the end of 10 minutes, the conductivity of iron being 0·2. (Latent heat of ice = 80 calories). (Pat. 1925)

[*Ans :* 300 gms.]

10. Find the difference in temperature between the two sides of a boiler plate 2 cm. thick, conductivity 0·2 C. G. S. units, when transmitting heat at the rate of 600 kilogram-calories per square metre per minute. (Pat. 1935)

[*Ans :* 10°C.]

11. Explain how heat is propagated through a given body by conduction and define co-efficient of conductivity. (C. U. 1932)

12. Calculate the amount of heat lost through each square metre of the walls of a cottage, assuming that the walls are 42 cm. thick, and that the conductivity of the material is 0·004 C. G. S. units, and that the temperature is 10°C. higher than outside. (L. M. B.)

[*Ans :* 9·5 cals./sec.]

13. Find how much steam per minute is generated in a boiler made of boiler-plate 0·5 cm. thick, if the area of the walls of the fire-chamber is 2 sq. metres ; the mean temperature of the plate-faces 200°C. and 120°C. respectively, the latent heat of steam 522, and the conductivity of the steel plate 0·164. (P. U.)

[*Ans :* 60321 gm.]

14. Heat is conducted through a slab composed of parallel layers of two different materials of conductivities 0·32 and 0·14, and of thicknesses 3·6 cm. and 4·2 cm. respectively. The temperatures of the outer faces of the slab are 96°C. and 8°C. Find the temperature gradient in each portion. (Pat. 1937)

[*Ans :* 6·67°C. and 15·24°C.]

Art. 95.

15. Spheres of copper and iron of the same diameter and of masses 8 : 7 are both heated to 100°C. and placed on a slab of paraffin wax. It is found that copper sinks in more quickly than the iron, but in the end the iron is level with the copper, having melted the same amount of wax ; give an explanation of this. (Pat. 1935)

[**Hints.**—Copper has less specific heat but greater conductivity than iron].

16. One end of a metal bar is heated. Indicate clearly the factors on which the rate of rise of temperature at any point on it depends. (Pat. 1925 ; All. '46)

17. Two metal bars A and B, of the same size but of different material, are coated with equal thickness of wax and placed each with one end in a hot bath. It is noted that at first the wax on A melts at a greater rate than that on B, but that when a steady state has been reached a greater length of wax has been melted on B than on A. Explain this. (C. U. 1941)

Arts. 104 & 106.

18. Discuss, as fully as you can, the grounds on which we conclude that radiant heat is but invisible light. (*See* Art. *102 b*) (C.U. 1912, '33 ; cf Pat. 1929)

19. Describe an experiment showing that thermal radiations are transmitted in straight lines. Show how to prove experimentally that the radiant heat received by a given surface is inversely proportional to the square of the distance of the surface from the source of heat. (Pat. 1920)

20. Describe a convenient apparatus for investigating the laws of reflection and refraction of heat and give the general results arrived at. (All. 1932 ; Pat. '45)

21. State the laws which govern the transmission of heat by radiation.

You are given two ordinary clear glass thermometers ; and the bulb of one of them is coated with lampblack : compare their readings when exposed (1) on a damp cloudy night, (2) on a clear dry night in the cold weather, (3) in the sun. (Pat. 1932 ; cf. '42 ; All. 1919)

[**Hints.** (1) On a cloudy night the free radiation from the earth being obstructed, the atmosphere becomes warmer, and as the lampblack-coated bulb absorbs heat to a greater extent, the temperature recorded by it is *higher* than that of the other. (2) On a clear night the black bulb radiates to a greater extent and so its temperature will be *lower* than that of the other : (3) In the sun the black bulb absorbs heat radiation to a greater extent and so will indicate a *higher* temperature.]

Art. 107

22. Explain (*a*) why it is of advantage to paint the roof of a house white in the hot weather ; (*b*) the principle and construction of the thermo-flask. (Pat. 1939 ; cf. All. '45 ; M. U.)

Art 108.

23. Enunciate Newton's law of cooling. How would you verify it experimentally ? (All. 1923, '25)

Art. 109.

24. Give an outline of Prevost's Theory of Exchanges. (All. 1945)

The bulbs of two identical thermometers are coated, one with lampblack and the other with silver. Compare their readings (*a*) when in a water bath in a dark room, (*b*) when in the sun, (*c*) when exposed on a clear night. (Pat. 1927, cf. '24)

[**Hints**. (*a*) Both will indicate the temperature of the bath ; (*b*) The temperature of the lampblack-coated thermometer will be higher as it will absorb more heat rays than the silvered surface ; (*c*) the temperature of the lampblack-coated thermometer will be lower than the other as it will absorb more cold when temperature of the atmosphere will go down at night.]

CHAPTER IX

Mechanical Equivalent of Heat. Heat Engines.

110. Nature of Heat. (*Caloric Theory*)—The old idea as to the nature of heat was that it was a weightless invisible fluid called *caloric*, which, according to the supporters of the caloric theory, was present in every substance in large or small quantities, which again rendered that substance hot or cold. The fluid was assumed to be given up by a hot body when placed in contact with a cooler one. The heat produced by compression or hammering was explained by supposing that caloric was squeezed out of the body like water of a sponge. Again, the heat produced by friction, as for example, by rubbing two bodies together, was explained by stating that in addition to the caloric squeezed out, the thermal capacity of a substance is less in the powder form than when taken in large masses, and so the particle did not require so much caloric to maintain the former temperature ; so some heat was given up which raised the temperature of the fine particles and the rubbed bodies.

Rumford's Experiments.—The first blow to the caloric theory was given by Count Rumford in 1798, while superintending the boring of cannon at the Munich Arsenal. He observed that a large amount of heat was developed both in the cannon and in the drill, which was apparently unlimited. He arranged to revolve a *blunt* drill in a hole in a cylinder of gun metal weighing 113 lbs., by which the temperature rose up to 70°F. though the weight of the metallic dust rubbed off the cylinder was only 2 ounces. It occurred to him that the only other source from which heat could be received was air. So, in order to avoid the effect of the atmosphere, he repeated his experiment by surrounding the cylinder with $2\frac{1}{2}$ gallons of water which began to boil after some time. It appeared impossible that such a large amount of heat could be liberated by such a small quantity of borings by a mere change in thermal capacity. This heat, he argued, could not also come from the water ; for water was only gaining heat. Rumford observed that the supply of heat produced by friction was unlimited, and he stated that anything which could furnish heat *without limitation* could not be a *material substance*. **Heat was**, according to him, not due to something material as the calorists thought, but **a kind of motion**.

Davy's Experiment.—The final blow to the caloric theory was given by Sir Humphery Davy by rubbing together in a vacuum two pieces of ice, which melted to form water even when the initial tem-

perature of the ice and its surroundings was 29°F., *i.e.*, below the freezing point. Davy states, "From this experiment it is evident that ice by friction is converted into water, and, according to the calorists, its capacity is diminished, but it is a well-known fact that the capacity of water for heat is much greater than that of ice, and ice must have an absolute quantity of heat added to it before it can melt. Friction consequently does not diminish the capacities for heat."

Inspite of these experiments scientists continued to support the caloric theory till 1850, when the **Dynamical Theory of Heat** was established by the experiments of Dr. Joule of Manchester, who not only showed that heat was a form of energy, but found the exact relation between heat and other forms of energy.

111. Mechanical Equivalent of Heat. (*Heat and Work*).—It has been found that when two bodies are rubbed against each other heat is produced at the expense of the work done. Similarly, when a body, falling from a height, strikes against the ground, it loses its kinetic energy acquired during the fall, which is converted into heat. Again, heat energy can be transformed into work as in the case of a steam engine. The mechanical work of the steam engine is performed at the expense of the heat energy due to the combustion of coal.

Every cyclist knows that at the time of pumping his tyres the pump grows hot. This is due partly to the friction of the piston against the walls of the cylinder, but chiefly to the fact that the downward motion of the piston is transferred to the molecules of air coming into contact with it, which has the effect of increasing the velocity of these molecules. These molecules colliding with the advancing piston rebound with increased velocity which is so great that the temperature of a mass of gas at 0°C., when compressed to one half of its former volume, rises to about 87°C.

Shooting stars, meteorites are also other interesting examples. These aro pieces of matter, cold to begin with, which are attracted by the earth through the surrounding space. They run through the atmosphere with such enormous speed that there is rapid compression of gases in the atmosphere and, as a result of the work done, the rise of temperature is so high that these pieces of matter become luminous, and very often burn away altogether.

As a result of experiments like these, we have the **First Law of Thermodynamics**, which states.—

When work is transformed into heat, or heat into work, the amount of work is mechanically equivalent to the quantity of heat.

That is, if W = work done ; H = heat produced, then $W \propto H$,

or, $\frac{W}{H} = J$, a constant, or $W = JH$,

where J is called the Mechanical Equivalent of Heat (or Joule's Equivalent). *It is the work equivalent to unit heat.*

The value of J is given by 4.2×10^7 ergs per calorie, if W be measured in ergs and H in calories. So, *4.2×10^7 ergs of work are necessary to produce one calorie of heat or one calorie of heat can develop 4.2×10^7 ergs of work.*

112 Determination of J. (*a*) (*Joule's Experiment*).

The first exact determination of the quantitative relation between heat and work, *i.e.* the *mechanical equivalent of heat* was made by Dr. Joule.

His apparatus (Fig. 66) consists of a special copper calorimeter C in which a paddle can be rotated, by means of a central spindle, between the stationary vanes soldered to the inner side of the calorimeter. The paddle is rotated by means of two cords wound round the wooden cylinder D at the top of the central spindle S. The other ends of the cords are attached to two large pulley P, P, which are rotated by two equal wheghts M and M. The spindle S is attached to the wooden cylinder D by means of the removable pin T.

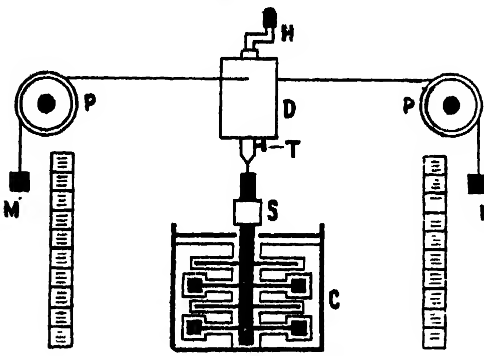


Fig. 66

A known quantity of water is put into the calorimeter and its temperature noted by a sensitive Hg-thermometer introduced into calorimeter through an aperture in the lid. To start with, the pin T is detached and cylinder D is rotated by the handle H till the wts. M, M are raised to a definite height. The pin is replaced and the wts. allowed to descend. The paddle turns and gives a churning motion to the water, which being resisted by the fixed vanes, is heated up. The final temperature of the water is noted, after a number of falls of the wts.

Calculation.—In order to produce appreciable rise of temperature, the weights are raised and allowed to fall serveral times.

Let m = mass of water in the calorimeter ; M = mass of each weight ; h = height through which each weight falls ; n = number of falls ;

w = water equivalent of the calorimeter ;

t° = rise of temperature of water in the calorimeter ;

v = velocity acquired by the weights on reaching the ground.

$\therefore$ Potential energy in the beginning, for both the weights $= 2Mgh$.

Kinetic energy just before striking the ground $= 2 \times \frac{1}{2}Mv^2$.

Total energy used $= n(2Mgh - Mv^2)$ ergs ; and heat produced $= (m + w)\ t$ cal.

$$J = \frac{W}{H} = \frac{n(2Mgh - Mv^2)}{(m + w)t} \text{ ergs per calorie.}$$

Errors and Corrections :—

Joule had to make various corrections in order to get a reliable result. He corrected for the kinetic energy of the scale pans with their loads at the end of each fall and made allowance for the energy converted into sound. Corrections were also made for the losses due to conduction, radiation, etc., and for the energy absorbed by friction.

The **defects** of his expt. were : (1) Joule, on the authority of Regnault, assumed Sp. heat of water to be the same at all temps., (2) the Hg -thermometer, he used, was not calibrated with reference to any standard thermometer, such as a gas thermometer, (3) the rise of temp. attained in his expt. was very small.

The final mean result of the value of J given by Joule was 773·4 ft.-lb. per B. Th. U. But, by later investigators, it was found that Jaule's result was rather low, and the accepted result was 778 ft.-lb. per B. Th. U.

(b) **Searle's (or Friction Cones) Method.**—The apparatus (Fig. 67) essentially consists of two conical brass cups A and B, one of which fits closely into the other. The lower cup B is fixed on a non-conducting base, which again is fixed on the top of a vertical spindle S. The spindle can be rotated by a hand wheel or a motor. A circular wooden disc CC is fixed to the inner cup A and a string wound round the circumference of the disc passes over a pulley and carries a suitable weight W at the other end. When the cup B is rotated, A is prevented from doing so by the tension of the string, and the speed of rotation, which is counted by a speed counter, attached to the spindle, is so

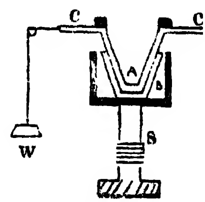


Fig. 67.

adjusted that the weight W hangs stationary, the tension of the string acting as a tangent to the disc. Now the work done against friction between the surfaces of the cups, *i.e.* the mechanical energy, is converted into heat which raises the temperature of the cups and a known mass of water taken in A. A thermometer in A records the rise in temperature.

Calculation.—Here for steady suspension the moment of the average frictional force F between the cups is equal to the moment due to the force $W(=Mg)$, where M is the mass of the wt. The former $=F.a$, where a is the mean radius of the surfaces of the cups which are in contact. So, $Fa = Mg \times r$, where r is the radius of the disc. Now the work done in ergs for each revolution $= 2\pi a \times F$. For n revolutions the work $= 2\pi naF = 2\pi nMgr$ ergs.

Again, if m be mass of water in the cup, w the water equivalent of the cups, and $t°C.$ the rise in temperature, the heat developed by rotation, $H = (m+w)t$ calories.

Hence the mechanical equivalent, $J = \dfrac{2\pi nMgr}{(m+w)t}$ ergs per calorie.

Otherwise thus:—The work can be calculated also thus :—The work done for n turns of the spindle in overcoming the friction between the cups is the same as would have been spent if the spindle and the outer cup B had been kept stationary and the inner cup A had been made to rotate by the wt. $W(=Mg)$ making n revolutions of the disc slowly.

The mechanical work done $=$ (force $\times$ distance) $= Mg \times 2\pi rn$

$$\therefore \quad J = \frac{\text{work done}}{\text{heat developed}} = \frac{2\pi nMgr}{H}.$$

Radiation is the chief source of error in this experiment to reduce which the cups must be brightly polished.

(c) Simple Laboratory Method for Determining J.—The following example illustrates another method for determining the value of J.—

A cylindrical tube 15 cms. long made of a non-conducting material, closed at both ends, contains 500 gms. of lead shot, which, when the tube is held vertically, occupy 6 cms. of the tube length. The tube is suddenly inverted so that the end originally above is now below, and the shots fall to the other end of the tube. The tube is then again quickly inverted and the process is repeated 200 times. At the end of this process the temperature of the shots is found by means of a thermometer to be 1·4°C. higher than it was at the beginning of the experiment. Find the value of the mechanical equivalent of heat. (Sp. ht. of lead is 0·03. It is assumed that no heat is lost by radiation or conduction). (C. U. 1910).

The lead shots fall through a height $=(15-6)$ cms. each time the tube was inverted. Hence the loss of potential energy for each time.

$$=500\times 981\times(15-6) \text{ ergs.}$$

$\therefore$ The total loss $=200\times 500\times 981\times(15-6)$ ergs.

Heat developed in lead shots $=500\times 0{\cdot}03\times 1{\cdot}4$ cal.

$\therefore$ Mechanical equivalent $(J)=\dfrac{\text{work done}}{\text{heat developed}}$

$$=\frac{200\times 500\times 981\times(15-6)}{500\times 0{\cdot}03\times 1{\cdot}4}\text{ ergs.}$$

$$=4{\cdot}2\times 10^7 \text{ ergs (nearly)}$$

(d) Electrical Methods.—See Art. 47, Ch. V, Part VII.

113. **Another Relation.**—The kinetic energy of a body of mass mass m moving with velocity $v=\frac{1}{2}mv^2$.

Heat developed H when the body meets an obstacle and stops suddenly $=mst$, where s is the specific heat of the body, and t the rise in temperature by the impact.

Then $\quad H \propto$ kinetic energy $\propto \frac{1}{2}mv^2$;

or $\quad JH=\frac{1}{2}mv^2$; or $J\times mst=\frac{1}{2}mv^2$;

or $\quad J=\dfrac{v^2}{2st}$.

The value of J obtained was 778 ft.-lbs., which means that 778 ft.-lbs. of work will produce unit amount of heat,—the heat required to raise 1 lb. of water through 1°F.

114. **The Value of J in Different Units.**—

J = 778 ft.-lbs. per pound-degree Fahrenheit unit of heat (*i.e.* per B. Th. U.)

$=778\times\frac{9}{5}=1400$ ft.-lbs. per pound-degree C. unit of heat (C. H. U).

$$=\frac{1400\times 30{\cdot}48 \text{ cms.}\times 453{\cdot}6 \text{ gms.-wt}\times 981}{453{\cdot}6}$$

$=4{\cdot}186\times 10^7$ ergs per calorie

$=4{\cdot}186$ Joules per calorie ($\because$ 1 Joule $=10^7$ ergs.)

The most approximate value of J is taken to be **$4{\cdot}2\times 10^7$ ergs per calorie.**

Example.—*1. How much work is done in supplying heat necessary to convert 10 gms. of ice at -5°C. into steam at 100°C.* (*All. 1917.*)

(1) Heat necessary to convert 10 gms. of ice at -5°C. into ice at 0°C $=10\times 0{\cdot}4\times 5=25$ cal. (Sp. ht. of ice $=0{\cdot}5$). (2) Heat necessary to convert 10 gms.

of ice at 0°C. into water at 0°C. $= 10 \times 80 = 800$ cal. (3) Heat necessary to convert 10 gms. of water at 0°C. into water at 100°C. $= 10 \times 100 = 1000$ cal. (4) Heat necessary to convert 10 gms. of water at 100°C. into steam at 100°C. $= 10 \times 536 = 5360$ cal.

$\therefore$ The total heat necessary $= 7185$ cal.

Work done $= J \times H = 4{\cdot}2 \times 10^7 \times 7185$ ergs. $= 3{\cdot}0177 \times 10^{11}$ ergs.

2. *If a lead bullet be suddenly stopped and all its energy employed to heat it, with what velocity must the bullet be fired in order to raise the temperature to 100° C. the specific heat of lead being 0·0314.*

Let m be the mass of the bullet in kilograms and v its velocity in cms. per second ; then its kinetic energy $= mv^2/2$ ergs.

And heat produced (H) when the bullet is stopped $= m \times 0{\cdot}0314 \times 100$ cal.

$$4{\cdot}2 \times 10^7 = \frac{mv^2}{2} + (m \times 0{\cdot}0314 \times 100) \text{ whence } v = 162{\cdot}407 \times 10^2 \text{ cm.}$$

per sec

3. *A lead-ball, dropped from an aeroplane at a temperature of 15°C, just melts on striking the ground. Supposing the whole of its kinetic energy is converted into heat, find out the height of the aeroplane at the moment at which the ball is dropped (Sp. ht. of lead = 0·03 ; melting point of lead = 350°C. ; latent heat of lead = 35 calories.)* (*Pat. 1932*)

If h be the height of the aeroplane, the loss of potential energy of the ball $= m \times 981 \times h$ ergs. This is converted into heat, which first raises the temperature of the ball $(350 - 15)$°C. and then melts it.

$\therefore$ The total heat developed $= m \times 0{\cdot}03 \times 335 + m \times 35$ cal. $= m \times 45{\cdot}05$ cal.

This is equal to $(m \times 981 \times h) + J$; So $4{\cdot}2 \times 10^7 = 981 \times m \times h + m \times 45{\cdot}05$ whence $h = 19287{\cdot}4616$ metres.

4. *If the mechanical equivalent of heat be 779 foot-pound-Fahrenheit unit, from what height must 10 lb. of water fall to raise its temperature by 1°C. ?* (*Pat. 1940*)

Let h ft. be the required height. Rise in temperature $= 1°C. = \frac{9}{5}°F.$

$\therefore$ Heat produced $H = 10 \times 1 \times \frac{9}{5} = 18$ (B. Th. U.)

Work done, $W =$ force $\times$ distance $= 10 \times h$ ft.-lbs.

$$\therefore \quad J = \frac{10 \times h}{18} \text{ ; or } 779 = \frac{10 \times h}{18} \text{ ; or } h = 1402{\cdot}2 \text{ ft.}$$

5. *With what velocity must a lead bullet at 50°C. strike against an obstacle in order that the heat produced by the arrest of the motion, if at all produced within the the bullet, must be just sufficient to melt it ? (Sp. ht. of lead = 0·031 : melting point of lead = 335°C., latent heat of fusion of lead = 5·37).* (*C. U. 1930 ; Dac. '32*)

The available energy $= \frac{1}{2}mv^2$. This is converted into heat by which m gms. of lead is raised in temperature from 50°C. to 335°C. and then melted. The amount of heat developed $(H) = m \times 0{\cdot}031 \times (335 - 50) + m \times 5{\cdot}37 = m \times 14{\cdot}2$ calories.

We have, $JH =$ work or energy. $\therefore$ $4{\cdot}2 \times 10^7 \times m \times 14{\cdot}2 = \frac{1}{2}mv^2$ whence $v = 34{\cdot}5 \times 10^4$ cms. per sec.

6. A meteorite weighing 2000 kilograms falls into the sun with a velocity of 1000 kilometres per second. How many calories of heat will be produced? ($J=4{\cdot}19 \times 10^7$ ergs per calorie). (*C. U. 1926*)

$$m = 2000 \text{ kilograms} = (2000 \times 1000) \text{ gms.}$$

The kinetic energy of the meteorite $= \frac{1}{2}mv^2$

$$= \tfrac{1}{2} \times (2000 \times 1000) \times (1000 \times 1000 \times 100)^2 \text{ ergs.} = 10^{22} \text{ ergs.} = JH.$$

$$\therefore \quad H = \frac{10^{22}}{4{\cdot}19 \times 10^7} = 2{\cdot}38 \times 10^{14} \text{ calories.}$$

7. The mechanical equivalent of heat is 4·2 Joules per calorie. By what number should it be expressed in the F. P. S. system; [Assume 1 kilogram = 2·2 lbs. and 1 inch = 2·5 cms.] (*Pat. 1942*).

1 calorie = Heat required to raise the temperature of 1 gm. of water through 1°C.; 1 gm. $= 1/10^3$ kg. $= (1/10^3) \times 2{\cdot}2$ lb.; $1°\text{C.} = \frac{9}{5}°\text{F.}$

$$\therefore \quad 1 \text{ calorie} = \frac{2{\cdot}2}{10^3} \times \frac{9}{5} \text{ lb.-degree Fahrenheit (or B. Th. U.)} \qquad \ldots \quad (1)$$

We have $J = 4{\cdot}2$ Joules per calorie $= 4{\cdot}2 \times 10^7$ ergs per calorie

$$= \frac{4{\cdot}2 \times 10^7}{981} \text{ gm. cm. per calorie} = \frac{4{\cdot}2 \times 10^7}{981 \times 10^3 \times 2{\cdot}5} \text{ kg. inch. per calorie}$$

$$= \frac{4{\cdot}2 \times 10^7 \times 2{\cdot}2}{981 \times 10^3 \times 2{\cdot}5 \times 12} \text{ ft.-lb. per calorie}$$

$$\therefore \text{ from (1)}, \quad J = \frac{4{\cdot}2 \times 10^7 \times 2{\cdot}2 \times 10^3 \times 5}{981 \times 10^3 \times 2{\cdot}5 \times 12 \times 2{\cdot}2 \times 9} \text{ ft.-lb. per B. Th. U.}$$

$$= 792{\cdot}9 \text{ ft.-lb. per B. Th. U.}$$

115. Work done by a Gas.—The work done *by* a gas where it expands at constant pressure is equal to the product of the pressure and the increase in volume. And, when a gas contracts at constant pressure, the work done *on* the gas is equal to the product of the pressure and the decrease in volume.

Let a certain amount of gas of volume v_1 c.c. be enclosed in a cylinder fitted with a piston of area A, and let p be the uniform pressure acting on the piston. Therefore, the force on the piston $= A \times p$. Suppose the gas expands, the increased volume being v_2, and the piston is moved through a distance x cms. Then the work done by the gas $=$ force $\times$ distance $= p \times A \times x$ ergs $= p \times (v_2 - v_1)$ ergs.

Thus, the *work done by a gas during expansion is equal to the product of the pressure and the increase in volume.* Similarly the other result will be obtained.

Example.—*How much work is done against uniform pressure when 1 gm. of water at 100°C. is converted into steam. Express your result in calories.* (*All. 1918*)

The pressure at which 1 gm. of water at 100°C. changes in to steam is 76 cms. of mercury. This pressure $= 76 \times 13{\cdot}6 \times 981$ dynes per sq. cm.

When water is changed into steam, its volume is increased 1670 times. So the volume of steam formed out of 1 c.c. of water is 1670 c.c. Hence the work done $= 76 \times 13{\cdot}6 \times 981 \times 1670$ ergs.

This is equivalent to $\dfrac{76 \times 13{\cdot}6 \times 981 \times 1670}{4{\cdot}2 \times 10^7}$ calories. $= 40{\cdot}52$ cal.

115. (*a*) **Energy given out by Steam.**—The high value of the latent heat of steam shows that when steam condenses, a tremendous amount of heat is given out, some of which is converted into work as in the case of a steam engine (Art. 116).

We have already seen (vide Art 53) that 1 lb. of steam in condensing at 100°C. would liberate about 965 B. Th. U. units of heat, which would raise the temperature of 965 lbs. of water through 1°F. Each B. Th. U. is equivalent to 778 ft.-lb. of work. So the enery given out $= 778 \times 965 = 750,770$ ft.-lb.

This means that the above amount of energy which is liberated by 1 lb. of steam is also derived by a mass of $= \dfrac{750,770}{2240}$ tons $= 335$ tons (nearly) in falling through 1 foot. So the same amount of energy must be necessary in boiling away 1 lb. of water.

We have seen also in Art. 53 that 144 B. Th. U. will be necessary in melting 1 lb. of ice, which is equivalent to 112,032 ft.-lb. This energy will be liberated by a mass of $\dfrac{112,032}{2240} = 50$ tons (nearly) in falling through 1 foot.

HEAT ENGINES

116. The transformation of mechanical energy into heat has already been explained. Now we shall deal with the reverse process, that is, the conversion of heat into mechanical energy. The machines by which this is done are called *Heat Engines*, which include the Steam Engines, Steam Turbines, Internal Combustion Engines, such as Oil Engines, Petrol Engines, etc.

The Steam Engine :—

In 1768 James Watt of England invented the Steam Engine. The

following must be the essential parts of a Steam Engine, though Engines of to-day may differ considerably in details of construction.

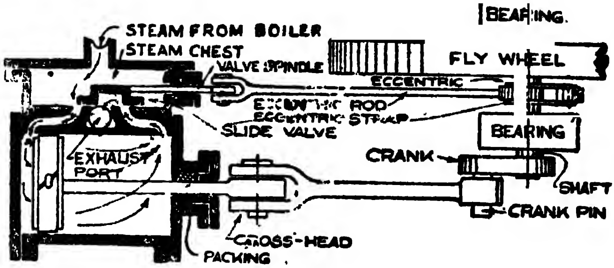


Fig. 68—Steam Engine.

(1). A Boiler.—

Steam is raised in this plant which may be either of the *smoke-tube* type or of the *water-tube* type. For engines of large horse-power, superheated steam at high pressure is produced by the boiler. The steam from the boiler is led through a tube into a chest, called the *steam-chest* or *valve-chest*. This supply-tube is provided with a valve, called the stop-valve for regulation of steam.

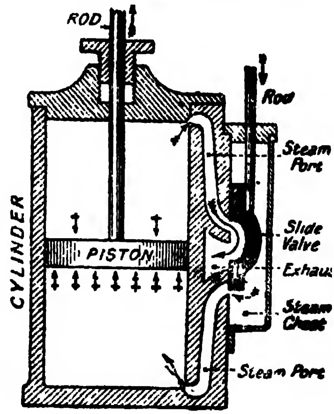


Fig. 69

(2) The steam or valve chest.—

It is a rectangular stout box (fig. 68) mounted on the *cylinder* of the engine. It has three openings or *ports*. The middle one is connected with the *exhaust pipe* while through the two side ones the steam chest communicates with the cylinder. These two communicating ports are alternatlely closed and opened by means of what is called the *Slide valve*.

(3). The slide valve.—

It has a variety of forms, D-valve (shewn in the figure), piston valve, drop-valve, etc. Its function is to direct the steam into the cylinder through the two communicating ports alternately so that the *piston* which works in the cylinder is acted on from both sides in turn producing a to-and-fro (reciprocating) motion. It is provided with a

spindle driven by a connecting rod joined to an *eccentric disc* mounted on the main shaft of the engine.

(4). The cylinder and the piston.—

A steam-tight *piston* usually of cast steel, works inside the *cylinder* which is a cylindrical vessel of high strength and which communicates with the steam chest through the two communicating ports. Its spindle called the piston rod works through a *packing* or *stuffing* box with which the front end of the cylinder is provided and is joined to the *driving rod* at the *cross-head* which moves along a fixed grove in a *guide* producing a straight line motion. The driving rod is connected to the *crank* by the crank pin. The crank which is mounted on the shaft is a contrivance for converting the to-and-fro motion of the piston rod into a circular motion of the shaft.

5. The fly-wheel.—

It is a large and massive wheel mounted on the shaft. The turning effort of the shaft produced by the crank is not constant during a revolution. It is this fly-wheel which keeps the speed of the shaft constant by smoothening down the variation by means of its large moment of inertia.

6. The governor.—

On change of load, the speed of the engine must vary. To keep the speed approximately constant on all loads, a self-acting machinery called the *governor*, driven by the main shaft is used. It is connected by a system of levers to a regulating valve, one form of which is called the *throttle valve.* The revolving balls with which the governor is provided rise or fall according as the rated speed increases or decreases. This rise or fall of the balls operates a sleeve which communicates through the lever system with the throttle valve and accordingly steam-supply is so reduced or increased as to keep the speed constant.

Principle of Action.—Here heat energy of steam is transformed into mechanical work through expansive action.

High pressure steam from the boiler is led into the steam-chest whence it enters into the cylinder. When steam enters the cylinder through the lower steam port (fig. 69). the slide valve covers the *exhaust* and the *upper port* so that these two are put into communication. The pressure of the steam due to its expansion action pushes the piston forward and forces out the cushion system on the other side through the exhaust. The movement of the piston rotates the crank shaft whereby the motion is communicated through the eccentric disc to the slide valve which moves *opposite to the piston.* The valve covers up the lower port and the exhaust by the time the piston reaches the forward end. The steam now enters through the upper

port and the same action as in the previous stroke occurs but the motions are all reversed. These two strokes, forward and backward forms a cycle of operations which is repeated successively. The to-and-fro motion of the piston is transformed into a rotatory motion of the shaft by means of the crank. Twice during each revolution of the shaft, once at the top and again at the bottom of each stroke of the piston, the crank and the connecting rod are in the same straight line, when there is no turning effect on the shaft. These positions are called the *dead centres* or *dead points*. Again at two positions during each revolution, the crank is at rt. angles to the connecting rod when the turning effect is maximum. The heavy flywheel carries the shaft through the dead centre positions and smoothens out the variation of speed. The change of speed due to change of load on the engine is regulated by the governor which is driven by the main shaft Its balls rise or fall as speed increases or decreases. This rise or fall of the balls operates a sleeve which communicates with a throttle valve and accordingly steam supply is reduced or increased so as to keep the rated speed constant. It is called a **Double-acting** engine, as the steam acts on both the faces of the piston.

Condensing and Non-condensing Engines.—The engine in which the steam passing through the exhaust-pipe escapes into the atmosphere is called a *non-condensing engine* ; and the engine in which the exhaust steam is led into a vessel called a *condenser*, where it is condensed at a low temperature and pressure, is called a *condensing engine*. When the steam exhausts into such a condenser where the pressure is kept low, (*i.e.* not more than a pound to the square inch), the back pressure against that end of the piston which is open to the atmosphere is reduced from 15 lb. down to 1 lb. and in that case the effective pressure, which the steam on the other side of the piston can exert, is increased.

Single and Double-acting Engines.—The engine, we have already considered, is a double acting one as here steam pressure acts on both side of the piston. In a single acting engine, steam pressure acts on one side and atmospheric pressure acts on the other side of the piston.

Efficiency of an Engine.--The **thermal** *efficiency of an engine is measured by the ratio of the mechanical work done per hour to the work equivalent of the energy supplied by steam per hour.*

$$i.e.\quad \text{Thermal efficiency} = \frac{\text{work units produced}}{\text{work units supplied by steam}}.$$

The thermal efficiency of a good steam engine is not even more than 20%, and that of an ordinary locomotive is only about 10%. The mechanical losses of energy are mainly due to friction of the moving parts of the engine.

Any device, which can do work at the rate of 550 ft.-lbs. per second, or 33,000 ft.-lbs. per minute, is said to be of one 'horse-power'. Thus a 10 H. P. engine will do work at the rate of (10 × 33000) ft.-lbs. of work per minute.

117. Internal Combustion Engines.—The engines used in aircraft, motor-cars, oil-engines, etc., are known as *Internal Combustion Engines*, so named because the combustion of the fuel is carried out inside the cylinder of the engine, and not separately as in the boilers of steam engines. So internal conbustion engines occupy less room and are specially suitable for small power purposes.

The general arrangement of the cylinder and piston in the case of internal combustion engines is almost the same as in the steam engine, but whereas in the steam engine the piston moves by the force of expanding steam, in the internal combustion engine the movement of the piston is produced by the explosive force generated by the combustion of a fuel, supplied in the form of a vapour, with air. The fuel used is either gaseous—such as *coal-gas, town gas, etc.* ; or a liquid, such as *petrol*, *benzene*, and *alcohol etc.*, which are readily vaporised, and every one of which, when vaporised forms an explosive mixture with air.

A *gun firing a bullet* is an example of a simple internal combustion engine. Here the spark produced by striking the horse against the cap explodes the powder and converts it into hot gases which drive the bullet with a great force.

Principle of Action.—The internal combustion engines are generally four stroke engines, *i.e.* they require four strokes of the piston to complete a cycle of operations within the cylinder. There are also *two-stroke* engines which are often used in motor boats. But the engines commonly used in automobiles, æro-planes, etc., are all four-stroke engines. The operations of the four-stroke internal combustion engines are understood by reference to Fig. 70.

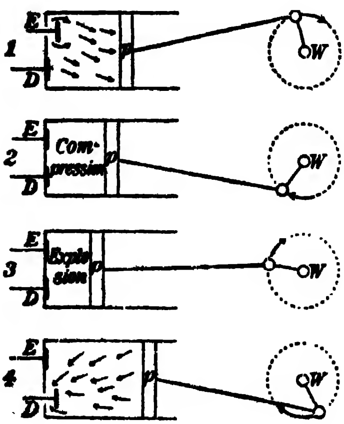


Fig. 70.—Exhaust Stroke.

(1) **First Stroke**—(*Charging stroke*).—The piston moves out-wards and draws into the cylinder an explosive mixture by air and gaseous fuel through the inlet valve E which then remains open.

(2) **Second Stroke**—(*Compression*

stroke).—The piston makes its return stroke, *i.e*, moves inwards and compresses the explosive mixture, the valves (Admission valve E and Exhaust valve D) being closed.

(3) **Third Stroke.**—(*Working stroke).*—At the beginning of this stroke, the mixture is ignited by electric spark and explosion occurs. The piston is driven outwards by the expansive action of the gases. and energy is communicated to the fly-wheel for the engine to do work.

(4) **Fourth Stroke.**—(*Exhaust stroke).*—The piston moves inwards and spent gases are forced out of the cylinder through the exhaust valve D which then remains open.

In the above sequence of operations, which is called a *cycle*, the engine is fired only once in the course of two forward and two backward motions of the piston. For this reason, an engine working on this plan is called a *four-stroke engine.*

The Petrol Engine.—There is no difference in principle between a petrol engine and any gas engine, but the former is more compact and light. So petrol engines are commonly used in motor-cars and airships. In Fig. 71 is shown the diagram of a petrol engine where there is a *piston* working in a *cylinder* as in a steam engine. Above the cylinder there is a chamber called **combustion chamber**, where a mixture of air and petrol vapour is ignited by means of electric sparks from **sparking plugs** fitted into the chamber. The entry of the fuel into the chamber by the *inlet pipe* and exit of burnt gases by the *exhaust pipe* are controlled by two valves (V_1 and V_2) of mushroom type held down on their seats by springs and lifted at proper moments by **cams** (C_1 and C_2) fixed on a rotating shaft driven by the engine itself. The cylinder is water-jacketed in order to prevent the temperature rising beyond about 180°F.

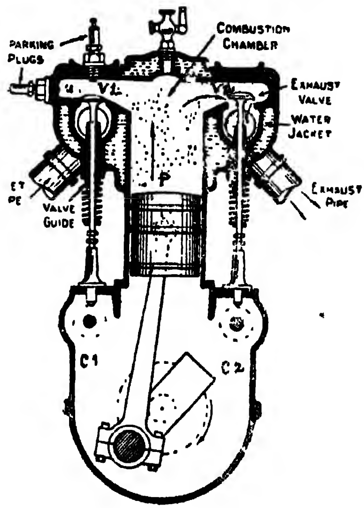


Fig. 71—Petrol Engine

The explosive mixture of petrol vapour with a correct proportion of air is formed in an arrangement known as a **carburettor**, and air charged with petrol vapour is said to be *carburetted*.

The current for the ignition of the charge is supplied by a **magneto** which is a magneto-electric machinery driven by the engine itself.

A petrol engine, as in a motor-car or æro-plane, is provided with a bank of cylinders, usually a multiple of two. The pistons of all the cylinders contribute their efforts to the same main shaft through their individual cranks (which are fitted on the shaft at equal angular intervals).

The Gas Engine.—A *Gas Engine* employing about one part by volume of coal gas and eight parts of air works like a petrol engine and is driven by properly timed explosions of the mixture of gas and air occurring within the cylinder. The ignition of the explosive mixture is effected by contact with the hot walls of a metal tube or by means of an electric spark.

A Gas Engine and a Steam Engine compared.—Though the fuel used in a gas engine is comparatively expensive, still a gas engine is better for the following reasons ;—(*a*) its efficiency is much higher than that of a steam engine ; (*b*) it occupies smaller space and it is free from smoke.

The Oil Engine.—In an *oil engine*, the oil which is used as fuel is supplied in the form of spray into a *vaporiser* tube—a red-hot metal tube, and at the same time air is also admitted there. The oil is converted into vapour, and the mixture of vapour and hot air explodes either with or without the help of a spark. Hot gases are produced in a small space due to which the pressure becomes high, and so the piston is drawn with considerable force.

In **Diesel Oil Engines,** named after the inventor, the cycle of operations works in the following way :—

At the *first stroke* air only is sucked in at a pressure less than the atmospheric pressure, and at the *second stroke* the air is very strongly compressed, keeping all the valves closed, so that much heat is developed within the cylinder. At the beginning of the *third stroke*, oil in the form of vapour is injected at very high pressure into the cylinder, which coming in contact with the intensely heated air of the cylinder takes fire spontaneously. After this, the volume expands and work is done. During the *fourth stroke*, the exhaust opens and the burnt gases escape.

Examples.—(1) *An engine consumes 4 lbs. of coal per horse-power per hour. The heat developed by the combustion of 1 lb of coal is capable of converting 15 lbs of water at 100°C. into steam at 100°C. What percentage of the heat produced is wasted ?*

The heat produced per hour $= 4 \times 15 \times 536$ pound-degree-centigrade units. The work equivalent to this heat $= 4 \times 15 \times 536 \times 1400$. ($J = 1400$ ft.-lbs. per pound-degree-centigrade unit)

The work done by the engine in 1 hour $= 33000 \times 60$ ft.-lbs.

(1 H. P. = 33000 ft.-lbs. per minute). $\therefore$ The efficiency of the engine

$= \dfrac{33000 \times 60}{4 \times 15 \times 536 \times 1400} = 0{\cdot}043$ or $4{\cdot}3$ per cent. That is, $4{\cdot}3$ per cent. of heat produced is converted into work. $\therefore$ $100 - 4{\cdot}3 = 95{\cdot}7$ per cent. is wasted.

(2) *What would be the horse-power of a steam-engine which consumes 2 lbs. of coal per hour assuming that all the heat supplied is turned into useful work? (1 lb. of coal gives 125000 B. Th. U., each of which is equivalent to 770 ft.-lbs.)*

Amount of heat available per hour = (12,5000 × 200) B. Th. U.

Equivalent amount of work = 12,5000 × 200 × 770 ft.-lbs. per hour.

$\therefore$ Work done per minute $= \dfrac{125000 \times 200 \times 770}{60}$ ft.-lbs.

Horse-power $= \dfrac{125000 \times 200 \times 770}{60 \times 33000} = 972$ (approx.).

Questions

Art. 110.

1. Explain what is meant by saying that heat is a form of energy. (Pat. 1926 ; Dac. '28 ; '30 ; C. U. '41)

2. Give an outline of the arguments which led to the conclusion that heat is a form of energy. (Cf. C. U. 1937 ; All. '18, '32)

Art. 111.

3. Explain why does a falling body become hotter when it strikes the ground (Dac. 1927)

4. Explain why does a bi-cyle pump get heated when the tyre pumped ? (Dac. 1932)

5. Describe experiments to establish the connection between heat and work and deduce from them the idea of mechanical equivalent of heat. (Dac. '27 ; Pat. 1930, '42)

6. State the First Law of Thermodynamics. What experiments would you perform to demonstrate the truth of the law ? (Pat. 1932. '42)

7. A mass weighing 2000 grammes falls from a height of 300 cm. If all the energy is converted into heat, find the amount of heat developed. (Mechanical equivalent of heat $= 4{\cdot}2 \times 10^7$). (C. U. 1920)

[*Ans :* 14 calories.]

Arts. 111 & 112.

8. Define mechanical equivalent of heat. Describe a method of finding it experimentally. (Dac. '27 ; All. 1916, '17, '24, '28 ; C.U. 1936, '39, '41 ; Pat. 42, '44).

9. Calculate the difference in temperature of the water at the top and the bottom of a waterfall where the height is 200 metres.

[*Ans :* 0·46°C.]

10. An engine of 1 horse-power is used in boring a block of iron of mass 1000 lbs. Assuming the whole of the work done by the engine is used up in heating the mass of iron, calculate approximately the rise in temperature of the iron after the engine has been working for 20 minutes. (The number of units of work required to raise the temperature of 1 lb. of water 1°F. = 772 ft.-lbs. The sp. heat of iron = 0·1 ; 1 horse-power = 550 ft.-lb. per sec.) (C. U. 1909)

[*Ans :* 8°·54F.)

11. (a) Calculate the work done by a gas in expanding against a uniform pressure.

(b) A ball of iron has its temperature raised through 0·6°C. through a fall of 25 metres. Calculate the value of *J*. (All. 1918)

[*Ans :* $4·09 \times 10^7$].

Art. 112.

12. A tube 6 ft. long containing a little mercury, and closed at both ends, is rapidly inverted fifty times. What is the maximum rise in temperature that can be expected ? (sp. ht. of mercury = $\frac{1}{30}$; 1 B. Th. U. is equivalent to 778 ft.-lbs). (L. G. S.)

[*Ans :* 11°·6F.]

13. A block of ice is dropped into a well of water, both ice and water being at 0°C. From what height must the ice fall in order that one-fiftieth of of it may be melted ?

[*Ans :* 685·01 metres approx.].

Art, 113

14. Two balls of equal weight, one of india-rubber, and the other of soft clay, are dropped on to a hard floor from the same height. Which would develop the greater amount of heat by impact on the floor ? (L. M.)

[**Hints.**—Though K. E. of both on reaching the floor would be the same, the amount of heat developed by soft clay would be greater as it would remain on the floor when the energy would be converted into heat. The rubber ball would at once rebound and so a large amount of its K. E. would be used up in overcoming *g* when going up].

15. Calculate the velocity of a lead bullet on striking an unyielding target if the temperature rises 200°C. and the whole of the heat generated by the impact remains in the lead. (Sp. ht. of lead is 0·03.) (C. U. 1937. '41, '44.)

[*Ans :* $22·45 \times 10^3$ cm. per sec.]

16. Explain why is it that while the value of the latent heat of water is less when expressed in terms of the Centigrade scale than when expressed in terms of the Fahrenheit scale, just the opposite holds in the case of numerical values of the mechanical equivalent of heat. (C. U. 1937)

Art. 116

17. Describe the principle and action of a steam engine giving a sectional diagram. (Pat. 1931 ; '38 ; C. U. 1923, '25, '38, '39 ; Dac. '30)

18. Describe, with a neat diagram, any form of a modern petrol engine. How does it act ? (C. U. 1933 ; '37)

19. A petrol éngine uses every hour 1 lb. of petrol, which produces 22000 B. Th. U. of heat, and has a efficiency of 30 per cent. What is its H. P. ?

(1 H. P. = 33000 ft.-lbs. per min. and 1 B. Th. U. = 778 ft./lbs.)

[*Ans* : 2·593 H. P.]

PART III

SOUND

CHAPTER I

Production and Transmission of Sound

1. Definition of Sound.—Sound is a kind of sensation received by means of our ears and carried to the brain which is responsible for the perception. The external cause which produces such sensations is a form of energy.

Acoustics is that branch of Physics which deals with the study of the nature and propagation of sound.

Whenever any sound is produced, on tracing its origin it will be found that it is due to the vibratory movement of a material body. The vibrations may usually be too rapid to be seen by our naked eye, but we can feel their existence by touching the source. When air is blown through a whistle, a nail is struck by a hammer, or ammunition explodes through a gun, we have instances where sounds are produced by matter in motion.

Expts.—(1) When we strike a metal vessel we hear a sound, and the indistinctness of the outline of the vessel shows that it is vibrating. By touching the body the vibrations are stopped, and sound also is stopped at the same time.

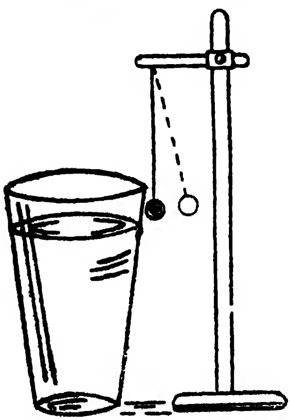


Fig. 1.

Pour water in a wide-mouthed thin-walled glass-tumbler until it is almost full and keep a pith ball suspended by a fine thread in touch with the rim of the vessel (Fig. 1). On bowing the edge of the tumbler with a violin bow, ripples will be produced in the water, and the pith-ball will be observed to jump forward by receiving a series of shocks from the rim on coming in contact with it, proving that the vessel is in a state of vibration.

A tuning-fork—a U-shaped steel bar provided with a handle at the bend of the *U* (Fig. 2) is taken and is made to vibrate by striking one of its prongs on the knee or on a hard cushion. It produces a musical sound. It

is then brought into contact with a pith-ball suspended by a thread, when the pith-ball will be thrown into vibration.

(III) On examining the string of a sounding violin it will be found to have a blurred outline due to its to-and-fro vibratory motion, which can be detected by placing a *V*-shaped paper rider on the string.

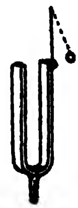


Fig. 2 – Tuning fork.

Thus a body must be made to vibrate in order to emit a sound, but, even when it is vibrating, the sound cannot be received or heard unless the mechanism of the ear also vibrates. We receive sound by the vibrations of a membrane in the ear, called the *ear-drum*, and these vibrations are transmitted to the brain and interpreted as sound.

It should be, however, noted that the rate of vibration must lie within a limited range in order to produce an audible sound. If the rate falls below about 30 per second or goes above 40,000 per second, the sound becomes inaudible.

2. **Propagation of Sound. (Medium necessary).**—In order that sound may be heard, the disturbance from the source must be carried to the ear through a space. This space is spoken of as the *medium.* Air is the usual medium through which sound travels, but it can also pass through any other material medium provided it is elastic and continuous. Thus an observer placing his ear against a continuous iron rail can hear distinctly even slight taps, given on the metal several hundred yards away. The ticking sound of a watch held agaist one end of a table is heard clearly by applying ear to the other end of it. Again, a diver within water distinctly hears any sound produced in water or in air. *Sound cannot, however, travel through a vacuum* and, in this respect, *it differs from light which can easily pass through a vacuum.* Light is propagated through a non-material medium, called the ether, whereas sound requires a *material* medium.

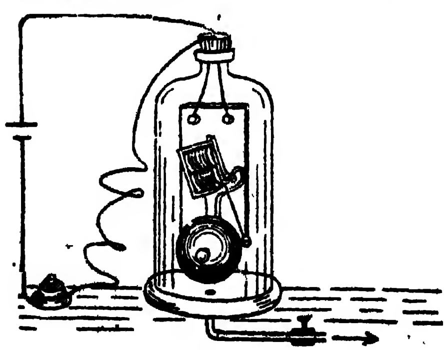


Fig. 3

That sound requires a material medium for its propagation and cannot travel through a vacuum may be demonstrated by the following experiment.

Expt.—An electric bell (Fig. 3) is placed inside the receiver of an air-pump and worked by a cell placed outside the jar. The bell is suspended inside the receiver by

means of a hook passing through a rubber stopper fitted tightly into the neck. The sound of the bell is distinctly heard so long as there is air inside the receiver, but as the air is gradually pumped out, the sound grows fainter and fainter, finally becoming quite inaudible. On re-admitting air, the intensity of the sound increases again.

It must also be noted that for the propagation of sound, not only the medium must be a material one but also it should be elastic and continuous. Non-elastic substances are not able to transmit sound to a great distance as the energy is dissipated very quickly. Again non-continuous substances, such as sawdust, felt, etc., are bad conductors of sound.

3. Requisites for Propagation of Sound.—

(*i*) A vibrating source to emit sound. (*ii*) A medium to transmit the sound ; the medium must be material, elastic and continuous. (*iii*) A vibrating receiver to receive the sound.

4. Mechanism of Sound Propagation—Let us examine the method by which sound is actually propagated through air. Suppose a body is struck. As a result of this, every particle constituting the body begins to vibrate—that is, to move to-and-fro, to a nearly equal distance on both sides of its mean position of rest. During this state, each of the extreme particles of the vibrating body in contact with air, at the time of moving to-and-fro between its extreme positions, strikes the line of air particles in contact with it, and starts them moving to-and-fro. These air-particles in their turn strike the particle beyond them, and set up similar vibrations in them, and this goes on from particle to particle. In this way a chain of vibrations is set up from the sounding body, each particle on the way begins to vibrate when it is struck by its neighbour, and in its turn strike its next neighbour, until the vibrations reach the ear of the listener producing a sensation of sound.

To consider it more closely, suppose a tuning-fork is vibrating. B is the position of one prong of it when at rest, and C and A being its two extreme positions on either side (Fig. 4). *The time taken by the prong to move from one extreme position to the other and back again to the first position, i. e. from A to C and back, is called* **the period of vibration**. Now, as the prong is moving from A towards C, it presses the air-particles in front of it, which in turn press their neighbouring particles, and this pressure passes on to the successive layers of the medium. So considering the effect of the movement upon a column of air on the right-hand side of the prong, it will be seen that, by the time the prong reaches C, the air-particles between A and some point C' will be compressed, and thus a pulse of *compression* will move forward with the velocity of sound. Again, during the back

stroke when the prong moves backwards from C to A, it tends to leave a partial vacuum behind it, due to which the layer in contact, being relieved of pressure, expands on the side of the prong and the pressure is consequently diminished.

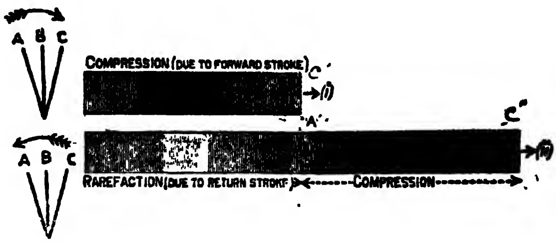


Fig. 4.—Propagation of Sound-waves.

Each succeeding layer acts in the same way and a *rarefaction* pulse is handed on from layer to layer, travelling forward with the same velocity as that of the compression pulse. This goes on upto the time the prong takes to reach A. During the time taken by the prong to travel from C to A, the compressed pulse has travelled onwards, and occupied a reigion $A' \; C''$ [Fig. 4(*ii*)] equal to $A \; C'$ in Fig. 4(*i*), which is now occupied by the rarefied pulse (Fig. 4*ii*). So in a complete period of vibration of the prong, the disturbance has travelled upto C'', one half of which ($A' \; C''$) is occupied by a *compressed pulse*, and the other half (AA') by a *rarefied pulse. A compressed pulse followed by rarefied pulse, together forms a complete sound-wave.* The amount of compression or rarefaction is not, however, equal at all points in the complete wave, though the compressed and the rarefied pulses occupy equal lengths in the air column. The reason is that the pressure communicated by the prong to the air at any instant depends on the velocity of the prong which varies from instant to instant in course of a period of vibration. The velocity of the prong being maximum at the mean position and zero at the extreme positions, the compression or the rarefaction is also maximum in the middle and zero at the ends of a zone of compression or rarefaction, as shewn in fig. 4(*ii*). If the displacements of the particles comprised in a wave at any instant of time be plotted in the ordinate against their distances as abscissa, the graph assumes the form of a wave. That is why a sound-wave is so called. *The Wave-length is the distance covered by one compression pulse and a rarefaction pulse together i.e. the distance through which the disturbance travels in a period of vibration of the source.* It is to be noted that each particle in the medium of propagation passes through all the phases of vibration as depicted in one wave-length in sound-waves.

When a body is sounded in a homogeneous medium, alternate pulses of compression and rarefaction start out in succession in all directions travelling with the same velocity. These pulses are like so many spherical shells of equal thickness spreading out with an expanding radius with passing of time (fig. 5).

They are analogous to the circular waves caused around a stone thrown into a calm sheet of water. Here a series of circular waves having alternate depressions and elevation are generated. They appear and disappear in a periodic time in succession. The depressions are called the *troughs* and the elevations the *crests*. They also spread out with an expanding radius till they reach the bank. The trough and the crest respectively correspond to the maximum rarefaction and maximum compression states in propagation of sound through a medium. The difference between the two cases is that a particle on the water sheet is displaced up and down at rt. angles to the path of propagation of the disturbance which travels along the surface towards the bank, where as a particle in the medium of propagation of sound is displaced to-and-fro along the same path in which the sound travels. That is, water-waves are **transverse** while a sound-wave is **longitudinal.**

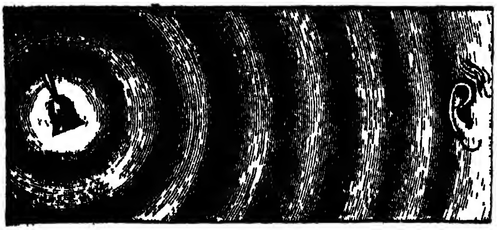


Fig. 5—Sound-waves caused by a bell.

The alternate compressions and rarefaction of air produced by the source of sound travel out in all directions and impinge upon the membrane of the ear, which vibrates exactly in unison with the source. The motion of the membrane is communicated to the brain by the mechanism of the ear and perception of the sound is caused.

5. (*a*) Graphical Representation of a Sound Wave.—Let a serits of dots in Fig. 6(*a*) represent a row of undisturbed particles of air. When a sound wave passes along this row, the particle in certain portions of the row will, at a given instant, come closer (*i.e.* compressed), and in certain other portions drawn further apart (*i.e.* rarefied), as represented in Fig. 6(*b*).

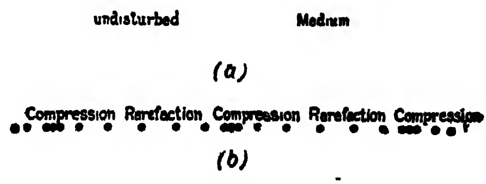


Fig. 6

The condition of these air-particles may be graphically represented by the curve $NA'B'C'$ (Fig. 7a), where the upper portions A', C', E', etc. represent the compressions and the lower portions B', D', etc. represent rarefactions. It is to be remembered that in a sound-wave the actual motion of the individual particles is in the direction NN, and not perpendicular to this direction, as in the case of water-waves or light-waves (see Fig. 9).

Fig. 7 (*b*) illustrates the condition of the air in a horizontal column in front of which a tuning fork is vibrating and due to which a series of sound-waves, both rarefied and compressed, produced by the fork, is maintained. In the darker portion [*i.e.* A, C, E] of Fig. 7 (*b*) the particles of air are crowded together. So A, C, E represent the condensed portions of the wave. Similarly the lighter portions, B, D represent the rarefied portions of the vaves.

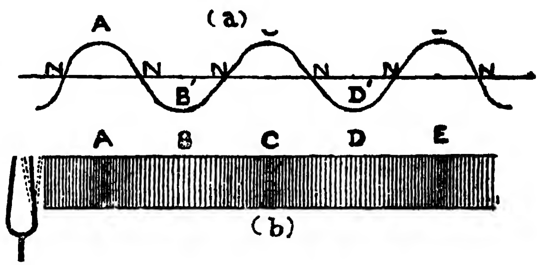


Fig. 7—Representation of a Sound-wave.

Fig. 7 (*a*) represents a sound-wave corresponding to the different conditions of air-particles in the tube Thus A' which is a crest of the wave represents the greatest compression corresponding to the position A in Fig. 7(*b*); and B' which is a trough of the wave represents the maximum rarefaction corresponding to the position B in Fig. 7 (*b*). It is to be noted that each particle in the column of air will pass through all the phases of vibration shewn in the graph during one period of vibration of the fork.

Similarly C', D', E' in Fig. 7 (*a*) represent positions corresponding to C, D, E in Fig. 7 (*b*). In the upper figure $NA'N$, $NC'N$ and $NE'N$ represent compressed waves, and $NB'N$, $ND'N$, represent rarefied waves. So $NA'B'N$, $A'B'C'$, $NB'C'N$, etc., each of which consists of one compressed and one rarefied wave represents one complete sound-wave.

(b) Demonstrations of Longitudinal Waves.—The propagation of longitudinal waves can be conveniently illustrated by a spiral spring suspended horizontally with threads from two parallel bars AB and $A'B'$ shown in Fig. 8. On pushing the end A of the spring suddenly forward, the nearest turns are compressed and the compression is

seen to move forward along the coil with a certain velocity towards the other end, each turn moving *forward* a little when the compression reaches it. This represents the state of the layer of air-particles when a wave of compression travels through it. Again if the end *A* be suddenly pulled outwards, the end turns will be separated from each other and this state of rarefaction, as we call it, is seen to be travelling along the coil to the farther end, each turn of the spiral moving *backward* a little when the extension reaches it. This represents the state of rarefaction travelling through air.

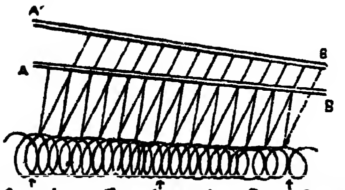


Fig.

Thus if one end of the coil be alternately pushed forwards and pulled outwards in a periodic manner, longitudinal wave-motion of compression and rarefaction will be seen to travel along the spiral with a constant velocity. Each particle of the spring executes a to-and-fro movement in the line of propagation of the pulse, but is not transferred from one position to another. In the same way, at the time of propagation of sound through air, the *particles* of the air only move about their mean positions of rest, and *are not bodily transferred from one place to another.* **It is the wave-form, or a succession of compressed and rarefied pulses, that travels forward** For this reason a blast of air is never felt to spread outwards even due to a loud report of a cannon.

Questions

Art. 1.

1. Describe experiments which prove that sound is due to vibrations. (Pat. 1921, '32, '33)

Art. 2.

2. Explain, why a medium is necessary for the propagation of sound and describe any experiment to prove the statement. (C. U. 1934.)

3. Discuss as far as you can the nature of vibrations in air when sound is transmitted through it. (Pat. 1930 ; C. U. '19, '26.)

4. Explain as far as you can, the mode of propagation of sound through air. (C. U. 1911, '18, '20, '24, '26 ; Cf. Pat. 1918, '31; '46.)

CHAPTER II

Wave-Motion : Simple Harmonic Motion

6. **Wave-Motion** —Every one is familiar with the circular waves when a stone is thrown into still water. The waves consisting of a series of *crests* and *troughs* travel outwards from the centre of disturbance in over-widening circles. But if some pieces of cork or bits of paper floating on the water, are carefully watched, it will be found that the floating objects, and therefore the particles of water, are only moving up and down, and that they do not travel outwards with the waves. It should be noted also that they rise and fall, not together but in succession, one after the other, showing that when waves pass over water, each separate particle of the medium must perform the same movement, not simultaneously, but each one a little later than the one preceding it. **It is the wave-form which travels forward,** while every particle of the water moves up and down about its means position of rest. Similarly when a wave crosses a cornfield, the tips of the corn blades do not move forward ; the form of the wave only moves forward. The vibratory motion of a series of particles through a medium constitutes what is called a *wave-motion.*

Thus we find that a wave-motion in water is, in fact, a particular state of motion handed on from one portion of water to another.

The water-waves are thus a kind of **progressive waves.**

In the case of water-waves, the motion of the water particles is at right angles to the direction of propagation of the waves. Such a wave is called a **Transverse wave**.

The other type of wave, as we have seen in Art. 4 in which the vibratory motion of the particles of the transmitting medium is along the line of propagation, is called a **Longitudinal wave.** This is a wave of compression and rarefaction. *Sound-waves in air, or* in any other fluid, are longitudinal, while radiant waves in ether, such as heat-waves and light-waves, are transverse. The electrical waves used in wireless telegraphy and telephony are also instances of transverse wave-motion.

It should be noted that *gases can transmit* only *longitudinal types of wave-motion,* because there being no cohesion between the molecules of a gas, transverse waves cannot be formed at all in gases, but solids and liquids can transmit both longitudinal and transverse waves.

Illustration.—In Fig. 9(*a*) AB represent a row of particles transmitting a traverse wave. As the wave passes, each individual particle of the medium will move up and down at right angles to the line AB (as shown by the double-headed arrow) along which the wave is propagated.

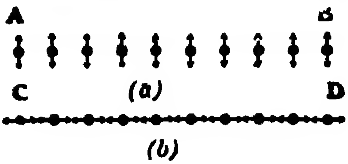


Fig. 9.—Illustration of Transverse and Longitudinal Wave Motion.

When a longitudinal wave passes along such a row of particles, the motion of each particle will be to-and-fro about a mean position along the line of propagation (CD), [Fig. 9(b)].

7. Some Important Terms.—

Frequency.—The number of complete vibrations made by a vibrating body in one second is called the frequency of the vibration. Thus, if n denotes the frequency of vibration, and T the periodic time of a vibrating body, we have, $nT=1$; *or*, $n=1/T$.

Amplitude.—It is the maximum distance or displacement to which a vibrating body moves from its position of rest during a vibration. In Fig. 13, BC or BD is the amplitude of the oscillating bob of the pendulum. (In Fig. 10, FE or GH is the amplitude).

Phase.—The phase of a vibrating particle at any instant is the state of the particle regarding its position and direction of motion in the path of vibration at that instant. *Two particles moving exactly in the same way are said to be in the same phase*, that is, particles which are at the same distance from their positions of rest, and are moving in the same direction, are said to be in the same *phase*. Thus, anything by which the direction of motion and displacement of a vibrating particle can be specified will be a measure of its *phase* at that time.

A water-particle A (Fig. 10) at the highest point of the crest of a water-wave is in the same phase with the particle B at the highest point of the next crest, and no other particle between these two positions are in the same phase.

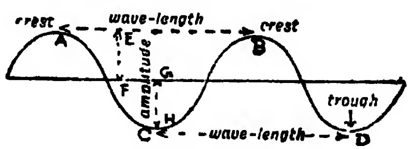


Fig. 10

Phase may be expressed in *three* ways.—(*i*) *By the fraction of the period that has elapsed since the vibrating body passed across the mean position of rest in a given direction.*

Thus in Fig. 13 the phase of the oscillating bob at C in the

direction BC is expressed by $\frac{1}{4}$ T, and at D in the direction BD by $\frac{3}{4}$ T, when B is its mean position of rest.

(*ii*) By the angle (as θ in Fig. 11) traced out by the generating point with reference to either of the co-ordinate axes (see Art. 9). Thus, the phase of the vibrating particle M is denoted by the angle θ (Fig. 11) traced out by the generating point P rotating about the circumference of the circle. Again it will be observed that the phases 90° and 450° are the same, while phases 90° and 270° are opposite to each other.

(*iii*) The phases of two points on a wave are also expressed by their path difference, *i.e.* by the fraction of the wave-length. In Fig. 10, A and B are in the same phase, and A, C are in opposite phases, their difference in phase being half the wave-length.

Wave-length.—It is the distance through which the wave motion travels in the time taken by a vibrating body or any one of the particles of the medium to make one complete vibration. It can also be defined as the *least distance* between two particles in the same phase of vibration.

In the case of a *transverse wave* the wave-length is the distance between one *crest* (or *trough*) and the next *crest* (or *trough*), as AB or CD in Fig. 10. In the case of a *longitudinal wave* it is the length occupied by a pulse of *compression* and a pulse *of rarefactiou*, as AC or BD in Fig. 6.

Wave-front.—It is defined as the trace drawn through all the points on a wave which are exactly in the same condition as regards displacement and direction of motion *i.e.* in the same phase. Thus, a surface drawn along the crests of a water-wave is a wave-front and also a surface drawn along the troughs would be another wave-front.

In a homogeneous medium a wave generated at a point travels out in all directions around the point with the same velocity. At any instant of time, the wave-motion lies upon the surface of a sphere whose centre is the generating point and radius equal to the product of the velocity and time. On this sphere the particles are all in the same phase of motion. This equi-phase surface is the wave-front at the time. At a very long distance from the source of disturbance, the spherical surface, over a limited region, may be treated as plane. So the wave-front may be taken as plane if the source of disturbance is at a very long distance.

Period.—The period of vibration is the time taken by a vibrating body to execute one complete vibration,

(*a*) **Velocity of Sound-Waves**—It is measured by the distance travelled over by a sound-wave in one second. If the Greek letter λ (pronounced "lambda") denotes the wave-length of a sound wave, and n the frequency of vibration, then in one second there will be n complete vibrations, and for each vibration the wave travels forward through a distance λ. Therefore the total distance travelled in one second $= n\lambda$. Hence, if V be the velocity of propagation of the wave, we have, $V = n\lambda$

Again, velocity $= \dfrac{\text{distance travelled}}{\text{time taken}}$, *i.e.* $V = \dfrac{\lambda}{T} = \dfrac{1}{T}.\lambda = n\lambda\ (\because n\,T = 1)$

Examples.—*1. A body vibrating with a constant frequency sends waves 10 cm. long through a medium A and 15 cm long through another medium B. The velocity of the waves in A is 90 cm. per sec. Find the velocity of the waves in B.* (*C. U. 1931*)

Let V be the velocity of the wave in B. Since velocity = frequency × wave-length, we have $90 = n \times 10$, where n is the frequency of vibration. $\therefore n = 9$ per second. Again, for the medium B, $V = n \times 15$ (n being constant in both the cases) $= 9 \times 15 = 135$ cms. per sec.

2. If the frequency of a tuning fork is 400 and the velocity of sound in air is 320 metres per second, find how far sound travels when the fork executes 30 vibrations.
(*C. U. 1913*)

In one second the sound travels 320 metres when the fork executes 400 vibrations. $\therefore$ In the time taken by the fork to execute 30 vibrations, the sound travels $\dfrac{320}{400} \times 30 = 24$ metres.

8 Simple Harmonic Motion.—A *vibration* is a motion which is continuously repeated and if the same series of movements is repeated at regular intervals of time, the motion is said to be **periodic**. Thus the motion of a particle continuously moving round a circle or an ellipse in a constant time is said to be *periodic*, and, in this sense, the motion of the earth round the sun is periodic. *The simplest type of periodic motion is that executed along a straight line by a particle moving to-and-fro.* This is called the *Simple Harmonic Motion* (also written S. H. M.). To understand the nature of a particle executing simple harmonic motion, let us imagine a particle P (Fig. 11) moving round a circle with uniform speed. The particle P is called the *generating point* and the circle $XYX'Y'$ round which it moves is known as the *circle of reference*.

Let PM be a perpendicular dropped from P on the diameter

XX' of the circle. Now as P moves round the circle in the direction of the arrow and describes a complete revolution, the foot M of the perpendicular PM moves to and-fro along the diameter XX' from the starting point upto X, then back to X', and then back to the starting point M.

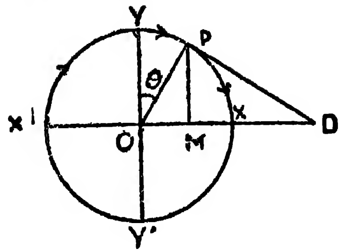


Fig. 11

This to-and-fro movement of M along XX' continues as P moves round the circle with uniform speed. The motion of M is known as simple harmonic motion. Thus a **particle may be said to be executing a Simple Harmonic Motion when it moves to-and-fro along a straight line, so that it describes each complete movement in the same constant period of time**

[**Note.**—The use of the term *harmonic* arose on account of the fact that the study of this was first made in connection with the study of musical vibrations.]

9. Equation of a Simple Harmonic Motion.—In Fig. 11, XPX' is a circle of radius OP which is equal to, say, a. Let P be a point which is travelling in the direction of the arrow round the circumference of the circle $XY'P$ with uniform speed, and let XOX' and YOY' be two diameters at right angles to each other. Let T be the period, *i.e.* the time for one complete revolution of P, and ω its angular velocity, *i.e.* the angular distance through which the radius OP revolves in 1 second. Then (see Art. 26, Part I),

$$\omega T = 2\pi \ ; \ or \ T = 2\pi/\omega \qquad \ldots \qquad \ldots \qquad (1)$$

As P moves round the circle, the point M, the foot of the perpendicular drawn from P on XOX', moves in S. H. M., and the frequency of vibration of M is the same as that of the point P. Hence the frequency of M, $n = 1/T$.

Let the time be counted from the instant when M is passing through its mean position O in the positive direction (*i.e.* from left to right when it is crossing the line OY). Let t be the time which has elapsed since M was last at O, *i.e.* the time taken by OP to make an angle θ with OY. The angle θ is called the *phase* of the vibrating particle M at that instant.

Then, $\theta = \omega t$. We have, $\dfrac{OM}{OP} = \cos POM = \cos\ (90 - \theta) = \sin\theta$

$\therefore$ The **displacement** x of P (*i.e.* OM) $= OP \sin\theta = a\sin\theta$ ... (2)

$= a\sin\omega t = a\sin\frac{2\pi}{T}.t$... , from... (1)

$= a\sin 2\pi nt$, where n is the frequency.

N. B. If time is recorded from an instant the generating point P is on the left of Y (*i.e* M is also on the left of O) to such an extent that the generating line OP makes an angle α with OY, then $\theta = \omega t - \alpha$. That is, $x = a\sin(\omega t - \alpha)$. This α, which is the *phase at the commencement of time*, is called the **Epoch.**

The greatest value of $\sin\theta$ is unity, hence the maximum value of x is a, which is, therefore, the **amplitude of vibration**. Thus the displacement has a positive maximum value at x when $\theta = 90^\circ$ and a negative maximum value at X' when $\theta = 270^\circ$.

The displacement of a body executing a S. H. M. is always given by an equation like (2).

10. Velocity and Acceleration of the Point.—

Velocity.—The velocity of M at any instant along XX' is the same as the component of the velocity of P parallel to XX' (Fig. 11.) Let PD be the tangent at P, meeting $X'X$ at D. The linear velocity of P at any instant is equal to v and is along the tangent PD. The component of v parallel to XX' *i.e.* in the direction $OD = v\cos PDO = v\sin POD = v\sin(90-\theta) = v\cos\theta$ ... ... (3)

Thus the velocity of M is zero at X, where $\theta = 90^\circ$,($\cos 90^\circ = 0$), and also at X', where $\theta = 270^\circ$. The velocity is a maximum at O, where $\theta = 0$, and $\cos 0 = 1$ (the maximum value of $\cos\theta$), and also it is a maximum in the negative direction when $\theta = 180^\circ$; and, after a complete swing, when $\theta = 360^\circ$, the velocity is again a maximum in the positive direction. Thus *in one complete oscillation* the velocities of M are *zero at the ends* of the swing, *i e.* at X and X', and *maximum* when passing *through the origin.* At O the velocity of M is parallel and so equal, to that of P.

Acceleration—It has been shown in Art. 46, Part 1, that the acceleration of P is directed towards O *i.e.* in the direction PO, and is equal to v^2/a. The acceleration of M is the component of the acceleration of P along OX. Hence the direction of the acceleration f of M is towards O and is given by

$$f = \frac{v^2}{a}\cos POM = \frac{v^2}{a}\sin\theta \quad \dots \quad \dots \quad (4)$$

But because v is the linear velocity of P which describes the distance $2\pi a$ in time T, we have $2\pi a = vT$

or, from (1) $\frac{2\pi}{T}.a = \omega a$ or $v^2 = \omega^2 a^2$

from (4), $f = \omega^2 a \sin\theta = \omega^2 \times displacement$ (5)

Hence, $\frac{\text{acceleration of } M}{\text{displacement of } M} = \omega^2 = \frac{4\pi^2}{T^2} =$ a constant.

Thus, *when a particle is describing a* S. H M., *the ratio of its acceleration to its displacement is constant, that is, when a particle (M) executes a S. H. M., its acceleration is proportional to its displacement (OM), and is directed towards a fixed point (O) in the line of vibration.*

The acceleration of M depends upon the sine of an angle just as displacement does and so the maximum and minimum values of acceleration occur exactly at times like those of displacement.

11. (a) Characteristics of Wave-motion —

Regarding the characteristics of wave-motion *three points* are to be noted.—(*i*) It is the *disturbance* which travels forward and not any particle of the medium.

(*ii*) The movement of each neighbouring particle begins a little later than that of its predecessor, or, in other words, there is a definite *phase difference* between two neighbouring particles.

(*b*) **Characteristics of a S. H M.**—(*i*) The motion is periodic. (*ii*) It is a vibratory (to-and-fro) motion. (*iii*) The motion takes place in a straight line.

(*iv*) The acceleration of the body executing a S. H. M. is proportional to its displacement and is directed towards a fixed point in the line of vibration.

12 The Displacement Curve of a S H M.—The displacement x of a particle executing a simple harmonic motion is given by the equation $x = a \sin wt$. If we plot a curve to show the relation between x and t, the curve will be a *sine curve*. Fig. 12 represents the displacement curve of a point M starting from O and moving with S.H.M. along YOY' due to the point P moving from X with uniform speed along the circumference of the circle with O as

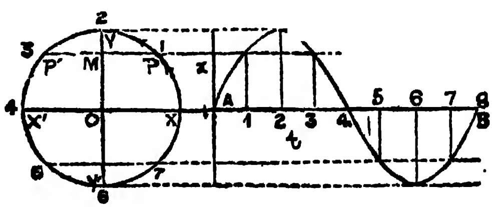


Fig. 12

centre in the direction XY as shewn by the arrow. Divide the circumference into any number of equal parts, say eight, and draw straight lines through the points of division P, Y, P', etc, parallel to XOX. If AB represents the period T, divide it into 8 equal parts. The time $(T/8)$ taken by P to move through each part of the circumference will then be represented by each division of AB. Draw ordinates at the point 1, 2, 3, etc., that are equal and proportional to the displacements OM, OY, etc. In plotting the distances, the points below O should be taken as of opposite sign to those above O. Now, joining the tops of these ordinate lines, the displacement curve is obtained which is identical with the well known *sine-curve*.

N.B.—Each particle in the medium transmitting a longitudinal sound-wave executes a S. H. M. with time. So the time-displacement curve for each particle in the medium or for the vibrating body itself, will be a sine-curve. The displacement, however, is in the line of propagation of the sound. The motion of the succeeding particles lying on the line of propagation reckoned in the same instant of time differs in phase from particle to particle. If the displacements of the particles are plotted in the ordinate against the distances of the particles concerned as abscissa (though they are in the same straight line), the graph will also be a sine-curve.

13. Examples of S.H.M.—The to-and-fro movement of one prong of a vibrating tuning-fork, the movement of a point in a stretched string when the string is plucked sideways, and also the motion of the bob of a simple pendulum oscillating with a small amplitude, are some of the familiar examples of simple harmonic motion.

Motion of a Simple Pendulum is a S H. M :—In Fig. 13, let the bob B of the pendulum of length l be displaced through an angle θ to the position C. If g be the acceleration due to gravity, and m the mass of the bob, the weight of the bob is mg, which can be resolved into two components, $mg \sin \theta$, which acting along CE at right angles to AC tends to bring the bob back to its original position B with an acceleration $g \sin \theta$. and force $mg \cos \theta$, which acts along CF and keeps the thread taut and is balanced by the tension of the string. After crossing the mean position B, when the bob moves towards BD in virtue of its inertia and acquired velocity, the acceleration acts in the opposite direction, and so the motion decreases and ultimately vanishes at the other

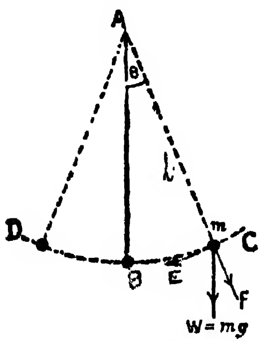


Fig. 13

extreme position D, when the direction of motion is reversed. This motion repeats itself. If the angle θ be very small (not greater than 4°) $\sin\ \theta$ may be taken to be equal to θ and so the acceleration of the bob $g \sin \theta = g\theta$. Similarly when θ is small and the thread is comparatively long, the arc BC is approximately a straight line, and then the displacement of the bob at C, that is, the distance of the bob from the mean position B = arc $BC = l\theta$ (where l = length of the pendulum).

$$\therefore \frac{\text{Displacement}}{\text{Acceleration}} = \frac{l\theta}{g\theta} = \frac{l}{g} = \text{a constant} \quad \ldots \quad \ldots \quad (1)$$

Thus the acceleration tending to bring the bob back to the original position is proportional to its displacement. Again the acceleration is always directed to the mean position B in the path of its movement. [At each of the extreme positions D and C, the displacement being the greatest, acceleration is also greatest]. These being the conditions of a S. H. M.,

the pendulum oscillates with S. H. M., and obeys the following law:

A particle executing a simple harmonic motion moves to-and-fro in a straight line such that its acceleration is always directed towards the mean position of rest and proportional to its displacement.

Here, if T be the period of oscillation of the bob.

we know, $$\frac{\text{acceleration of the bob}}{\text{its displacement}} = \frac{4\pi^2}{T^2}.$$

$$\therefore \quad T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{l}{g}}$$

The simple harmonic motion is of great importance in the study of sound, as a vibration of this type only gives the sensation of a pure tone with no admixture of other tones. Any other kind of vibration gives rise to a compound note which is composed of two or more simple tones.

14 Sound is a Wave motion—Sound is produced by the vibration of a sounding body, and the assumption that it is conveyed to the ear by means of waves is based on the consideration that the characteristics of wave propagation also apply to the case of transmission of sound.

(1) A wave takes time to travel from one place to another.

Sound also takes time to travel from one place to another, *i.e.*, it has a definite velocity.

(2) In solids and liquids the velocity of sound is greater than in gases. This can be explained only if sound is conveyed by wave-motion (Art. 117, Part IV).

(3) A wave requires a medium to pass through.

Sound also similarly requires an elastic medium to pass through.

(4) Waves are reflected or refracted obeying certain laws.

Sound is also reflected or refracted according to the same laws.

(5) Two sets of waves affecting the same place of a medium at the same time may destroy the effect of each other under certain conditions. This is the phenomenon of **interference**.

Sound also shows interference by the phenomenon of beats. (See Art.36).

6. Sound can bend round an obstacle. Moreover sounds of different acuteness show this effect by different amounts. The phenomenon is known as **diffraction**. Diffraction is possible owing to the wave character of sound. Since sounds of different acuteness have different wave length, the amount of diffraction caused by them should be different.

7. A wave of condensation started from a source has been actually photographically detected by R. W. Wood. The reality of *secondary wavelets*, first conceived by Huyghens in his wave theory, has been thus proved.

8. The phenomenon of polarisation is shown by transverse waves only. Light-waves being transverse show the phenomenon of polarisation but the fact that sound-waves fail to show the phenomenon of polarisation prove that the vibration in this case is longitudinal and not transverse.

14 (a). Expression for Progressive Wave motion.—Assuming the motion of any particle in the progressive wave to be simple harmonic, the displacement at any instant is given by,

$$x = a \sin(wt - \alpha)$$

$$= a \sin\left(\frac{2\pi}{T} . t - \alpha\right) = a \sin\left(\frac{2\pi}{\lambda/v} . t - \alpha\right).$$

$= a \sin\left(\frac{2\pi v.t}{\lambda} - \alpha\right)$ where $v =$ velocity of the wave, $\lambda =$ wave-length, and $T =$ time-period.

The wave lags a phase-angle α behind the origin *i.e.* a distance (r) given by the following,

$$r=\frac{\lambda}{2\pi}\alpha, \text{ since a distance } \lambda \text{ corresponds to a phase-angle } 2\pi.$$

That is, $\alpha=\frac{2\pi r}{\lambda}$.

$$\therefore \qquad x=a\sin\left(\frac{2\pi v.t}{\lambda}-\frac{2\pi r}{\lambda}\right)$$

$$=a\sin\frac{2\pi}{\lambda}(vt.-r)$$

where r = distance of the wave from the origin.

Questions

Art. 7.

1. Define amplitude, frequency and wave-length. What is the relation between velocity and wave-length ? (All. 1920).

2. When are two particles said to have the same phase ? (C. U. 1910 ; Pat. '18)

3. Describe and explain the terms frequency, amplitude and wave-length as applied to sound-waves in air. What are the differences in sensations perceived which correspond to differences in these quantities. (All. 1923)

Art 8.

4. Describe the motion of a sounding body. How would you demonstrate the nature of this experimentally. (C. U. 1910)

[**Hints.**—For the first part see Art. 8. For the second part see Ch. V. The nature of the motion of the vibrating body will be represented by the wavy line on the smoked paper].

5. Define the angular velocity of a body moving uniformly in a circle. Find its periodic time. Show that the foot of the perpendicular drawn from the body to a fixed diameter of the circle describes Simple Harmonic Motion and hence define such a motion. (C. U. 1933)

[*See* also Art. 26, Part I]

6. Define Simple Harmonic Motion, and explain it with reference to any familiar example. (*See* Art. 13). (C. U. 1921, '35 ; Cf. A. U. 1936 ; Pat. '41).

Arts. 11 & 13.

7. What are the principal characteristics of a simple harmonic vibration as illustrated by the motion of a pendulum ? In what respect is the motion of a pendulum similar to the vibration of a tuning fork ?

Art. 14.

8. Describe experiments to demonstrate that sound consists of a wave-motion in air. What is the nature of the wave constituting sound? (Pat. 1927)

9. What reasons are there for believing that sound is conveyed by wave-motion? (All. 1913, '15; C. U. '29; Dac. '32, '40).

10. What are the evidences in support of the view that sound is propagated by means of wave-motion, and that some matter is essential for its propagation? (Pat. 1933, '40).

[*See* Art. 2]

11. What are the main characteristics of wave-motion? Point out the chief resemblances and differences between waves of sound and waves of light. (L. M.)

(*See* Arts. 5 & 11. Also see wave-theory in Part IV.)

CHAPTER III

Velocity of Sound

15. Velocity of Sound in Air—Numerous examples can be cited to show that sound takes an appreciable time to travel from one place to another. Thus, though lightning and thunder are produced together, the flash of the lightning is seen much before the report of the thunder is heard. When a gun is fired at some distance, the flash is seen before the sound is heard; the puff of steam issuing from the whistle of a distant locomotive is seen some time before the sound is heard; so also the striking of a cricket ball with the bat is seen before hearing the sound. In each of these cases the time-interval between seeing and hearing is due to the difference between the times taken by light and sound to travel from the source to the observer. As light-waves travel almost instantaneously (186,000 miles per sec.), the time taken by light can be neglected in the determination of the velocity of sound. The velocity of sound at 0°C is generally accepted to be 332 metres per sec.

Experimental Determination of Velocity of Sound :—

(A). Open-air method.—The members of the Paris Academy first determined the velocity of sound in open air in 1738. Their

findings show that the velocity of sound (i) does not depend upon changes of the atmospheric pressure ; (ii) increases with temperature and humidity ; (iii) increases in the direction of the wind and decreases against it. According to the Dutch physicists Moll, Van Beck and others, the velocity of sound at 0°C is 332·26 metres per sec. Bravais and Martins determined the velocity of sound along a slope, the difference of altitude between Faulhorn, the upper station and the Lake of Briez the lower station being 2079 metres while their distance was 9560 metres. They found velocity corrected to 0°C to be 332·37 metres per sec. During the Arctic expeditions of Parry and Greely, the expts. were done at very low temperatures and almost the same result was found.

Arago did the following expt. in 1829. Two observers were stationed on the tops of two hills several miles apart. One of them was provided with a gun while the other had an accurate stop-watch. The first man fired his gun, the second man started his watch on seeing the flash and kept a continuous record of the time until the sound of the firing was heard. A large number of observations under similar atmospheric conditions were taken and the mean value (t secs,) of the recorded times was taken. If x ft. is the distance between the two stations, the velocity of sound (v) is given by,

$$v = x/t \text{ ft. per sec.}$$

such determinations suffer from the two principal errors, viz., (1) *the error due to the wind velocity* and (2) *the personal equation of the observer.*

The first error is that the velocity of sound is affected, though slightly, by the velocity of the wind, it being greater in the direction of the wind and less against it. It is corrected by the *method of reciprocal observations*, in which both the observers are provided with a gun as well as a stop-watch. When one fires, the other records the time and vice-versa. Suppose t_1 and t_2 are the mean values of the time recorded by the first and the second observers respectively. If the wind is blowing in the direction of the second station from the first at the rate of e ft./sec,

$$v + e = x/t_1 \text{ and } v - e = x/t_2$$

$$\therefore \quad v = \tfrac{1}{2}\left(\frac{x}{t_1} + \frac{x}{t_2}\right) \text{ ft./sec.}$$

That is, the effect of the wind is eliminated. The second error is that every man is apt to delay some fraction of a second to start the watch after he actually sees the flash of the firing and this delay period varies from person to person and is a personal factor of the person

making the expt. This error can be avoided by making electrical arrangements for recording of the exact moment of the gun-fire at one station and the report of the sound at the other.

Regnault used both of these precautionary measures in the determination of the velocity of sound in open air in 1864 at Versailles. He found the velocity greater in the case of *sounds having great loudness*, such as explosions of bombshell etc. *Sound ranging methods* (Art. 28) used during the Great War of 1914—18 for the location of enemy guns etc. give the most recent and modern means of determining the velocity of sound in open air.

(B). Laboratory method :—

By resonance of air column. (see Art 55).

(C). Method for Rare gases :—

Kundt's tube method. (see Art. 57).

16. Newton's Formula—Sir Isaac Newton was the first to formulate a law that the velocity of sound in a gas varies directly as the square root of its bulk-elasticity (Ch. VII, Part I) and inversely as the square root of its density. That is,

$$\text{Velocity}=\sqrt{\frac{\text{Elasticity}}{\text{Density}}}\ ; \quad i.e. \quad V=\sqrt{\frac{E}{D}}\ ,$$

where E is the *modulus of bulk-elasticity* of the medium, and D the *density of the medium*.

Now, the modulus of bulk-elasticity $E=\dfrac{\text{stress}}{\text{strain}}$.

In the case of gases, *stress* is the change in *pressure per unit area*, and *strain* in the corresponding change in *volume* produced *per unit volume* (see Ch. VII, Part I).

Consider a gas of volume V c.c. under a pressure P dynes per unit area. Let the pressure be now increased by a very small amount p per unit area, and consequently let the volume be decreased by a small amount v.

$$E=\frac{\text{stress}}{\text{strain}}=\frac{\text{increase of pressure per unit area}}{\text{consequent change of volume per unit volume}}$$

$$=\frac{p}{v/V}=V\frac{p}{v}\ \dots\dots\dots\dots\ (1)$$

Newton assumed that the change of pressure takes place under isothermal condition, i.e. it takes place slowly so that there is no appreciable change of temperature. So we have, according to Boyle's law,

$PV=(P+p)(V-v)=PV+pV-vP-pv.$

Since in the case of sound-waves the changes in pressure and volume are very small, p and v are very small, and so the product pv is negligible.

$$\therefore \quad pV = vP, \qquad or \; pV/v = P$$

$$\therefore \quad E = P, \text{ from (1).}$$

Hence by Newton's law, velocity of sound in a gas is given by

$$V = \sqrt{\frac{P}{D}}.$$

N. B. *In gases obeying Boyle's law the elasticity is equal to the pressure.*

Calculation of the Velocity of Sound in Air at N. T. P.

Normal pressure is the pressure exerted by a column of Hg., 76 cms. in height, at $O°C.$ in the sea-level at $45°$ latitude *i.e.*, $P = 76 \times 13{\cdot}596 \times 980{\cdot}6$ dynes/cm.^{2} $= 1{,}013, 250$ dynes/cm^2. Again density of air at $O°C.$

$$= 0{\cdot}001293 \text{ gms./c.c.}$$

$$\therefore \quad \text{Velocity of sound at N. T. P.} = \sqrt{\frac{P}{D}}$$

$$= \sqrt{\frac{76 \times 13{\cdot}596 \times 980{\cdot}6}{0{\cdot}001293}}$$

$$= 280 \text{ metres/sec. approxly.}$$

But this value of the velocity of sound at $O°C.$ is not in agreement with the value obtained by actual experiment, viz. 332 metres per second.

17. Laplace's Correction.—The calculation of the elasticity of a gas, according to Newton, involved Boyle's law according to which changes of pressure and volume of a given mass of gas take place at a constant temperature. Newton assumed that the changes in the air taking place in wave-motion had no effect on the temperature, *i.e.* the changes were *isothermal.* About 20 years later Laplace pointed out in 1817 that the changes of pressure, when sound-waves are travelling through a gas, are so *rapid, and the radiating and conducting powers of a gas are so poor*, that equalisation of temperature is improbable. So Newton's assumption that the temperature remains constant is not correct. According to him the changes that take place in a gas in which sound-waves are travelling are *adiabatic, i.e.* in which no heat is allowed to enter from outside, or leave from inside, the gas. That is, Laplace held that the alternate compressions and rarefactions take place so rapidly that the heat developed in compressed air remains fully confined in the compressed layer, and has no time to be dissipated into the body of the gas, and similarly the cold

produced in the rarefied layer remains wholly confined in it. So Boyle's law does not apply in this case.

[When sound travels in air or in any other gas, the particles of the gas are suddenly compressed at the condensed part of the wave, and suddenly separated at the rarefied part of the wave. If a gas is compressed, or allowed to expand, suddenly its temperature rises or falls momentarily, and with the rise or fall of temperature the gas expands or contracts. Now consider the effects of changes of temperature on the elasticity of a gas. During compression the temperature of the gas rises owing to which the volume of it tends to increase, and so a greater increase of pressure is necessary to produce a given diminution of volume than what is necessary, if the temperature of the gas remained constant (*i.e.* Boyle's law held good) during the compression. So the *elasticity* in the first case (when temperature increases) is *greater* than that in the second (when temperature is constant). Similarly, during rarefaction the temperature of a gas falls owing to which the volume of it tends to diminish, and so a greater diminution of pressure is necessary to produce a given increase in volume than what is necessary if the temperature of the gas remained constant. So, here also the *elasticity is greater* than that in the first case. Considering the above, Laplace said that the value for the elasticity E under *adiabatic conditions* should be substituted in Newton's formula for the velocity of sound.]

It is known that the elasticity of a gas, under adiabatic conditions $= \gamma \times$ pressure,

$$\text{where } \gamma = \frac{Cp}{Cv} = \frac{\text{specific ht. of the gas at constant pressure}}{\text{specific ht. of the gas at constant volume}}$$

The value of γ for a *di-atomic* gas like oxygen, nitrogen, or air is 1·41 (for a tri-atomic gas like CO_2 it is 1·33). Therefore, the formula of velocity of sound in air with Laplace's correction becomes,

$$V = \sqrt{\frac{\gamma \times P}{D}} = \sqrt{\frac{1{\cdot}41 \times P}{D}} \text{ metre per sec.}$$

$$= 280 \times \sqrt{1{\cdot}41} = 332{\cdot}5 \text{ metres per sec.}$$

18. Effect of Pressure, Temperature, and Humidity on the Velocity of Sound in a Gas.

(I) *Effect of Pressure.*—If P_1 and P_2 be the pressures of a given mass of gas, v_1 and v_2 the volumes, and D_1 and D_2 the corresponding densities, we have, by Boyle's law, $P_1v_1 = P_2v_2$.

$$or \quad \frac{P_1}{P_2} = \frac{v_2}{v_1}.$$

But volume varies *inversely* as density, *i.e.*, $\frac{v_2}{v_1}=\frac{D_1}{D_2}$ because $v_2D_2=v_1D_1=\text{mass}=\text{a constant}$.

Hence $\frac{P_1}{P_2}=\frac{D_1}{D_2}$; *or* $\frac{P_1}{D_1}=\frac{P_2}{D_2}=\text{a constant}$.

Therefore, in the formula $V=\sqrt{\frac{1{\cdot}41P}{D}}$, the fraction $\frac{P}{D}$ remains unchanged. Hence, *the velocity of sound in a gas is independent of any change of pressure.*

(II) *Effect of Temperature.*—Let v_o, v_t be the volumes of a gas at $O^\circ C.$ and $t^\circ C.$ respectively, and D_o, D_t the corresponding densities, then we have

$\frac{v_o}{v_t}=\frac{273}{273+t}$ (because, according to Charle's law, the volume of a gas is directly proportional to its absolute temperature).

But $\frac{v_o}{v_t}=\frac{D_t}{D_o}$. $\therefore \frac{D_o}{D_t}=\frac{273+t}{273}$ ... (1)

If V_o and V_t be the velocities of sound in the gas at $O^\circ C.$ and $t^\circ C.$ respectively and P be the pressure of the gas, we have

$$V_o=\sqrt{\frac{1{\cdot}41P}{D_o}},\text{ and } V_t=\sqrt{\frac{1{\cdot}41P}{D_t}}.$$

$$\therefore \frac{V_t}{V_o}=\sqrt{\frac{D_o}{D_t}}=\sqrt{\frac{273+t}{273}}=\sqrt{\frac{T}{T_o}},$$

when T and T_o are the absolute temperatures corresponding to $t^\circ C.$ and $O^\circ C.$ respectively.

Therefore, the velocity of sound in a gas is directly proportional to the square root of its absolute temperature. So velocity of sound in a gas increases with rise of temperature.

We have $V_t=V_o\left(1+\frac{1}{273}t\right)^{\frac{1}{2}}=V_o\left(1+\frac{1}{2}\times\frac{1}{273}t\right)$ neglecting terms containing t^2 and higher powers of t.

In the case of air, $V_o=332$ metres per second.

$$\therefore \quad V_t=332\left(1+\frac{1}{546}t\right)\text{ metres per second.}$$

$$=(332+0{\cdot}61t)\text{ metres per second.}$$

Hence, *for each centigrade degree rise in temperature, the velocity of sound in air increases by about 0·61 metre or 61 cms. (or about 2 ft.) per second.*

(III) *Effect of Humidity.*—The density of water-vapour is less than the density of dry air at ordinary temperatures in the ratio of 0·62 : 1. Therefore, the presence of water-vapour in the air lowers the density of air and so increases the velocity of sound in it. Hence for a given temp., *the velocity of sound in damp air is greater than that in dry air.*

19 Correction for the presence of Moisture in the observed value of the Velocity of Sound in Air :—

If V_m = velocity in moist air at pressure P mm. and temperature $t°C$.

V_d = velocity in dry air at pressure 760 mm. and temperature $t°C$.

D_m = density of moist air at pressure P mm. and temperature $t°C$.

D = density of dry air at pressure 760 mm. and temperature $t°C$.

then, $$V_m = \sqrt{\frac{\gamma P}{D_m}}; \quad V_d = \sqrt{\frac{\gamma \times 760}{D_d}}.$$

Now, if f = saturation pressure of water-vapour at $t°C$, we have D_m = weight of 1 c.c. of moist air at pressure P mm. and temperature $t°C$.

= wt. of 1 c.c. of dry air at pressure $(P-f)$ mm. and temperature $t°C$. + wt. of 1 c.c. of moisture at pressure f mm. and temperature $t°C$.

We know that the mass of 1 c.c. of water-vapour = 0·622 × mass of 1 c.c. of dry air.

Now, because the density of a gas at a constant temperature varies directly as its pressure, we have,

$$D_m = \frac{P-f}{760} \times D_d + 0{\cdot}622 \times \frac{f}{760} \times D_d = \frac{D_d}{760}(P - f + 0{\cdot}622 \times f)$$

$$= \frac{D_d}{760}(P - 0{\cdot}378 f) \ldots\ldots\ldots\ldots(1)$$

$$\therefore \frac{V_d}{V_m} = \sqrt{\frac{760 \times D_m}{P \times D_d}} = \sqrt{\frac{P - 0{\cdot}378f}{P}} = \sqrt{1 - 0{\cdot}378\frac{f}{P}}$$

$$\therefore V_d = V_m\sqrt{1 - 0{\cdot}378\frac{f}{P}}$$

20. Velocity of Sound in various Gases.—We know that

the velocity of sound in air $V_a=\sqrt{\frac{\gamma P}{D_a}}$, where D_a is the density of air and V_a the velocity of sound in it. Under similar conditions of pressure and temperature the velocity in another di-atomic gas (for which the value of γ is the same), say hydrogen,

$$V_h=\sqrt{\frac{\gamma P}{D_h}}\,;\ \therefore\ \frac{V_a}{V_h}=\sqrt{\frac{D_h}{D_a}}.$$

So the velocity of sound in a gas is inversely proportional to the square root of its density. Thus, if V_o, V_h be the velocities, and D_o, D_h the densities of oxygen and hydrogen respectively under the same conditions of temperature and pressure,

$$\frac{V_o}{V_h}=\sqrt{\frac{D_h}{D_o}}=\sqrt{\frac{1}{16}}=\frac{1}{4}.$$

21. Velocity of Sound in Water.—The velocity of sound in water was determined by Colladan and Surm in 1827 in the lake of Geneva, where a large bell, hung below the surface of water from the side of a boat, was struck by a hammer. The sound was received through a sort of ear-trumpet fixed in the water to another boat, which was placed at a distance of 2 miles. There was an arrangement in the first boat such that, at the instant the hammer was struck a charge of gunpowder was ignited giving a flash in the air which could be seen by the observer in the second boat. The interval between the flash and the report was noted and the velocity was calculated in the usual way.

Theoretical Calculation.—

Velocity of sound in water, $V_w=\sqrt{\frac{\text{adiabatic elasticity}}{\text{density}}}$ (For water, density = 1 gm. per c.c. and adiabatic elasticity = $2{\cdot}1\times10^{10}$ dynes per sq. cm.)

$$\therefore\ V_w=\sqrt{\frac{2{\cdot}1\times10^{10}}{1}}\ \text{cms. per sec.} = 1449\ \text{metres per sec.}$$

This agrees fairly well with the experimental result. Note that this is nearly 4 times the velocity in air.

22 (a) Velocity of Sound in Solids.—Sound travels much faster in solids than in air. The velocity of sound in cast iron was determined by Biot by striking with a hammer one end of a long series of cast iron pipes of total length 951 metres joined end to end. The

sound travels both through the walls of the pipes and through the air inside them with unequal speeds. An observer at the other end noted the interval between the sound transmitted by the metal and that by the air. The interval between the sounds was 2·5 second

Therefore, if V = velocity of sound in cast iron, and V_1 that in air, the time taken by sound to travel 951 metres through cast iron $= 951/V$,

and that through air $= \frac{951}{V_1}$, $\therefore \frac{951}{V_1} - \frac{951}{V} = 2·5$.

Assuming the value of the velocity of sound in air at the particular temperature, the velocity in cast iron was determined, but the result was not quite accurate.

Theoretical Calculation.—When a compression wave is transmitted along a solid, its velocity is given by $V = \sqrt{Y/D}$, where Y = Young's modulus of elasticity for that material. For annealed steel $Y = 21·4 \times 10^{11}$ dynes per sq. cm. and $D = 7·63$ gm /c.c.

$$\therefore \quad V = \sqrt{\frac{21·4 \times 10^{11}}{7·63}} \text{ cms.} = 5296 \text{ metres per sec.}$$

The present accepted value of the velocity of sound in iron is 5130 metres per second.

(b) Velocity in other forms of Solids—The velocity of longitudinal waves in solids, when in the form of a string, can be experimentally determined in the laboratory as explained in Art. 45. When the solid is in the form of a rod, the velocity is conveniently determined by Kundt's method (*vide* Art. 55) which is based on the principle of resonance.

From the table of velocities of sound it will be seen that sound travels faster in solids and liquid than in air. If the ear is applied to one end of a long wooden or metal board while somebody lightly scratches the other end, the sound of the scratching will be clearly heard, but it may not be audible when the ear is removed from the contact of the board, *i e.*, when sound travels through air. Similarly any sound made under water may be easily heard at a considerable distance by means of a submerged **hydrophone,** which is an underwater microphone receiver with a sensitive metal diaphragm for recording sound-waves. *But sounds do not readily pass from one medium to another when they differ greatly in density.* For this reason when your ears are under water you will not be able to hear the shouts of people around you made in air.

The sounds made by a running horse's hoofs will be heard from a very long distance if the ear is applied to the ground though they may be inaudible when the listener is standing up, and similarly the ear in contact with a railway line will catch the sounds of an approaching train long before they may be heard by others. This principle is applied by the water company's inspector in detecting leaks in the water main under the street. This is done by applying a rod to the ground above the pipe and pressing the ear to the rod, that is, by making a continuous solid connection from the pipe to the ear the sound of water running in the pipe will be readily audible. Similarly the doctor presses his stethoscope on the chest in order to make a solid connection between the chest and the ear so that the sound in the lungs and of the heart beating may be audible.

This principle may be applied for preventing sound from passing from one room to another of a building by making cavity walls, that is, walls with an air space between them.

Hydrophone.—It is a microphone receiver used for reception of sound under water and finding the direction of the sound. It is largely used in echo-depth-sounding, location of submerged objects and ice-bergs by methods of echo-soundings, location of submarines by the method of sound ranging in the sea-water and similar acts of sound reception under water.

Ordinarily, it is a carbon-granule type of transmitter adapted for use under water. It consists of a heavy annular metallic ring provided with a central thin diaphragm made also of metal. A stylus is fixed at its one end to the centre of this diaphragm and to a carbon-granule-box at the other. The diaphragm and the back-end of the box are separately joined to two wires from a cable by which the receiver is dipped into tha sea. The ends of the wires at the other end of the cable are connected in series to a headphone and a battery of cells. The back end of the ring is closed by a stout screen called the baffle (or the deaf-side), since it cuts off the reception of sound at that end. The movement of the diaphragm, due to incidence of any vibratory disturbance on it, causes a fluctuation of resistance in the carbon-granule-box and of the current in the headphone circuit. For correct reception of sound, the receiver is rotated in all possible directions until the maximum sound is heard in the headphone. The direction of the sound is normal to the plane of the diaphragm at this position.

23. Sound Ranging.—In war the position of an enemy gun can be located by noting the times taken by the report of the gun to reach several sound-detecting stations. Suppose there are three different

stations A, B and C If the report of the gun reaches B a second later than it reaches A, then taking 1100 ft. per sec. for the velocity of sound, the gun at G say, must be 1100 ft farther from B than from A or $GB - GA = 1100$ ft If again the report reaches C three-fifths of a second later than at A, then $(GC - GA) = \frac{3}{5} \times 1100 = 660$ft. If now circles with radii 1100 ft. and 600 ft. respectively are drawn with centres B and C, the gun will be at the centre of a circle passing through A and touching each of the other circles

In practice more than three stations may be selected and at each of the stations is placed a **hot-wire microphone** for detecting the sound. This is a very sensitive apparatus to receive electrically both continuous and discontinuous sounds

24. Determination of Ship's Position —In foggy weather when a ship finds it difficult to get its bearing, it sends out simultaneously two signals,—a wireless signal and an under-water sound signal—to the stations on the coast, which are equipped for receiving them and which in turn informs the ship by wireless the interval between receiving the two signals. Thus, if the interval is 2 sec at the station A, then the ship is $(\ \times 4714) = 9428$ft from A, while an interval of 4 sec. at B would indicate that the ship is at a distance of $(4 \times 4714) =$ 18,856 ft. from B where the velocity of sound in sea-water is 4714 ft/sec. Therefore ship's position will be obtained by intersecting arcs drawn on the chart with centres A and B and radii 9,428 ft and 18,856 ft. respectively.

Examples—1. *10 seconds have elapsed between the flash and the report of a gun. What is its distance, the temperature being 15°C? (Velocity in air at 0°C = 332 metres per second).*

From formula $V_t = V_0(1 + \frac{1}{2} \times \frac{1}{273}t)$, we have $V_{15} = 332(1 + \frac{1}{546}t) =$ $(332 + 0.61t)$

$$= 332 + 0.61 \times 15 = 332 + 9.15 = 341.15 \text{ metres.}$$

Hence in 10 seconds the sound would have travelled $341.15 \times 10 = 3411.5$ metres. The distance required $= 3411.5$ metres

✓ **2.** *A piece of stone is dropped into a well and the splash is heard after 1.45 seconds. Calculated the depth of the well assuming the velocity of sound in air to be 332 metres per second* *(Pat 1919).*

If t be the time taken by the stone in falling, the depth of the well $x = \frac{1}{2}gt^2$. Hence the time taken by the report to reach the mouth of the well from the water $= (1.45 - t)$ sec So the distance travelled by the sound,

$x =$ velocity × time $= V(1.45 - t)$, $\therefore V(1.45 - t) = \frac{1}{2}gt^2$

or $332(1.45 - t) = \frac{1}{2} \times 9.81 \times t^2$, ($\because g = 981$ cm $= 9.81$ metre).

or $332 \times 1.45 - 332t = 4.9t^2$, *or* $4.9t^2 + 332t - 481.4 = 0$ $\therefore t = 1.42$ seconds.

Hence the depth of the well $x = 332(1{\cdot}45 - 1{\cdot}42) = 332 \times 0{\cdot}03 = 9{\cdot}96$ metres.

3 Calculate the velocity of sound in air at 10°C when the pressure of the atmosphere is 76 cm

A sound is emitted by a source at one end of an iron tube 950 metres long and two sounds are heard at the other end at an interval of 2·5 secs. Find the velocity of sound in iron.

We know that $V = \sqrt{\dfrac{1{\cdot}41\,P}{D}}$

$\therefore$ The velocity of sound in air at 0°*C*. and 76 cm. pressure,

$$V_o = \sqrt{\frac{1{\cdot}41 \times 76 \times 13{\cdot}6 \times 981}{{\cdot}001293}} \text{ cms. per sec.} = 332{\cdot}5 \text{ metres per sec.}$$

$\therefore$ The velocity of sound at $10°C. = 332{\cdot}5 + 0{\cdot}61 \times 10 = 338{\cdot}6$ metres per sec.

If V be the velocity of sound in iron expressed in metres per sec., the time taken by the sound to travel 950 metres along the iron tube is $950/V$ sec. The time taken by the sound to travel through the same distance in air at 10°*C*. is 950/338·6 sec., where the velocity of sound in air at 10°C = 338·6 metres per sec.

The velocity of sound in solids is greater than that in air, hence the time taken by the sound to travel through the iron of the tube is smaller than the time taken to travel through the air inside the tube.

$$\therefore \quad 2{\cdot}5 = \frac{950}{338{\cdot}6} - \frac{950}{V}, \text{ whence } V = 3107{\cdot}92 \text{ metres per sec.}$$

4. A man sets his watch by the noon whistle of a factory at a distance of 1 mile. How many seconds is his watch slower than the time-piece of the factory? (Velocity of sound—332 metres sec. per). **(*Pat. 1941.*)**

The man when setting his watch by the whistle did not take the time taken by the sound to travel over a distance of 1 mile into consideration. Hence his watch is slower than factory time-piece by the above time.

Velocity of sound = 332 metres per sec. = 1088 ft. per sec.

1 mile = 5280 ft. Therefore the time taken to travel over 1 mile $= \frac{5280}{1088}$ $= 4{\cdot}85$ seconds. Hence the watch is 4·85 seconds slower.

Questions

Arts. 15 & 18

1. How will you determine the velocity of sound in air? Will the result be the same when strong wind is blowing? How will you eliminate the effect due to wind? Will the result be the same in summer and in winter? Give reasons for your answer. (All. 1931)

[**Hints.**—As velocity increases with temperature, the value of it will be found to be greater in summer than in winter.]

Arts. 16 & 20.

2. State the law connecting the velocity of sound through a gas with its

density. Compare the velocities of sound in hydrogen and oxygen under similar conditions. (C. U. 1912).

3. Calculate the velocity of sound in air on a day when the barometer stands at 75 cm. (the density of air is ·00129)

[*Ans* : 329·6 metres per second.]

Arts. 17 & 18.

4 (a). Give Newton's expression for the velocity of sound in a gas. (All. '46).

(b). Explain clearly the different steps in the reasoning which led to the introduction of Laplace's correction in Newton's original expression for the velocity of sound in air. (Pat. 1927 ; cf. All. '46.)

5. How can the velocity of sound in atmospheric air be measured ? Give **any two** methods. How is the velocity affected by changes of pressure and temperature ? (C. U. 1917, '37, '41 ; All. '45, '46 cf. Pat. 1921, '30, '40, '43).

6. Indicate how you could find the distance of a storm by noting temperature of the air and the interval between the flash of lighting and the sound of thunder coming from the storm.

What evidence could you give that the velocity of sound is practically independent of the amplitude and frequency of the air vibrations. (Pat. 1934).

[**Hints**.—When some one speaks or sings, the sound is not a simple one. It is a compound sound, *i.e.* it consists of notes of different amplitudes and frequency. But as every note takes the same time to reach us, evidently the velocity of sound does not depend on the amplitude and frequency of the air vibrations.]

7. The interval between the flash of lighting and the sound of thunder is 3 secs. when the temperature is $10°C$. How far away is the storm ? (Velocity of sound in air at $0°C$. is 1090 ft. per sec.)

[*Ans* : 1110 yds.]

8. A cannon is fired from a station A at the top of a mountain and observers are placed at two points B and C equidistant from A. B is at the top of another mountain, while C lies in the valley between the two. Assuming the temperature of air to fall as we descend, explain which of the observers will hear the cannon first. (Pat. 1922).

9. An observer sets his watch by the sound of a gun fired at a fort 1 mile distant. If the temperature of the air at the time is $15°C$. what will be the error ? Mention other causes which are likely to lead to errors in the setting. (Velocity of sound in air at $0°C. = 1090$ ft. per sec). (L. M.)

[*Ans* : 4·667 secs.]

10. If the velocity of sound in air at $0°C$. and 76 cm. of mercury pressure is 330 metres per sec., calculate the velocity at $27°C$. and 74 cm. pressure.

[*Ans* : 346·47 metres per sec.] (C. U. 1935).

11. On what factors, and how, does the velocity of sound in a given medium depend ? (Pat. 1932).

12. The densities of dry air and moist air are in the ratio 1 : 8. On a dry day a sound travels a certain distance in 6 secs. How long will the sound travel the same distance on a moist day.

[*Ans :* 5·37 secs.]

13. On one occasion when the temperature of air was $0°C$. a sound made at a given point was heard at a second point after an interval of 10 seconds. What was the temperature of the air on a second occasion, when the time taken to travel between the two points was 9·652 seconds.

[*Ans :* $19·7°C$.]

14. An observer sets his watch by the sound of a signal gun fired at a distant tower. He finds that his watch is slow by two seconds. Find the distance of the tower from the observer. Temperature of air during observation is 15°C. and the velocity of sound in air at 0°C. is 332 m./sec. (Pat. 1939).

[**Hints.**—$V = 332(1 + 0·61 \times 15) = 341·15$ m/sec.

$\therefore$ Distance $= 341·15 \times 2 = 682·3$ metres.]

Art 22.

15. How would you show that sound travels faster in air than in carbon dioxide and slower in air than in iron ? (Pat. 1918)

[**Hints.**—Repeat the resonant air column experiment (Ch. VII) by replacing air by carbon dioxide. For the second part see Art. 22].

Art. 24.

16. Explain how sound-waves have been used to determine the position of a ship in a sea in a very foggy weather ?

CHAPTER IV

Reflection and Refraction of Sound

25. Sound and Light Compared.—When a disturbance occurs in open air, sound-waves proceed radially outwards in all directions from the source as the centre. Light also is a wave-motion, but *light* waves are *transverse*, whereas *sound* waves are longitudinal waves of compression and rarefaction. It is important to remember that the

term *rays of light* does not mean that light consist in so many straight lines, but this is only a convenient way of expressing the directions in which light waves are preceeding forward Similarly any straight line along which a sound-wave is propagated may be called a *sound ray*. We have seen that light waves are reflected from plane and spherical surfaces obeying certain laws ; so also sound waves are reflected according to the same laws—*viz*., that the angles of incidence and reflection are equal and that they are in the same plane ; but conditions under which reflections of these two waves take place are widely different on account of the great difference between the lengths of light waves and the lengths of sound waves, and also because light waves can travel through vacuum whereas sound waves require a material medium for their transmission.

Under favourable conditions sound waves can also be *refracted* like light waves. and there may be also *interference* due to two waves of sound as due to two similar luminous sources in the case of light.

We shall see later on (Ch. VI) that the quantity of sound depends upon the number of the simple tones and also on the intensities of these tones. Similarly light from a luminous source is composed of a number of simple colours mixed up in a definite proportion.

Frequency of vibration of a sounding body determines the pitch of the sound emitted ; similarly a particular colour, say, red or blue, of light depends upon the frequency of vibration of ether particles and consequently on the wave length of ether waves.

Sound waves are detected by the *auditory* nerves of the ear while light waves are detected by the *optic* nerves.

26 Reflection of Sound.—In order that appreciable reflection of a wave may take place from any surface, the area of the surface should be fairly large in comparison with the wave-length of the wave incident on it Sound waves are much larger than light waves. The lowest audible note has got a wave-length of about half-an-inch, and the highest audible note has got a wave-length of about 36 ft.—for example, the wave-length corresponding to the note C is nearly 4 ft., whereas the wave-lengths of visible light are included between 16 and 30 millionths of an inch. Consequently it is evident that larger surfaces are required for complete reflection of sound waves than are required for light waves. On the other hand, the sound waves being larger do not require the surface to be so smooth as may be required for light waves. For this reason, a brick wall, a wooden board, or a hill-side all serve as reflectors of sound waves. The following

experiments will illustrate the reflection of sound waves like light waves.

(*a*) **Reflection at a Plane Surface.**—Fix a large plane wooden board AB vertically and place a long hollow tube T_1 with its axis pointing at some point C on the board making a definite angle with the plane of the board (Fig. 14). Now place another similar tube T_2 with its axis pointing towards C. Hold a small watch just in front of the tube T_1 and put your ear at the end of the receiving tube T_2 which is rotated with the point C as centre in all possible positions till the sound of the watch appears maximum, a board S being placed between the tubes to cut off the direct sound. It will be found that sound obeys the same laws of reflection as light *viz.*—

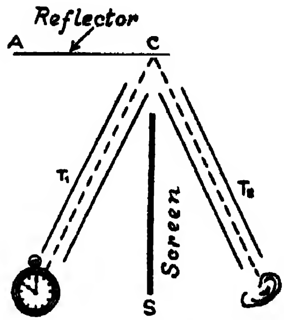


Fig. 14

(1) The angle of reflection is equal to the angle of incidence, that is, the axes of T_1 and T_2 make equal angles with the normal at C.

(2) The reflected sound ray, the incident ray, that is, the axes of T_1 and T_2, and the normal at the point of incident C of the board lie in one plane.

(*b*) **Reflection by Concave Surfaces** :—Two large concave spherical mirrors M and M', are placed coaxially on a table facing each other. A watch is placed at the focus of one of them M. The sound waves proceeding from the watch being reflected from the mirror will fall on the second mirror, and will be converged at the focus of M', where the sound waves can be received in the ear E by means of a funnel tube. The ticking of the watch will be distinctly heard at the focus, and it will be inaudible at other points, or at the same point, by turning the mirror a little.

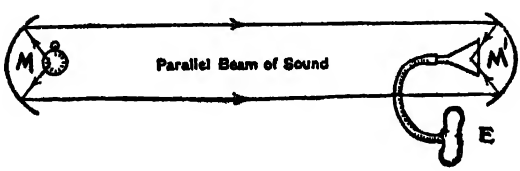


Fig. 15

26. (*a*) **Practical Examples.**—The principle of reflection of sound

is applied in *speaking tubes*, *ear-trumpets*, *doctors' stethoscopes*, etc.

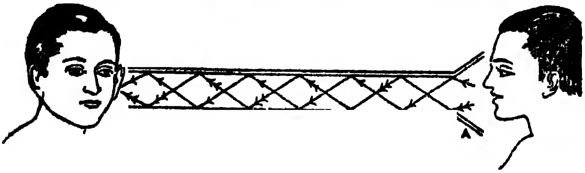


Fig 16.—Reflections in a tube.

In these cases the sound waves are reflected repeatedly from side to side of the tubes (Fig. 16). Here the sound waves can not spread, so the energy of the waves, instead of being distributed through a rapidly increasing space, remains more or less confined within the limits of the tubes, and so an ear, placed at the distant end, can hear the sound distinctly.

Reflection in an Auditorium.—Sometimes the rooms and halls of buildings with arched ceiling serve as reflectors of sound waves, and also the walls of large halls often reflect the sound waves and interfere with the words of a speaker, which becomes very troublesome. This may be avoided by the hanging up of screens and curtains which are bad reflectors for sound waves. The interference is also avoided to a certain extent when the hall is filled with an audience whose bodies serve to damp the sound, and for this reason it is often easier to speak before a large audience than in an empty hall. On the other hand it has been found that in the open air, where there is no echo, it is rather difficult to make oneself heard to a large crowd, and this is not so in a big hall as a certain amount of reflection helps in increasing the volume of sound. It has been practically seen that the effect is better when echo is heard nearly about 2 seconds after the original sound.

In churches there is often a concave reflecting board above the pulpit which reflects the sound made by the preacher down to the congregation.

It is known to everyone that the hollow of the hand held at the back of the ear in a curved way serves to concentrate the sound waves and thus helps to hear a distant sound.

27. Echo—*An echo is a case of repetition of a sound caused by the reflection of the wave back along the same direction.* A sound made near a wall or a hill-side will be reflected and heard as two distinct sounds, provided the distance between the observer and the reflecting surface is big enough to allow the reflected sound to reach him without interfering with direct sound. The impression of a sound persists for about $\frac{1}{10}$th of a second after the exciting cause ceases to exist. Taking the velocity of sound to be 1100 ft. per second under ordinary conditions, a sound wave can travel 110 ft. in $\frac{1}{10}$th of a second. So,

in order that the echo of sound may be distinctly heard, the reflecting surface should be at a distance of about 55 ft. from the observer, so that the reflected sound wave returns to the ear of the observer $\frac{1}{10}$th of a second after the incident wave left it.

Velocity of Sound.—*By means of echoes it is possible to obtain a rough estimate of the velocity of sound.* Suppose you stand some hundreds of yards from a hill and try to find out the time between a shout and its echo. If you are 500 yds. from the hill and the echo comes back in 3 seconds, the sound has travelled twice 500 yds. or 3000 ft. in 3 seconds and therefore has travelled 1000 ft. in a second. So the velocity of sound is 1000 ft. per second.

Series of Echoes.—Suppose a person at A is placed between two reflectors B and C situated at a distance of 330 ft. from each other, so that the distance AB is 110 ft. and AC 220 ft. Now if a pistol is fixed at A, the wave travels to B, is reflected and comes back to A reaching in $\frac{2\times110}{1100}=\frac{2}{10}$ seconds. The wave then travels to C and comes back to A in $\frac{2\times220}{1100}=\frac{4}{10}$ seconds after the first echo, *i.e.* $\frac{6}{10}$ seconds from the beginning. The wave again travels to B and is reflected. This goes on.

But in the beginning the sound wave also travels to C, is reflected and comes back to A in $\frac{4}{10}$ seconds It then goes to B and comes to A, $\frac{2}{10}$ second later and so on. So we get a series of echoes, beginning from B, in $\frac{2}{10}$, $\frac{6}{10}$, $\frac{8}{10}$, $\frac{12}{10}$, $\frac{14}{10}$, etc., seconds, and another series of sound beginning from C in $\frac{4}{10}$, $\frac{6}{10}$, $\frac{10}{10}$, $\frac{12}{10}$, $\frac{16}{10}$, etc., second.

Articulate Sounds.—In the case of *articulate* sounds, however, the distance of the obstacle should be at least twice, that is, 110 ft. instead of 55 ft., as observed above. It is so because a person cannot pronounce more than 5 syllables distinctly in one second, and the ear also cannot make out what is said if more than 5 syllables are pronounced in one second. If a person pronounces a he takes $\frac{1}{5}$th of a second for it by which time sound can travel through 220 ft. taking the velocity of sound to be 1100 ft. per second. So echo will be heard only if the reflecting surface be at least at a distance of 110 ft. from the observer. If the person pronounces any 5 syllables, say, a, b, c, d, and e, and if the reflecting surface be at a distance of 110 ft., then he will hear the echo of the first syllable just as he is about to pronounce the second syllable b. Similarly the echoes of b, c, d would come to him by $\frac{1}{5}$th of a second, just as he is about to pronounce the next one. So only the echo of the last syllable will be distinctly heard. This echo which enables us to hear only one syllable distinctly is called **mono-syllabic**. If the reflecting surface be at a distance of two or three times 110ft., the echo will be **di-syllabic, tri-syllabic,** and so on. Evi-

dently if the distance be n times 110 ft., then the echo of the last n syllable can be heard. Echoes which enable us to hear two or more syllables are sometimes called **polysyllabic.**

28 Echo Depth Sounding.—The reflection of sound has been applied in measuring the *depth of the sea.* For this purpose a hydrophone is placed under water and a small underwater charge of some explosive is placed near it. Two sounds are heard when the charge is fired, the direct sound of explosion coming through the hydrophone and the echo of it coming a little after being reflected from the sea-bed. The interval between the two sounds is recorded by some electrical device. If this is t sec., then taking the velocity of sound in water to be 4714 ft. per sec., the distance from the sea-bed and back must be $4714.t$. ft. and the depth of the sea $2357.t$. ft.

Examples.—*1 A man stationed between two parallel cliffs fires a gun. He hears the first echo after two seconds and the next after 5 secs. What is his position between the cliffs and when he hears the third echo?* (*All. 1910*)

Let V be the velocity of sound in air, x the distance of one of the cliffs from the man, and y the distance of the other cliff. Then, if the first echo be heard after two seconds, $2=\frac{2\times x}{V}$; or $V=x$.

The sound wave will also be reflected by the other cliff and come back after 5 seconds. $5=\frac{2\times y}{V}$; or $V=\frac{2}{5}y$. $\therefore$ $x=\frac{2}{5}y$; or $\frac{x}{y}=\frac{2}{5}$.

That is, the position of the observer divides the distance between the cliffs in the ratio of 2 : 5.

The third echo will be heard 7 seconds after the firing of the gun, for the sound wave reflected from either of the cliffs will be reflected from the other cliff and take 7 seconds to come to the man.

2. An engine is approaching a tunnel surmounted by a cliff, and emits a short whistle when half a mile away. The echo reaches the engine after $4\frac{1}{2}$ seconds. Calculate the speed of the engine assuming the velocity of sound to be 1100 ft per second. (*L M*)

Let A be the first position, B the second position when the echo of the whistle is heard and C the position of the cliff.

Then $AC=\frac{1}{2}$ mile $=2640$ ft. In $4\frac{1}{2}$ seconds the distance travelled by sound $=1100\times\frac{9}{2}=4950$ ft. $\therefore$ The distance $(AC+BC)=4950$ ft.

So $BC=4950-2640=2310$ ft. and $AB=(2640-2310)\times 330$ ft.

This distance is travelled by the train in $4\frac{1}{2}$ secs.

$\therefore$ Speed of engine $=\frac{330\times 2}{9}$ ft. per sec. $=50$ miles per hour.

3. An echo repeats 5 syllables, each of which requires $\frac{1}{5}$ of a second to pronounce and $\frac{1}{5}$ a second elapses between the time the last syllable is heard and the first

syllable is echoed. Calculate the distance of the reflecting surface, the velocity of sound being 332 metres per second.

The five syllables will take $(5 \times \frac{1}{5})$ second to pronounce. As $\frac{1}{2}$ a second elapses after the last syllable is pronounced in order that the echo of the first syllable is heard, so the time taken by the sound of the first syllable to travel to the reflecting surface and back to the observer is $1 + \frac{1}{2} = \frac{3}{2}$ seconds.

In $\frac{3}{2}$ sec. the sound will travel $332 \times \frac{3}{2} = 498$ metres. This distance is twice that between the observer and the echo; therefore the required distance $= \frac{498}{2} = 249$ metres.

4. *A man standing before a cliff repeats syllables at the rate of 5 per second. When he stops he hears distinctly the last 3 syllables echoed. How far is he from the cliff? (The velocity of sound in air is 1109 ft. per sec.)*

It has been explained that in the above case the distance of the reflecting surface must be 110 ft. Now because the last 3 syllables are heard distinctly the man must be at a distance of about $3 \times 110 = 330$ ft. from the cliff.

29. Nature of Reflected Longitudinal Wave.—Whenever a longitudinal wave passing through one medium meets another medium of different density, it will be partly reflected, *but the nature of the reflected wave will depend upon the density of the second medium. This* can be understood by the following illustration.

Expt.—Let a number of light and heavy steel balls be arranged in one line, the light balls representing the particles of a lighter medium and the heavy balls those of the denser medium. If a forward push be given to one of the lighter balls, it will strike the next ball, which in turn will strike its neighbour, and in this way energy will be handed on from one to the other, untill the last light ball strikes a heavy ball. After the impact, the light ball will rebound and strike a ball just behind it, and thus set up a *reflected pulse* backwards. It should be noticed that at the time of proceeding forwards, one ball was pressing against another and it appeared as if a compression wave was moving onwards.

After the impact, also, the same process is repeated backwards. Therefore the nature of the pulse is not changed. Similar thing happens in the case of longitudinal sound waves. *When such a wave meets a fixed end, or the surface of a denser medium, a wave of compression is reflected back as a wave of compression, and a wave of rarefaction is reflected as a wave of rarefaction,—that is, in reflection from a fixed and rigid surface, there is no change of type of the wave.*

Now, in the above experiment, if a forward impulse be given to one of the heavy balls, the direction of motion of the heavy ball after impact with a light ball will remain the same, *i.e.*, forward. But the second ball being lighter, the striking heavy ball after impact will

move with greater speed, so it will create *rarefaction* behind it. Consequently, *in the case of a longitudinal wave meeting a less dense medium, the reflected wave suffers a reversal of type ; a compressed wave is reflected back as a rarefied wave, and vice versa.*

If both ends of a spiral are free, a pulse of condensation travelling to the other end is reflected along the same path as a pulse of rarefaction So also a pulse of rarefaction returns as a pulse of condensation.

30. Refraction of Sound.—When sound-waves cross the boundary separating two media in which the velocity of transmission is different, they are refracted, obeying the same laws of refraction as for light. The refraction of sound may be demonstrated by taking a lens-shaped india rubber bag filled with any gas say, carbon dioxide, whose density is different from that of air. Refraction of sound, however, has got very little important application.

Atmospheric Refraction.—As the density of air changes due to change of temperature it follows that change of temperature of air causes refraction of sound-waves. During the day time the lower layers of air are at a higher temperature than those higher up. So the sound-waves, as they travel will be refracted upwards, *i e* their line of advance will be bent away from the ground , and hence the intensity at a distance will be diminished due to this effect. On the otherhand, at night time when the lower layers are colder than those above, as with layers of air over the surface of water, the bending of the line of advance will be towards the ground and the intensity will be increased. So in this case, sound from a longer distance will be heard much more clearly than in day time.

Questions

Art. 26.

1. Describe an experiment to demonstrate the reflection of sound. (C. U. 1946)

Name a few appliances based on the reflection of sound-waves. (Pat. 1944, cf. C. U. '46)

Art. 27.

2. What is an echo ? (C. U. 1946 ; Pat. '47)

Why is a succession of echoes sometimes observed ?

A man fires a gun on the sea-shore in front of a line of cliffs, and an observer, equidistant from the cliffs and 300 ft. away from the firer, notices that the echo takes twice as long to reach him as does the report. Find by calculation or graphically the distance of the man from the cliffs. (Pat. 1922)

[**Hints.**—A is the position of the firer and C that of the observer, B is the place on the cliffs where reflection takes place. (See Fig. 13. Part. III.)

From the question, $AN = NC = 150$ ft. if A, N and C are in the same st. line; and $AB = BC = 300$ ft. Hence calculate NB, which is the distance of the man from the cliffs.]

3. Explain how *echoes* are produced. How may the phenomenon be used to measure the velocity of sound in air? (L. M.)

4. A boy standing in a disused quarry claps his hands sharply once every second and hears an echo from the face of the opposite cutting. He moves until the echo is heard midway between the claps. How far is he then from the reflecting surface if the velocity of sound at the time was 1120 ft. per sec.

[*Ans :* 280 ft.]

5. At what distance from the source of sound must a reflecting surface be placed so that an echo may be heard 4 secs. after the original sound. (The velocity of sound in air is 1100 ft. per second.)

[*Ans :* 2200 ft.]

6. A man standing between two parallel cliffs fires a gun. He hears one echo after 3 secs. and another after 5 secs., what is the distance between the cliffs?

[*Ans :* 4400 ft.]

7. Six syllables are echoed by a reflecting surface placed at a distance of 650 ft. What is the temperature? ($V_0 = 1090$ ft. per sec.)

[*Ans :* -3°C.]

8. A cannon is placed 550 yards from a long perpendicular line of smooth cliffs. An observer at the same distance from the cliffs hears the cannon shot 4 seconds after he sees the flash. If the velocity of the sound is 1100 ft. per second, when will he hear the echo from the cliffs.

[*Ans :* 1 second after hearing the direct report.]

9. Explain the production of echoes. An echo repeated six syllables. The velocity of sound is 1120 ft. per sec. What was the distance of the reflecting surface?

[*Ans :* 672 ft.] (C. U. 1940).

Art. 28.

How is echo employed to measure depths of oceans? (C. U. 1946).

10. An echo repeats 4 syllables. Find the distance of the reflecting surface, if it takes one-fifth of a second to pronounce or hear one syllable distinctly. (Vel. of sound = 1120 ft. per sec.) (Pat. 1944)

[*Ans :* 448 ft.]

11. A man standing between two parallel cliffs fires a rifle. He hears the first echo after $1\frac{1}{2}$ secs., then a second $2\frac{1}{2}$ secs. after the shot, then a third

echo. Explain how these three echoes are produced. Calculate how many seconds elapsed between the shot and the third echo, and calculate the distance apart of the two cliffs. (C. U. 1944)

[*Ans :* $t=4$ secs ; Distance $=2\times$ vel. of sound].

CHAPTER V

Resonance : Interference : Stationary Waves

31. Free and forced Vibrations.—All bodies, no matter what their size, shape, or structure, vibrate in their own natural periods, when slightly disturbed from their positions of rest and left to themselves. Such vibrations are called *free vibrations.* A bob of a simple pendulum, when slightly moved to one side and then released, vibrates with its own period depending on its length ; so also large structures like bridges, tall chimneys, and large ships on oceans have got their natural periods of vibration.

If a periodic force be applied to a body capable of vibration, and if the period of the force be not the same as the free period of the body, the body will ultimately vibrate in a period equal to that of the applied force. Such vibrations of the body are called *forced vibrations.*

Examples.—If a vibrating tuning-fork is held by the stem in the hand, the sound will be almost inaudible even from a small distance, but if the stem be pressed on a table, the sound is much intensified. The reason is that the vibrations of the fork are communicated to the table which is thus forced to vibrate at the same rate Due to the vibrations of the table a large volume of the air in contact is made to vibrate, and the waves thus set up are added to those originating from the fork, and, consequently, the sound becomes louder.

The diaphragm of a gramophone sound box is a common example of forced vibration, where the diaphragm vibrates with frequencies corresponding to the tones conveyed from the record. The vibrations from the sounding boards of musical instruments like violin, piano, etc., are also forced vibrations. The sounding board of a violin, is first set into forced vibration by the vibration of the strings, and then the large mass of air inside the board also vibrates and intensifies the sound.

32. Resonance.—When a body is forced to vibrate, due to an applied external source, it vibrates with a very small amplitude, if the period of the applied force is different from that of the free period of the body; but when these two periods are the *same*, the body vibrates with a much greater amplitude. The latter phenomenon is known as **resonance**. *Thus resonance is a particular case of forced vibration and is produced when one body forces vibrations on a second body whose natural frequency of vibration is equal to that of the first.* The principles of forced vibration and resonance may be illustrated by the following experiment.

Experiment—Four simple pendulums *A*, *B*, *C*, and *D* are suspended from a flexible support. The lengths of *A* and *B* are equal, and so they have got the same period of vibration; *C* is slightly shorter, and *D* slightly longer than *A* or *B* (Fig. 17). When *A* is set in vibration, the flexible support is also set in forced vibration of the same period, but of similar amplitude. As a result of the vibration of the support, a periodic force of the same period is applied to each of the pendulums *B*, *C* and *D* which are made to vibrate. It will be found that *B*, whose length is equal to that of *A*, readily vibrates with an equal amplitude. *This is the case of resonance.* The pendulums *C* and *D* at first swing slowly, then come to rest, but ultimately vibrate steadily with the same period as that of *A*, but with smaller amplitude. They show forced vibration.

Fig. 17.

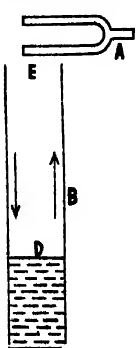


Fig. 18.—Resonant Air Column.

33. Resonance of Air Column.—The air-column within a tube may also be made to vibrate by resonance when a vibrating tuning-fork is held close to the upper end of the tube.

Take a vibrating tuning-fork *A* and hold it horizontally over a tall glass jar *B*. (Fig. 18). Now gradually pour water into the jar and note that for a certain length *ED* of the air column inside the jar a maximum sound is heard. Pour more water in, the sound disappears. This strengthening of the sound is called *resonance*, which, in this case, takes place when the period of vibration of the tuning-fork is equal to the natural period of vibration of the enclosed column of air.

It will be found, that for forks having different frequencies of vibration, the lengths of air-column giving maximum

resonance will be different. It will be greater or less as the frequency of vibration of the fork is lower or higher (for explanation see Art 55).

Sounding or resonance boxes—Tuning-forks are often mounted on hollow wooden boxes, called sounding or *resonance boxes*. The sizes of these boxes are so arranged that the enclosed mass of air has a free vibration, the period of which is equal to that of the forced vibration of the wood of the box. When the fork is struck, it sets the wood into forced vibration in the same period, and this agrees with the natural period of vibration of the enclosed mass of air; so the sound becomes louder due to resonance.

34. **Resonators**—The great German scientist Helmholtz (1821-1894) constructed globes of brass, each having a large aperture B for receiving sound-waves and a small one A at the other side against which the ear is placed. (Fig. 19). He utilised the principle of resonance in his investigations on the quality (see Art. 39) of notes emitted by various sources. These globes of various sizes are called **Helmholtz resonators**. In a given set of these resonators the size of each resonator is such that it can respond to a tone of given frequency and the tuning is so perfect that the particular tone if present in a complex note can be picked up with distinctness, by placing the ear at the small aperature A.

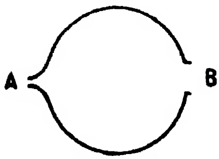


Fig. 19.—A Resonator.

35. **Sympathetic Vibration.**—

It is due to sympathetic vibration that if two stringed instruments are tuned to the same frequency and if one of them is sounded, the other also is automatically excited when placed close by.

Let two turning-forks of the same vibration frequency fitted to two resonance boxes be placed near each other. One of them is bowed strongly and then the vibration is stopped by touching it, when the other will be found to emit the same note, although it has not been bowed at all. This is a case of resonance. The vibration of the second fork is called *sympathetic vibration*. The phenomenon will not happen if the frequencies of the forks are not exactly the same.

If a sequence of small repeated impulses be applied to a vibrating pendulum, and if each push be given exactly at the end of one complete swing, or, in other words, if the period of the impulse be exactly equal to the period of vibration of the swing itself, the pendulum will vibrate so that each succeeding swing will be greater than the previous one. It is for this reason that soldiers are ordered to break step when

crossing a suspension bridge, as otherwise the regularity of the impulse due to the steady marching would agree with the natural period of vibration of the bridge, which will set up dangerous oscillations. Similarly a ship at sea may be thrown into dangerous oscillations when the frequency at which the ship is struck by the wave is equal to its natural frequency of vibration.

36. (a) Interference of Sound.—When two systems of waves travel through any medium simultaneously, the actual disturbance at any point of the medium at any instant is the resultant of the component disturbances produced by the waves separately, *the displacement at any point in the medium being the algebraic sum of the displacements due to the separate waves.* This is known as the **principle of Superposition.** If the crests of the two waves arrive simultaneously at the same point, *i.e* if they are in the *same phase,* then they will combine to produce large crests; and similarly two troughs arriving at the same point at the same instant will produce deeper troughs. But if the two waves are exactly *similar,* and if conditions are such that the *troughs* of one wave fall upon the *crests* of the other, *i.e.* if they are in *opposite phases,* then they will completely annul one another and the result will be the absence of any disturbance in the medium at that place and the *two sound-waves,* in such a case, *will produce silence. This is the* **principle of interference of wave-motion.**

By dropping two stones into a pond simultaneously at two neighbouring points, two sets of circular ripples are produced and when these ripples intersects one another, a definite *interference pattern* is observed. Some lines can be seen along which the water particles are undisturbed and there are other intermediate lines, along which a maximum disturbance occurs. Similarly in sound-waves the compressions of one set serve to neutralise the rarefactions of the other set at other points and to reinforce compressions in the other set at other points.

Beats. When two sounds of the same type and intensity of very nearly the same frequency of vibration are produced together, a fluctuation of loudness (waxing and waning of sound) occurs due to the mutual interference of the two notes. In the resulting sound-wave the component waves reinforce each other at some regions, and destroy each other at other places, and so the sound heard possesses a characteristic throbbing or beating effect. *This phenomenon is known as* **Beats.** *The phenomenon may be presented graphically as follows.*

In Fig. 20, the dotted curves represent two waves (arranged on

the same axis) produced by two vibrating tuning-forks of slightly different frequencies. At the beginning of a given second, the two forks are swinging together so that they simultaneously send out condensations, and the result of the two condensations will produce a double effect upon the ear (as at A in Fig. 20). But as the frequencies of the forks differ, the subsequent effect upon the ear is represented by the continuous curve which is the result of combining these two waves, and is obtained by finding the algebraic sum of the separate displacements at each point.

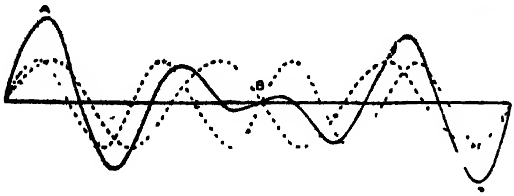


Fig. 20.—Formation of Beats.

It is evident from the nature of the continuous curve that its amplitude varies in a periodic manner, being maximum at A and C, and minimum at B due to which there is a periodic change in the intensity of the sound heard. At A, where the vibrations are in the *same phase*, the resultant displacement is the sum of the displacements of the two component waves, and at B these are in *opposite phases* and the resultant displacement is given by their difference. As the loudness depends upon the amplitude of vibration, the sound heard through small intervals near A and C is the loudest, where the amplitudes are maximum, and it is minimum at B where the amplitude is minimum. Such fluctuations of loudness of the sound are known as **beats**.

Suppose two tuning-forks having frequencies, say, 256 and 257 per second respectively, are sounded together. If, at the beginning of a given second they vibrate in the *same phase* so that the compressions and rarefactions of the corresponding waves reach the ear together, the sound will be strengthened. Half a second later, when one makes 128 and the other $128\frac{1}{2}$ vibrations, they will be in *opposite phases i.e.* a compression of one wave will unite with a rarefaction of the other and will tend to produce silence. At the end of one second they will again be in the *same phase* and the sound will be augmented, and by this time, one fork will gain one vibration over the other. Thus, in the resultant sound the observer will hear the maximum of loudness at every interval of one second. Similarly a minimum of loudness will be heard at an interval of one second. As we may consider a single beat to occupy the interval between two consecutive maxima or minima, *the beat produced in the above cases is one in each second.* It

is evident, therefore, that when two sounds of nearly the same vibration frequency are heard together, the number of beats per second is equal to the *difference* of the frequencies of the two vibrating sources. Thus, if n_1 and n_2 be the frequencies of the two sources, then the number of beats per second is equal to $(n_1 - n_2)$. Thus *the number of beats heard each second is numerically equal to the difference in frequencies of the two sounds.*

Number of beats per sec. is the difference between the two frequencies

Let the smaller of the two frequencies be n_1 and the other differ from it by n. Assuming that they start with the same phase, displacements produced by the two wave-systems at a point at some instant of time t will be given by,

$$y_1 = a \sin 2\pi\, n_1 t \text{ and } y_2 = b \sin 2\pi(n_1 + n)t.$$

By the principle of superposition, the resultant displacement will be given by,

$$y = y_1 + y_2 = a \sin 2\pi\, n_1 t + b \sin 2\pi(n_1 + n)t.$$
$$= \sin 2\pi n_1 t(b \cos 2\pi nt + a) + b \cos 2\pi n_1 t. \sin 2\pi nt.$$

This equation represents a wave-equation which may be condensed into the form $y = F \sin (2\pi n_1 t + \alpha)$ where F is its amplitude and α, the epoch. The values of F and α can be found by comparing the two equations and equating the co-efficients of $\sin 2\pi n_1 t$ and $\cos 2\pi n_1 t$. That is,

$$F \cos \alpha = b \cos 2\pi nt + a \quad \text{and} \quad F \sin \alpha = b \sin 2\pi nt.$$

By squaring both sides and adding,

$$F^2 = b^2 \sin^2 2\pi nt + b^2 \cos^2 2\pi nt + 2ab \cos 2\pi nt + a^2$$
$$= a^2 + b^2 + 2ab \cos 2\pi nt \qquad \ldots \quad (1)$$

Also, $$\tan \alpha = \frac{b \sin 2\pi nt}{b \cos 2\pi nt + a} \qquad . \quad (2)$$

It is evident from (1) that the amplitude of the resultant waves varies with time. It assumes maximum and minimum values as follows,

when $t = 0$, $\cos 2\pi nt = 1$, $F = a + b$ (maximum),

when $t = \frac{1}{2n}$, $\cos 2\pi nt = -1$, $F = a - b$ (minimum);

when $t = \frac{1}{n}$, $\cos 2\pi nt = 1$, $F = a + b$ (maximum);

Thus in an interval of $\frac{1}{n}$ second, two maxima and an intermediate minimum take place. Similarly it can be shewn that between two minimum sounds, a maximum occurs in a period of $\frac{1}{n}$ sec. So the number of beats (two successive maxima or two successive minima

produce one beating effect) per sec is $= n =$ the difference of the frequencies.

(*b*) **Tuning Instruments**—It should be *remembered* that beats can be heard only when the frequencies of the notes are nearly equal to each other; if their difference is greater than 15 or 16, separate beats cannot be heard and a discordant unpleasant noise is the result It is for the above reason that musical instruments are **tuned by means of 'beats'**. If beats are heard between the first overtone (see Ch. VII) of a lower note and its octave, say, in the case of a piano or organ, it is a sure test that the instrument needs tuning. Beats are not heard when the frequencies of the two sounds are exactly equal.

37 Determination of the Frequency of a Fork by the Method of Beats.—Two forks having nearly the same frequency are mounted on sounding-boxes and sounded together. The number of beats in any time is counted by means of a stop-watch, and, from this, the number of beats per second is determined, which is equal to the difference of the frequencies of the forks. By knowing the vibration frequency of one of them, that of the other can be determined To know whether the frequency of the given fork will be higher or lower than that of the other, one of the prongs of the given fork is loaded with a little wax, and the number of beats per second is again determined. The frequency of the fork is diminished by loading its prong. Hence, if the number of beats per second obtained after loading the fork is *greater* than the number obtained before, the frequency of the given fork must be *less* than that of the known fork, if the number be *less*, then the frequency of the unknown fork is *greater* than that of the known fork.

N B The frequency of a fork is increased by filing it.

The **uses of beats** are in (*a*) *finding frequencies*; (*b*) *tuning instruments*.

Examples—*1. Two tuning forks A and B, the frequency of B being 512, are sounded together and it is found that 5 beats per second are heard. A is then filed and it is found that beats occur at shorter intervals. Find the frequency of A.*

(*All. 1916, C U 1936.*)

Since A is filed, its period is diminished, and its frequency is increased; but because beats occur at shorter intervals, *i.e.* the number of beats increases by increasing the frequency of A, it is clear that the frequency of A is greater than that of B.

If n_1 and n_2 be the frequencies of A and B respectively, we have

$$n_1 - n_2 = 5\text{ ; or } n_1 - 512 = 5\text{ ; } \therefore\ n_1 = 512 + 5 = 517.$$

2 The interval between two tones is $\frac{15}{16}$ and the higher tone makes 64 vibrations per second. Calculate the number of beats occurring per second between the tones (L.M.)

The interval is the ratio of the two frequencies (see Art. 41). Let the frequency of the first be n, then we have

$\frac{15}{16} = \frac{n}{64}$; or $n = 60$. $\quad\therefore$ The number of beats $= (64 - 60) = 4$ per sec.

3. A fork of unknown frequency, when sounded with one of frequency 288, gives 4 beats per sec. and when loaded with a piece of wire again gives 4 beats per sec. How do you account for this and what was the unknown frequency? (*Pat. 1945*)

The experiment shows that the unknown frequency n in the beginning was higher by 4 and after loading the fork with a piece of wire the frequency n' was lowered by 4, *i.e.* it became $(288-4)=284$. So the unknown frequency $n=288+4=292$.

37(a). The conditions for interference of two sounds :—

(1) The component waves must have the same frequency and amplitude.

(2) The type of the two waves must be similar.

(3) The displacements caused by them must be in the same line.

37(b). Experimental demonstration of accoustical interference

Two separate sources producing waves satisfying the conditions for interference can not be realised in practice. That is why, in practice, the waves from a single source are divided at a point and made to reunite again at some other region after travelling a path of different length. **Quinke** based his arrangement on this principle and his apparatus consists of a mouth-piece A connected to the two limbs B and C which combine, again into one tube EF against which the ear is placed. D is a sliding tube, by drawing it in or out the length of the path $ACDE$ can be suitably altered. A vibrating tuning fork T is held at A and the resulting sound at F is heard. When the sliding tube is at D, the paths ABE and $ACDE$ are equal so that the two waves passing through them meet in the same phase at F and produce a maximum sound. The path $ACDE$ is then increased by drawing out the sliding tube D till a position D_1 is obtained when a minimum sound is produced. The diff. in path between $ACDE$ and ACD_1E is half the wave-length. By further drawing out the tube D from D_1 to D_2, again a maximum sound is obtained, the shift so made being equal to half the wave-length again. Thus the full wave-length of the sound used is obtained.

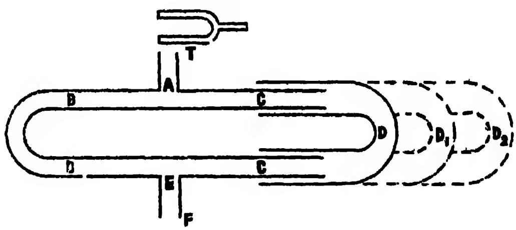


Fig. 20(a)

38. Progressive and Stationary Waves —

Progressive Waves.—In a *progressive wave* all the particles of the medium execute simple harmonic motions about their mean positions. The phases of the particles change continuously from one to another

along the direction of propagation of the wave. So each particle in turn goes through a similar movement. The wave consisting of alternate crests and troughs, or compressions and rarefactions, advances onwards with a fixed velocity.

An ordinary sound-wave in air is an example of longitudinal progressive wave and an ordinary water-wave is a transverse progressive wave.

Stationary Wave —When two exactly similar waves, that is, waves of the same type, period and amplitude, travel through a medium in *opposite directions* with the same velocity and along the same line, the resultant wave is known as a *stationary wave*, where the vibration at each point is fixed or stationary in character, that is, there is apparently no transmission of vibratory motion from one particle to another, and the compressions and rarefactions in the wave merely appear and disappear at certain regions without progressing in any direction, the amplitude of the resultant stationary wave being twice as great as that of either of its components. In such a wave, therefore, *the positions of* **nodes**, *i.e. the points of maximum change of pressure and minimum amplitude of vibration, and those of* **antinodes** *or loops, i.e. the points of the minimum change of pressure and maximum amplitude of vibration, are fixed.*

A stationary wave, results from interference between an incident and its reflected wave, in the case of a longitudinal wave directly reflected at a plane surface.

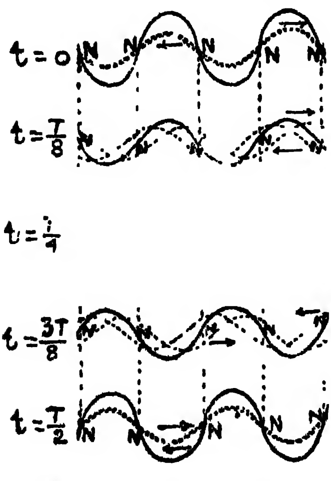


Fig. 21.—Stationary Waves.

The longitudinal vibration of air-particles in organ pipes (Art. 52) sets up *stationary longitudinal waves*, and the waves set up in a string tightly clamped at both ends (see Art. 45) are examples of *stationary transverse waves*.

In Fig. 21 is shown graphically the addition of two tranverse harmonic waves travelling in the opposite direction. The full curve represents the resultant waves obtained by adding the ordinates, *i.e.* the displacements of the two dotted curves. The second diagram in the figure shows the two waves and their resultant, a time $\frac{1}{8}T$ later than the first ; that is, each wave has advanced one-eighth of a wave-length (λ), one $\frac{1}{8}\lambda$ to the right and the other $\frac{1}{8}\lambda$ to the left. The third diagram shows the waves $\frac{1}{8}T$ later than the

second, *i.e.* $\frac{1}{4}T$ later than the first, and one of the dotted curves has moved $\frac{1}{8}\lambda$ to the right farther than the preceding one and the other $\frac{1}{8}\lambda$ to the left farther than the preceding one. The dotted curves exactly neutralize one another and the resulting disturbance is represented by a straight line. Similarly the fourth and the fifth diagrams represent the waves and their resultants respectively after times $\frac{3}{8}T$ and $\frac{1}{2}T$. By taking times $\frac{5}{8}T$, $\frac{6}{8}T$ etc., it will be seen that the same changes are produced in the reverse order.

Note that the points of the full curves marked N through which dotted vertical lines pass are always at rest. These points are called *nodes*. The points midway between the nodes are called *antinodes* or loops. These are points of maximum disturbance. The resultant disturbance simply changes the form given by the full curve, but there is no forward motion of the wave as a whole. Waves of this character in which the positions of nodes and antinodes are fixed are called **Stationary Waves.**

Nodes and Antinodes.—In a stationery wave, the vibration at each point is fixed or stationery in character. Those points at which there is maximum change of pressure and minimum amplitude of vibration are called nodes. The point midway between any two nodes is a point of minimum change of pressure and maximum amplitude of vibration—such a point is called an antinode. The distance between two successive nodes or two successive antinodes is equal to half the wave-length of either of the two interfering waves

Progressive and Stationary Waves Compared.—

Progressive Waves	*Stationary Waves*
(*i*) All particles of the medium execute simple harmonic motions about their mean positions, and have the same amplitude.	(*i*) All particles of the medium (except at some equidistant points) execute simple harmonic motions of varying amplitudes. The points where the amplitude is minimum are called Nodes. At some other equidistant points midway between the nodes, the amplitude is maximum. These are antinodes. The period of the S. H. M. is that of the component waves,
(*ii*) The wave travels onwards with a definite velocity.	(*ii*) The wave is not bodily transferred from one part of the

Progressive Waves	*Stationary Waves*
	medium to another ; and the compressions and rarefactions of the wave merely appear and disappear at certain regions without progressing in either direction.
(*iii*) The movement of one particle begins just a little later than its predecessor, or in other words, the phases of the particles change continuously from one particle to another.	(*iii*) At any instant all the particles in any one segment, *i.e.* between two consecutive nodes, are in the same phase, but the particles in two consecutive segments are in opposite phase.
(*iv*) Each particle of the medium in turn goes through a similar movement, *i.e.* similar changes of pressure and density, as a complete wave passes through it and is restored to its initial condition after the periodic time.	(*iv*) The particles at nodes undergo maximum change of pressure while those at antinodes undergo minimum change of pressure throughout the motion.
(*v*) In a complete vibration all the particles of the medium are not stationary at any instant.	(*v*) Twice in each complete vibration all the particles are at rest at the same moment (see line 3, Fig. 21).

Questions

Art. 31.

1(*a*). Explain clearly the difference between Forced vibration and Resonance. Give mechanical and acoustical illustrations. (Cf. C. U. 1909) (See also Art. 32).

(*b*) Write notes on "Forced vibrations". (Pat. '47)

2. Describe experiments to illustrate the principle of forced and free vibrations and give illustrations in case of sound. (Pat. 1931)

Art. 32.

3. Explain the principle of resonance.
(All. 1925, '29, '45 ; Pat. '29, '30 ; C. U. '29).

Art. 33.

4. Explain why, when the handle of a vibrating tuning-fork is pressed against a wooden board, the intensity of sound is greatly increased ?
(C. U. 1915 ; cf. 1920, '31, '47).

Art. 34

5. Explain what do you mean by 'resonance' and 'resonators'.
(Pat. 1929 ; cf. '31, '33 ; All. '18).

6. Explain how resonators are used for the analysis of sound.

Arts. 46 & 37.

7. What are beats ? (Pat. 1947).

How are they produced ? If two tuning-forks sounded together produce beats, how would you determine which was of the higher pitch ?

(All. '25, '32, '44 ; Dac. '30 ; cf. Pat. '32, '40, '41, '45 ; cf. C. U. '33, '39.)

8. A standard fork A has a frequency of 256 vibrations and when a fork B is sounded with A there are four beats per second. What further observation is required for determining the frequency of B ? (C. U. 1933)

[*Ans :* The frequency is either 260 or 252. To know exactly the frequency of B, we should know whether the frequency of A is greater or less than that of B.]

9. You are provided with two tuning-forks of nearly equal frequencies. Explain how you would proceed to find out which of the two has the greater frequency. (Pat. 1941)

Art. 38.

10. Distinguish between a progressive and a stationary wave giving an example of each and illustrating your answer by diagrams.

(C. U. 1923, cf. '10, '39 ; cf. Pat. '31, '35 ; All. '31, '46)

11. What are beats and stationary vibrations ? Explain by composition of vibrations the production of beats and stationary vibrations. (Pat. 1937) (see also Art. 36).

12. What are stationary waves ? (C. U. 1947).

CHAPTER VI

Musical Sound : Musical Scale : Doppler Effect

39. (a) Musical Sound and Noise.—Sound may be divided into two classes ; (*i*) *Musical Sound*, and (*ii*) *Noise*.

A *musical sound* is a pleasing continuous sound which is produced by regular and periodic vibrations ; sounds produced by a tuning-fork, a violin, or a piano are all musical sounds.

Noise is a general term including all sounds other than musical sounds. It is discordant and unpleasant to the ear.

The essential *difference* between a musical sound and a noise lies in the fact that in the former case the vibrations are *regular* and

periodic ; while in the case of a noise, the vibrations are *irregular* and *non-periodic* in character. It is, however, difficult to draw up a clear line of demarcation between musical sounds and noises, as in practice, musical sounds are seldom free from some irregularities of vibration, while, on the other hand, in noises sometimes there is regular periodicity of the vibratory motion producing some regular wave-motion in the irregular disturbance. Very frequently noise is accompanied by musical vibrations as in the clang of a bell. Moreover the difference is subjective The same sounds may appear to be musical or noisy to different persons and under different conditions. Therefore, the difference is more artificial than real.

(*b*) **Characteristics of Musical Sound.**—Musical sounds may be said to differ from one another in three particulars. These are

(1) *Intensity* or *loudness* ; (2) *Pitch* : (3) *Quality* (or *Timbre*).

(1) Intensity.—It is a measure of the loudness or volume of a note. It is an objective consideration and depends on the energy contained per unit volume of the medium through which the sound waves are passing. It may also be measured by the energy which passes through unit area placed normal to the direction of propagation. It is a characteristic of all sounds, whether musical or not

(*i*) The loudness depends upon *the square of the amplitude* or *the extent of vibration* of the sounding body. When the body vibrates with greater amplitude, it sends forth a greater amount of energy to the surrounding medium, and, hence, energy received by the drum of the ear is also greater. So the sound appears to be louder.

The energy e of a body of mass m vibrating with velocity v and amplitude a is given by,

$$e = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{2\pi a}{T}\right)^2 = \frac{2\pi^2 ma^2}{T^2} \text{ (see Art. 10)}, \quad \therefore\ e \propto a^2.$$

Therefore *the loudness of a note* which depends upon the energy of the vibration *is proportional to the square of the amplitude of the vibration.*

(*ii*) The loudness of a sound is *inversely proportional to the square of the distance* of the observer from the source. **(Inverse Sq. Law).**

Thus the energy received by the observer at a distance of 2 metres from the source is only one-fourth the energy which the observer would experience when at a distance of 1 metre from the source.

[Supposing it is required to compare the intensities of the sound at two points A and B, distant r_1 and r_2 from a source of sound from

which the total sound energy emanating per sec. uniformly all around is E. Draw two spheres with the source as centre with radii r_1 and r_2 respectively. The amount of energy flowing per unit area normal to the surface of the sphere $= I_A =$ intensity at $A = \frac{E}{4\pi r_1^2}$. Similarly, the intensity at $B = I_B = \frac{E}{4\pi r_2^2}$. $\therefore \frac{I_A}{I_B} = \frac{r_2^2}{r_1^2}$. That is, intensity at a point is inversely proportional to the sq. of the distance.]

(iii) The loudness of a sound depends upon *the density of the medium* in which the sound is produced. It is seen that the greater the density of the medium, the greater the loudness of the sound heard.

It is seen that some effort is to be made to make oneself heard by another in aeroplanes or balloons when flying high up from the surface of the earth as the density of air there is much less. For the same reason the sound is more intense in carbon-dioxide than in air.

(iv) The loudness of a sound depends upon the *size of the vibrating body*.

If the size be larger, then a larger volume of the medium is put into vibration, and hence, a greater amount of energy will pass through each unit area. So the sound heard will be louder.

(v) The loudness of a sound is increased by the *presence of resonant bodies*.

The sound of a tuning-fork, or a vibrating string in air, is much intensified when placed on a sounding-box which undergoes forced vibration.

(2) **Pitch.**—The *pitch* of a note depends on the *frequency of vibration of the sounding body*. Pitch refers to musical sounds only. Two notes of the same intensity sounded on the same instrument will differ in pitch when their vibration frequencies are different. The greater or smaller the frequency, the higher or lower is the pitch of the sound.

(3) **Quality or Timbre.**—The *quality* or *timbre* is that characteristic of a musical note which enables one to distinguish a note sounded on one musical instrument from a note of the same pitch and loudness sounded on another instrument.

Experimentally it has been known that a musical note is a mixture of several simple tones ; of these the one having the lowest frequency, called the fundamental, is relatively the most intense. Its

frequency determines the pitch of the note. *The notes of the same pitch and loudness sounded on two different musical instruments differ in quality from each other owing to the difference in the number of other tones (or overtones) besides the fundamental, their order of succession and their relative intensities.* Helmholtz investigated on the physiological effects of overtones. He found that a note possessing the fundamental and the first few overtones not exceeding the sixth is very pleasing to the ear while a note in which the fundamental has mixed up in it more overtones than the sixth and which are relatively more intense, produces a metallic and harsh effect. *He also found that the phase difference between the constituent overtones does not affect quality.*

Since the quality of a note depends on the number of overtones, their order and their relative intensities, two notes similar in pitch and loudness but differing in quality, will have different wave-forms, though the wave-length and amplitude of their fundamentals may be the same and their pitch and loudness also the same So the nature of the displacement curve of a note represents its quality.

40. Determination of Pitch.—The pitch of a musical note is determined by the frequency of vibration of the source of the note. Determination of frequency can be done by the following methods.—

(*1*) **Savart's Toothed Wheel.**—This consists of four toothed wheels of equal diameter mounted concentrically on a spindle fitted to a whirling table (Fig. 22). The number of teeth on each wheel conforms to a certain ratio, *e.g.* 20, 30, 36, 48.

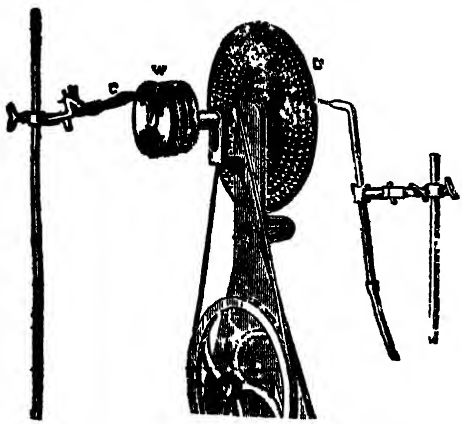


Fig. 22.—Savart's Toothed Wheel (W) and Seebeck's Siren (D)

A thin metal plate or a card-board *C* is clamped in front of the wheel so that it lightly presses against the teeth of one of the wheels *W* when it is in motion, and a sound formed by a series of taps is heard. On increasing the speed of rotation, a musical sound is produced, the pitch of which depends on (*a*) *the number of taps made in a given time*, and (*b*) *the speed of rotation.*

To determine the pitch of a note. the speed of rotation of the wheel is gradually altered by keeping the card against a particular wheel until the note emitted by the wheel is in unison (see Art. 41) with the given note. Now, if m be the number of teeth in the wheel, and n the number of revolutions per second, the frequency N of the note is given by N = number of taps made per sec. = mn.

(2) **Seebeck's Siren**—Seebeck's siren or Puff siren consists of a circular metal disc D (Fig. 22) through which a number of equidistant small holes have been drilled along concentric circles of varying diameter. The disc is mounted on a whirling table. A stream of air blown through a narrow tube ending in a nozzle by means of foot-bellows is directed to pass through the holes in one of the rings. As the disc rotates, the stream of air through the tube is alternately stopped and allowed to pass through the holes producing a series of puffs at regular intervals. To determine the pitch of a note, the rotation of the siren is adjusted until the note produced by the siren is exactly in unison with the given note. Now, if m be the number of holes in the ring used, and n the number of revs. per sec. made by the whirling table, the frequency N of the given note is given by N = number of puffs made per sec. = mn.

Note.—The highest frequency up to which a note is audible varies from 20,000 to 30,000 per second and the lowest is about 20 per second.

Example.—*The disc of a siren is making 10 revolutions per second. How many holes must it possess in order that it may produce four beats per second with a tuning-fork of frequency 484? Which has the greater frequency, the siren or the fork?*

The number of beats per second is numerically equal to the difference of frequencies of the two notes. Hence the note emitted by the siren must have a frequency of $(484+4)=488$; or, $(484-4)=480$.

The frequency of the note emitted by the siren N = no. of holes in the siren × no. of revolutions per second. $\therefore$ N = no. of holes × 10.

As the number of holes must be a whole number, N must be a multiple of 10. So the value 488, which is not a multiple of 10, cannot be accepted, Hence $N=480$.

$\therefore$ 480 = no. of holes × 10. $\therefore$ The no. of wholes $=\dfrac{480}{10}=48$. Evidently the fork has the greater frequency.

(3) **Cagnaird de la Tour's Siren**—This is a much improved form of siren by which the pitch of a note can be fairly accurately determined. In this siren (Fig. 23) a current of air is blown through a pipe into a wind-chest A, from which it issues through a ring of

equidistant holes cut in the circular top of the wind-chest. Another disc having holes exactly corresponding with holes in the top of the box is supported on the top of the wind-chest, and very close to it, in such a way that it can rotate freely about a vertical axis. The two sets of holes are drilled so as to slant in opposite directions, as shown in Fig. 23, so that the pressure of the air at the time of escaping through the holes causes the upper disc to rotate, the number of rotations being counted by a speed-counter S attached to the axle of the disc.

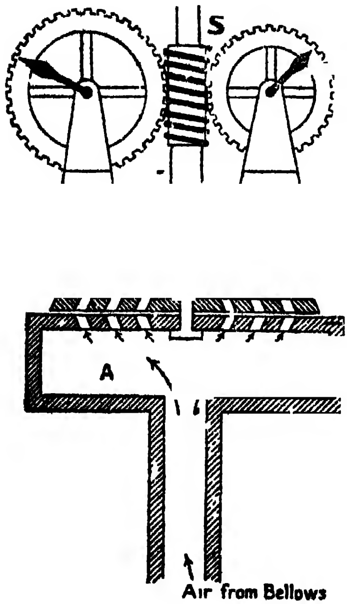


Fig. 23.—Cagniard de la Tour's Siren.

Every time the holes in the two rows coincide at the time of rotation of the upper disc, a jet of air escapes from each hole in the upper disc, and, if there are m holes in each of the discs and the lid, there will be m puffs for one revolution of the disc. Of course each puff will consist of m separate jets, but, as they take place simultaneously, they are regarded as a single puff. Now, if n be the number of revolutions of the disc per second, the frequency of the note emitted is equal to mn.

(*4*) **Resonance of Air Column.**—In the relation $V=4nl$ in Art. 55, if V and l are known, the frequency n can be determined.

(*5*) **Method of Beats.**—The frequency can also be determined by the method of beats, as explained in Art. 37.

(*6*) **Sonometer.**—By tuning a vibrating string on a sonometer in unison with a given note, the frequency of the note can be determined by the formula

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}}. \qquad \text{(See Art. 48).}$$

(*7*) **Direct or Graphical Method. (Duhamel's Vibroscope)**—The frequency of a vibrating fork can be determined by the graphical method. A sheet of smoked paper is wrapped round a cylindrical which can be rotated uniformly by means of a handle attached

to it (Fig 24). A thin metal style is attached to one prong of the tuning-fork which is so arranged that it can vibrate parallel to the axis of the drum and the style just touches the smoked paper. As the drum is rotated, the style will trace a wavy line on the paper. If, at the time of the vibration of the fork, two points can be marked on the wavy line of the smoked paper at an interval of half-a-second, or one second, the frequency of the fork can be determined by actually counting the number of complete vibrations between the points.

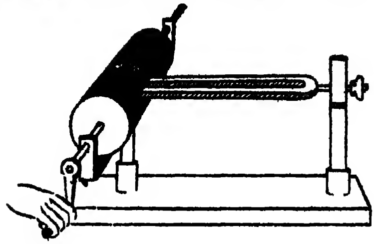
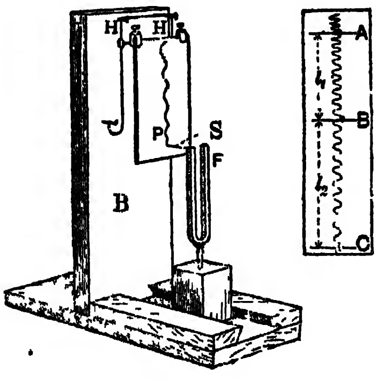


Fig. 24

In order that the amplitude of the waves traced out by the style may not decrease owing to effects of friction, in actual practice, the fork is excited and its vibrations are maintained electromagnetically. The tracing time is recorded by means of an electric pendulum which is so arranged as to produce a spark at the expiry of a rated interval. Corresponding to successive sparks, spots are made on the wavy line traced on the smoked paper. The number of vibrations made by the fork in the rated interval (and hence the frequency of the fork) is given by the number of complete waves found between any two consecutive spots.

(8) **Falling Plate Method**—In this expt. an arrangement is so made that a plate may fall freely under gravity. A glass plate P blackened preferably by camphor smoke is suspended vertically by means of a thread from two hooks fixed on a vertical piece of wood B as shown in Fig. 25(a). A tuning-fork F to one prong of which a very light style S is fixed is clamped in front of the plate in such a way that the style just touches the smoked plate during the fall. The fork is set into vibration by striking it with a violin bow and then the plate is released by burning the thread between the hooks. As the plate falls under gravity the style draws a wave-trace [Fig. 25(b)] of steadily increasing wave-length upon the smoked glass.



(a) (b)
Fig. 25.—Falling Plate Method.

Ignoring waves at the beginning, which are very crowded (and not suitable for counting), choose two lengths AB and BC having the same number of complete waves.

Let the velocity of the plate at the point $A = u$, and the time required for the plate to fall through the distance AB or $BC = t$ Calling $AB = l_1$ and $BC = l_2$, we have $l_1 = ut + \frac{1}{2}gt^2$ The velocity at B being $(u + gt)$, we have $l_2 = (u + gt)t + \frac{1}{2}gt^2$

$$\therefore \quad (l_2 - l_1) = gt^2 \ , \quad \text{or} \quad t = \sqrt{\frac{l_2 - l_1}{g}} \qquad \dots \quad (1)$$

If n be the frequency of the fork F and m the number of complete waves between AB or BC, we have $nt = m$.

$$\therefore \quad n = \frac{m}{t} = m \div \sqrt{\frac{l_2 - l_1}{g}} \quad \text{from (1).}$$

N. B This method has the disadvantage that by attaching the style to the prong, the frequency of the fork is altered. Also there may be some friction between the plate and the slide by which the free rate of fall of the plate is affected.

Example.—*A small pointer, attached to one of the prongs of a tuning fork presses against a vertical smoked glass plate The fork is set vibrating and the glass plate is allowed to fall If 30 waves can be counted in the first 10 cm find the frequency of vibration of the fork* ($g = 980$ *cm* $/sec.^2$) *(Pat 1913)*

We have, the distance fallen, $S = ut + \frac{1}{2}gt^2$, but here $u = 0$, $S = 10$

$$\therefore \quad 10 = \tfrac{1}{2}gt^2 \ ; \ \text{or} \ t^2 = \frac{20}{g} = \frac{20}{980} = \frac{1}{49} \ ; \ \text{or} \ t = \frac{1}{7} \, .$$

As 30 waves are counted in $\frac{1}{7}$ second, we have frequency $n = 30 \div \frac{1}{7} = 210$.

41. Musical Scale.—We express the pitch of a note by the number vibrations per second, but pitch can also be expressed by what is known as the musical method In this method certain sounds constitute what we call a *musical scale* The musical scale used for many centuries by most of the European countries is called the **Major Diatonic Scale**, which affords the simplest and the most pleasing succession of notes in an ascending order of frequency. This scale consists of eight notes, the lowest one, *i e.* the fundamental note, is named *do*, and others *re*, *me*, etc., and they are generally designated by the letters C, D, E, F, G, A, B, C'. The note from which the scale starts is called the **tonic or key note**.

Interval.—The ratio of the frequencies of two notes expresses the *interval* between them. Thus the interval of two notes having frequencies 256 and 192 is $\frac{256}{192}=\frac{4}{3}$; 512 and 256 is $\frac{512}{256}=\frac{2}{1}$; and so on. It is the interval which is detected by the ear. In changing from one frequency to another, the change-over is not recognised by the ear provided the ratio between them is constant, whatever might be the actual frequencies concerned. Certain intervals have names ; thus $\frac{2}{1}$ is called the **octave** ; $\frac{3}{2}$ the **fifth** ; $\frac{4}{3}$ the **fourth** ; $\frac{5}{4}$ the **major third** ; $\frac{6}{5}$ the **minor third**.

Any two intervals are added together by taking the product of their frequency ratios. For example, major third and minor third $= \frac{5}{4}\times\frac{6}{5}=\frac{3}{2}=$ *fifth.*

Again *fifth* and *fourth* $=\frac{3}{2}\times\frac{4}{3}=\frac{2}{1}=$ *octave.*

41. (a). Some Terms.—When the two notes have the same frequency, *i.e.* their interval is 1, they are said to be in *unison.* Two notes, when sounded together, are said to be in *concord* or *consonance* when they give a pleasing sensation to the ear. This happens when the interval between them is a simple ratio such as 2 to 1, 3 to 2, etc. But if the ratio is complicated such as 9 to 8, 15 to 8, etc., they produce an unpleasant or harsh effect and they are said to be in *discord* or *dissonance.*

According to Helmholtz, the cause of dissonance is the production of beats by the interference of the notes. The beats produce a jerking effect on the ear-drum and are discordant, just as flickering of light is disagreeable to the eye.

The pleasing effect produced by sounding two notes, which are in concord, one after another, is called **melody** : and when they are produced simultaneously, the pleasing effect is called **harmony**. When three notes of frequencies in ratios 4 : 5 : 6 are sounded together they form a concordant combination which is called a musical **triad** (*e.g.* $C : E : G$) and if a triad is sounded with an additional note which is the octave of the lowest note of the triad the combination is known as a **chord**. When one musical instrument alone, such as a violin or a flute, is played upon, the performance is called a *Solo.*

Octave.—One note is an *octave* (Gk. *okto*, eight) higher than another when their interval is 2 : 1. These notes when played together produce the most pleasing combination in the scale.

The names and the relations between the notes of an Octave are given as follows :

Name (Western system)	Do	re	me	fa	sol	la	te	do
„ (Indian „)	Sa	re	ga	ma	pa	dha	ni	sa
Symbol	C	D	E	F	G	A	B	C′
Actual Frequency...	256	288	320	341.3	384	426.6	480	512
Relative Frequency...	24	27	30	32	36	40	45	48
Interval between C and each note	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Interval between each note and its predecessor		$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

Intervals and their special names.

1 : 1	Unison			4 : 3	Fourth
16 : 15	Semitone (or limma)			3 : 2	Fifth
10 : 9	Minor tone			5 : 3	Major sixth
9 : 8	Major tone			8 : 5	Minor sixth
6 : 5	Minor third			15 : 8	Seventh
5 : 4	Major third			2 : 1	Octave

It will be noticed that there are five black keys inserted between all the consecutive notes except the 3rd and 4th, and 7th and 8th The first of these is named *C sharp*, second *D sharp*, third *F flat*, fourth *G flat* and fifth *A flat*.

N. B The intervals in the **Major Diatonic Scale** are a major tone, minor tone or a semi-tone As there are two major tones ($D : C$ and $G : F$), the major diatonic scale is so-called.

Example —*Taking the frequency of vibration of C to be 256, find the note which makes 320 vibrations per sec*

Let x be the vibration ratio or the interval between the two notes, then

$$256 \times x = 320 \; ; \; \therefore \quad x = \frac{320}{256} = \frac{5}{4} \; ;$$

$\frac{5}{4}$ is the interval between the note E and C, hence the required note is E.

41(b). Tempered Scale :—Modern music requires frequent change of the *tonic*. For such changes to be possible in the Major Diatonic Scale, a very large number of keys has to be employed and this will make the instrument unmanageable. The problem was, therefore, how to maintain flexibility of the scale without undue complication being necessary. The solution was the **tempered scale** in which besides usual eight keys, five additional intemediate keys have been introduced and the intermediate keys have been slightly altered in frequencies to make all successive intervals equal. Thus to a very large extent, change of tonic (**modulation of scale**) has been made possible and at the same time, the simplicity of the instrument has been retained.

42. Doppler Effect.—It will be noticed that the pitch of the whistle of a train appears to rise when the train approaches the hearer and it falls as the engine recedes from him. Similar effect is noticed when a motor car passes at a high speed. This apparent change in the pitch of a note as perceived by an observer due to any relative motion of the source, the observer or the medium, is called the **Doppler effect** after the name of the Austrian Physicist Christian Doppler (1803-53).

Apparent frequency :—The apparent change in pitch perceived by an observer due to motion of the source, observer and the medium is calculated as follows.—

Let S and O represent the position of the source and the observer respectively and the distance SM or OW be equal to the velocity of sound in still air (V). Suppose that the source, the observer and the medium are all moving in the same direction from left to right. Let the velocity of the source (V_s) be equal to SS_1 *i.e.* the distance passed over by the source in one sec. and similarly the velocity of the observer $OO_1 = V_o$ and the velocity of wind $MM_1 = WW_1 = w$.

At some instant of time when the observer is at O, let a wave reach him for the first time. After one sec., that wave will be at W_1, for the wave travels a distance OW in still air and the air-medium moves through a distance WW_1 in that sec. in that direction. All the waves received by the observer in that sec. are confined between O_1W_1,

since the position of the first wave of that sec. is at W_1 while the last wave is received by the observer when at O_1. The length occupied by the waves $O_1W_1 = V + w - V_0$.

Now turning to the source end, the first wave was sent out by the source while at S and the last wave while at S_1 in the particular sec. under consideration. All the waves emitted in that sec. are confined between S_1 and M_1, because the first wave reaches M_1 in that sec., it having travelled over SM in still air and the air medium having moved through MM_1. Thus all the waves sent out by the source in that sec. are contained within the length $S_1 M_1 = V + w - Vs$.

If n be the real frequency of the source, it emits n waves in one sec. which occupy a length $V + w - Vs$. If the apparent frequency as perceived by the observer under the circumstances as stated above be n_1, n_1 waves are contained within the length $V + w - V_0$

So we have, $n_1 : n = \dfrac{V + w - V_0}{V + w - Vs}$ or $n_1 = n \times \dfrac{V + w - V_0}{V + w - Vs}$.

N.B.—The velocity of the source, or the observer or the medium will be zero if it is at rest. Proper signs, positive or negative, shall have to be assigned to them depending on the directions in which they move. **Remember**, the observer moving away from the source is positive, the source moving towards the observer is positive and the wind moving towards the observer is positive in the above calculations.

Examples.—*1. What is the apparent frequency of the sound of a whistle of frequency 600 from an engine which is approaching an observer at rest at 10 metres per sec.? (Velocity of sound = 332 metres per sec.)*

Here $V_o = 0$, $w = 0$ and $V_s = 10$.

$\therefore$ Apparent frequency, $n_1 = n \times \dfrac{V + w - V_o}{V + w - V_s}$

$$= 600 \times \frac{332 + 0 - 0}{332 + 0 - 10} = 600 \times \frac{332}{322} = 619 \text{ (approximately) per sec.}$$

2. Calculate the apparent frequency of a note of a whistle of frequency 1000 per sec. heard from a train which is approaching the station at 44 ft/sec. where the whistle is blown (vel. of sound = 1100 ft/sec.)?

Here $V_s = 0$, $w = 0$ and $V_o = -44$ ft./sec.

$\therefore$ Apparent frequency, $n_1 = n \times \dfrac{V + w - V_o}{V + w - V_s}$

$$= 1000 \times \frac{1100 + 0 - (-44)}{1100 + 0 - 0} = 1000 \times \frac{1144}{1100} = 1040 \text{ per sec.}$$

Questions

Art. 39.

1. Distinguish clearly between the loudness and the pitch of a musical note. On what physical conditions of the sounding body do they respectively depend ? (C. U. 1909, '12, '14, '19, '21 ; Pat. 1924, '28 ; All. '24 ; Dac. '29).

2. On what do loudness, pitch and quality of a musical sound depend ? (C. U. 1931 ; All. 1926 ; Dac. '28, '31 ; Pat. '28, '39).

3. How would you distinguish between (*a*) musical sound and noise, and (*b*) one note from another ?

4. How will you explain the difference between pitch and loudness of sound by comparing the roar of a lion and buzzing of a mosquito ? (All. 1927).

[**Hints.**—The buzzing of a mosquito is due to the motion of its wings which vibrate several hundred times a second, so the frequency and consequently the pitch of this sound is high ; but as the energy put forth is small and as the loudness depends upon this energy, the sound is feeble. On the other hand the lion being very strong, energy put in the roar is great and so the sound is of high intensity or loudness, but of low pitch as the frequency of vibration of the sound is small.]

5. How do you explain why audible notes from different sources can generally be distinguished one from another, even when they have the same intensity or pitch ? Describe experiments in order to demonstrate the correctness of your answer.

6. (*a*) What is meant by musical scale ? (All. '46).

Trace the sounds coming from a violin, flute, a harmonium, and a piano to their ultimate sources. How do these sounds differ from one another, and why ? (Pat. 1932)

(*b*) Write notes on "Timbre" (Pat. '47).

Art. 40

7. What do you understand by the pitch of a note ?

Explain a method of experimentally determining the pitch of the note emitted by a given tuning-fork. (C. U. 1917, '32 ; Pat. 1926, '47 ; All. 1919, '21, '24)

8. Give a brief account of the various methods of determining the frequency of a fork and discuss their merits. (All. 1928 ; Pat. 1936)

9. Describe a siren, giving a diagram and explain how you would use it to determine the frequency of a given tuning fork. (C. U. 1921, '28, '30, '40 ; Pat. 1920, '21, '33, '37, '40 ; Dac. '27)

10. The disc of a given siren has 32 holes. A tuning-fork makes 512 vibrations per second. What must be the speed of rotation per minute of the siren disc so that the note emitted by the siren may be in unison with that emitted by the tuning-fork. (C. U. 1910)

[*Ans* : 960 per min.]

11. The disc of a siren is making 10 revolutions per second. How many

holes must it possess in order that it may be in unison with a tuning-fork of frequency 480 ? (Dac. 1932)

[*Ans* : 48]

12. A cog-wheel containing 64 cogs revolves 240 times per minute. What is the frequency of the musical note produced when a card is held against the revolving teeth. Find also the wave-length corresponding to the note if the velocity of sound is 1126·4 ft. per second.

[*Ans :* 256 ; 4.4 ft.]

13. How would you determine experimentally the absolute value of the frequency of a tuning-fork ? Illustrate your answer with a neat sketch of the arrangement described. (Pat. 1941)

Art. 41.

14. Define musical interval, harmony, melody and chord. Show that the interval *sa* and *ga* is obtained by multiplying the intervals *sa* and *re* and *ga*, but not by adding them. (Pat 1928).

15. Explain what is meant by pitch of a note. A note of frequency 384 is said to be a 'fifth' higher in pitch than one of 256. What is the frequency of the note a 'fifth' higher than the 384 note, and what is the difference in pitch between it and the 256 note ? (L. M.)

[*Ans :* 576 ; a major tone above the octave of 256].

16. A siren having a ring of 200 holes is making 132 revolutions per minute. It is found to emit a note which is an octave lower than that of a given tuning-fork. Find the frequency of the latter. (C. U. 1944)

[*Ans :* (2×440)]

CHAPTER VII

Vibration of Strings

43. Vibration of Strings.—In sound a *string* is usually understood to mean a wire or a cord of any material, which is flexible and supposed to be uniform in cross-section. These conditions are found to be satisfactorily fulfilled by thin metal wires or catgut. Strings may vibrate in two ways, *transversely* and *longitudinally*. A string can be vibrated longitudinally by rubbing it along the length with a piece of chamois leather covered with resin, or by a piece of wet flannel. It can be vibrated transversely by plucking it aside, or by

bowing it with a violin bow. When a string is plucked to one side it tends to return to its original (straight) position of rest. But owing to inertia that it possesses, it overshoots the mark like the motion of a pendulum, and goes over to the other side and goes on swinging to-and-fro with gradually decreasing amplitudes and hence after some time it stops. The vibration in this case is mainly due to the tension in the string, which, when the string is deflected, tends to bring it to its initial straight position. *In stringed musical instruments, only the transverse vibrations of strings* are employed.

44 Reflection of Transverse Waves. (*a*) *Reflection of Wave in a string.*—Let a wave travelling along a wire, say from left to right, meet a fixed support and let the wave meet the support in the form of a crest. The end of the wire will exert a force on the support tending to move it in the direction of the force. Then. according to Newton's third law of motion, the support will react and exert an equal and opposite force on the wire which causes a rebound, so that the pulse is thrown over on the other side of the string and starts a reversed pulse travelling back along the string from right to left. Thus, **in this case, reflection takes place at the fixed ends with change of type** ; *a crest is reflected back as a trough and a trough is reflected back as crest.* It should be noted, however, that in the case of water-waves, which are also transverse waves, a crest meeting a rigid wall is reflected back as a crest and a trough is reflected back as a trough like longitudinal sound-waves (See Art. 29), and the important difference between the reflections of sound-waves at the closed and open ends of a pipe should also be noted (Ch. VII).

(*b*) *Reflection of Water-Wave.*—When a water-wave travels along, it has both potential and kinetic energy. Part of the energy is *potential* because a force must have been applied to and work done upon the water to raise it above its normal level, and a part is *kinetic*, because the molecules are in motion. When the waves strike a rigid wall or a denser medium, the motion of the molecules towards the wall is arrested, their kinetic energy is reduced, which is then converted into the potential, thus increasing the amount of potential energy. So the average elevation of the water in the crest is increased and the water is piled up against the obstruction, which then runs down and away from the wall producing a crest like the original wave and travelling in the opposite direction. Thus, in the case of a water-wave meeting a rigid wall, a *crest is reflected as a crest and similarly a trough is reflected as a trough.*

45. Stationary Waves in the String.—When a stretched string is plucked aside, a wave will travel along its length with a definite

velocity. This transverse wave will be propagated to both ends and will be reflected at these points. If a complete wave consisting of a crest and a trough is sent along a string crest first, it will return as trough first after reflection at the fixed end. These reflected waves will return to the centre of the string when they pass each other, and go on to the ends to be once more reflected, and so on. Therefore the incident and reflected waves, travelling to-and-fro along the string in opposite directions with equal velocities, combine to form *transverse stationary waves whose position of nodes and antinodes are fixed* (see Art. 38).

46. Velocity of Transverse waves along a string.—When a string, stretched under tension, is displaced laterally, transverse waves are set up in it. The waves travel along the string with a velocity dependent on the tension and the linear density of the string.

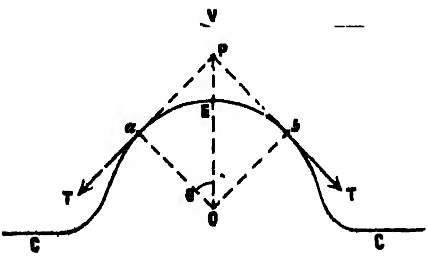


Fig. 25 (N)

Suppose the string *cc*, stretched under tension T is displaced perp. to its length so as to make transverse vibrations (through plucking, bowing or striking), due to which, suppose, the summit aEb of the displaced position, is bent into the arc of a circle. The transverse wave travelling along the string from left to right with velocity V may be imagined to be due to the hump also travelling with the same velocity. For the circular motion of an element near the summit E of the hump the necessary centripetal force is supplied by the tensions at a and b.

Suppose $a\,E = E\,b$ and O, the centre of the curvature $a\,E\,b$. Let the angle $a\,o\,E$ be θ. Join $o\,a$ and $o\,b$. Draw tangents T at a and b, representing tension of the string which produced backwards meet at P on oE. suppose the length aEb is S, mass per unit length of the string is m and the radius of curvature is R.

The components of the tensions T at a and b in the direction PO (each equal to $T \sin\theta$) constitute the centripetal force $\dfrac{m.\,s.\,V^2}{R}$ on the hump, the components of T at rt. angles to PO cancel each other.

Therefore,

$$\frac{m.\,s.\,v^2}{R} = 2\,T\sin\theta = 2\,T\,\theta \text{ approxly.}$$

$$2T.\frac{S/2}{R} = \frac{TS}{R}$$

$$\therefore \sqrt{\frac{T}{m}}.$$

46. (a) Frequency of Transverse Vibration of Strings.—The velocity of a transverse wave along a stretched string is given by

$$V = \sqrt{\frac{T}{m}} = \sqrt{\frac{Mg}{m}} \quad \dots \quad \dots \quad (1)$$

where T = tension of the string expressed in dynes ; m = mass in grams per unit length of the string ; M = mass of load on the string.

When the string gives out its fundamental, *i.e.* note of the lowest pitch, the length of the string, l cm. = distance between two consecutive nodes $= \lambda/2$.

$\therefore$ From Art. 7, $V = n\lambda = 2nl$.

Substituting the value of V in (1) we get,

$$2nl = \sqrt{\frac{T}{m}}\ ;\ or\ n = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \dots \quad \dots \quad \dots \quad (2)$$

Again, if ρ be the density of the material of the wire and r be its radius, then $m = \pi r^2 \rho$, snd so we have from (2)

$$n = \frac{1}{2l}\sqrt{\frac{T}{\pi r^2 \rho}} = \frac{1}{2rl}\sqrt{\frac{T}{\pi\rho}} = \frac{1}{dl}\sqrt{\frac{T}{\pi\rho}} \quad \dots \quad \dots \quad \dots \quad (3)$$

where d is the diameter of the wire.

Laws of transverse vibration of strings.—From formula (2) we get the following laws of the transverse vibration of strings.

(1) *Law of Length.*—The frequency of a note emitted by a string varies inversely as the length, the tension remaining constant, that is, $n \propto 1/l$, when T and m are constant.

(2) *Law of Tension.*—The frequency of a note varies directly as the square root of the tension, the length being kept constant, that is, $n \propto \sqrt{T}$, when l and m are constant.

(3) *Law of Mass.*—The frequency of a note varies inversely as the square root of the mass per unit length of the string, that is, $n \propto 1/\sqrt{m}$, when l and T are constant.

Again, from formula (3), the law of mass may be put into two additional laws for round strings as given below.

(*a*) *Law of Diameter.*—The frequency of the note produced by a string varies inversely as the diameter of the string, that is, $n \propto 1/d$, when l and T are constant.

(*b*) *Law of Density.*—The frequency varies inversely as the square root of the density of the material of the string, that is, $n \propto 1/\sqrt{\rho}$, when l and T are constant.

47. Experimental Verification of the Laws of Transverse vibration of strings. (*By Sonometer*).—The laws of transverse vibration of strings can be verified by means of an instrument, called a **Sonometer**

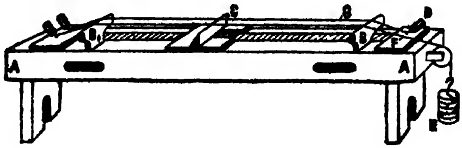


Fig. 26—Sonometer.

or *Monochord*. It consists of a hollow wooden box AA, on which one or more wires can be stretched. Each wire is attached to a peg at one end and passes over two wedge-shaped hard woods B, B_1, called *bridges*, and also a pulley at the other end. The string is kept by a number of weights E attached at the end. A third bridge C can be placed in any position between the other two in order to set any desired length of the string in vibration (Fig. 26)

Law 1. *To verify* $n \propto 1/l$.—To verify the law of length, two tuning-forks of known frequencies n_1 and n_2 are taken. One of the forks is made to vibrate, and, altering the position of the movable bridge C, the length of the sonometer wire is so adjusted that the note emitted by that length of the wire, when plucked in the middle, is in unison with the note yielded by the fork. Then the frequency n_1 of the fork is equal to the frequency of the wire of length l_1. Repeating the experiment with the other tuning-fork, another length of the wire is similarly determined. Let n_2 be the frequency of this fork, and l_2, the corresponding length of the wire. It will be found by experiment that $\frac{n_1}{n_2} = \frac{l_2}{l_1}$ or, $n_1 l_1 = n_2 l_2$. Repeating the experiment with other forks if will be found that $n_1 l_1 = n_2 l_2 = n_3 l_3 = ..$ *i.e.* $nl =$ a constant, which verifies the law.

Law. 2. *To verify* $n \propto \sqrt{T}$. Stretch another wire, called the comparison wire, by the side of the first wire. Let T_1 be the tension on the first wire. A length of the comparison wire is then adjusted which is in unison with the note yielded by the first wire.

Let the length be l_1. Now increase the tension on the first wire to T_2; so the frequency of the note emitted increases. Again another length l_2 of the comparison wire is found which is in unison with the note of the first wire. Let n_1 and n_2 be the frequencies of the notes of the comparison wire of lengths l_1 and l_2, and so of the first wire corresponding to tensions T_1 and T_2 respectively. We have by the law of length, $\frac{n_1}{n_2} = \frac{l_2}{l_1}$. Again, it will be found by the experiment that $\frac{l_2}{l_1} = \sqrt{\frac{T_1}{T_2}}$. So, $\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$.

By applying different tension T_1, T_2, T_3, etc., to the first wire and determining corresponding attuned lengths l_1, l_2, l_3, etc., for which the respective frequencies are n_1, n_2, n_3, etc., it may be shown that $n/\sqrt{T}$ is constant. This verifies the law of tension.

Law 3. *To verify* $n \propto 1/\sqrt{m}$.—To verify the law of mass, two wires of different mass per unit length are taken. The wires may be of the same material or of different materials. One of them is stretched by the side of the comparison wire by a suitable load. Taking any length of the wire whose mass per unit length is m_1, a length l_1 of the comparison wire is determined, which is in unison with the note of the first wire. Replacing the first wire by the second wire of mass m_2 per unit length, and keeping the *tension* the *same*, the above experiment is repeated, taking the *same length* of the second wire as that of the first. A length l_2 of the comparison wire is found which is in unison with the note of the second wire. Then a measured length of each of the wires is taken, and each of them is weighed. From these wts., mass per unit length (m_1 and m_2) for the two wires is found.

Let n_1 and n_2 be the frequencies of the lengths l_1 and l_2 of the comparison wire. We have, by the law of length, $\frac{n_1}{n_2} = \frac{l_2}{l_1}$, and it is found by the experiment that $\frac{l_2}{l_1} = \sqrt{\frac{m_2}{m_1}}$.

Hence $\frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$. or $n_1 \sqrt{m_1} = n_2 \sqrt{m_2}$. Repeating the experiment with other wires of different mass per unit length it will be found that $n \sqrt{m}$ = a constant. This proves the law of mass.

Law 3a. *To verify $n \propto 1/d$*—Take two wires of different diameters but of the same material, and proceed just as in the above experiment (Law 3). Let l_1 and l_2 be the lengths of the comparison wire which are found to be in unison with the notes produced by equal lengths of the two wires having diameters d_1 and d_2 respectively. Now measure d_1 and d_2 with a screw-gauge. From Law 1, we have $n_1/n_2 = l_2/l_1$, and it will be found by experiment that $l_2/l_1 = d_2/d_1$. Hence $\frac{n_1}{n_2} = \frac{d_2}{d_1}$, which verifies the law.

Law 3b. *To verify $n \propto 1/\sqrt{\rho}$.*—Take two wires of different materials but of the same diameter, and repeat the experiment exactly as above (Law 3a). It will be found that $\frac{n_1}{n_2} = \sqrt{\frac{\rho_2}{\rho_1}}$. This verifies the law.

N. B. It should be noted that this experiment gives a method of determining acoustically whether two wires are made of the same material or not.

Notes on Tuning.—In tuning two strings, a tuning-fork and a string, or any two notes, two methods may generally be adopted :

(a) **"By Resonance."**—Tune as nearly as possible by ear. Then place a *V*-shaped paper rider, or a thin wire rider, on the middle of the string, and place the stem of the vibrating tuning-fork on the sonometer box It will set the string in vibration by resonance and the rider will be thrown off, if the tuning be accurate If however, this does not occur, adjust the length of the string by moving the movable bridge until the rider is thrown off.

(b) **"By Beat."**—By adjusting the length of the string by the movable bridge until the two notes (of the string and the fork) are very nearly of the same frequency, beats will be heard, *i e.* the resultant sound appears to give alternate maxima and minima of loudness. On adjusting the length still further beats will become slower, and will cease entirely when tuning is exact, *i e.* when the frequencies of the two notes are exactly equal.

48. Determination of Pitch by Sonometer —(a) The frequency of a note can be determined either by keeping the length of the sonometer wire constant and adjusting the tension, or by adjusting the length of the wire keeping the tension constant, until the string is in unison with the note, the pitch of which is to be determined. The latter method is, however, convenient. If the frequency of a tuning-fork is to be determined, it is pressed against the sonometer box after it is made to vibrate. The length of the wire is then adjusted

by means of the movable bridge until the note emitted by this is in unison with that of the fork. The length of the wire is then measured and the mass of the string per unit length is also determined. The stretching weight is noted, and the tension is calculated by multiplying it by the acceleration due to gravity. The frequency n is then calculated by the formula, $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$.

N.B.—By knowing n, the *density* of the material of the wire can be determined from formula (3) Art. 46.

(b) The pitch of a tuning-fork can also be determined by taking another standard fork of known frequency and then determining as above a length of the same wire stretched by the same wt until this fork and the wire are in unison again. If n be the frequency of the standard fork, n' the unknown frequency; and l and l' be the corresponding lengths of the wire, then, we have, $\frac{n}{n'} = \frac{l'}{l}$, whence n' can be determined.

49. Certain Terms.—

Note, Tone—A *note* is a general term denoting any musical sound. It is, however, a complex sound made up of two or more simple sounds of different pitches. Each of the simple component sounds is called a *tone*. A tone cannot further be divided into components.

Fundamental, Overtone, Harmonic, and Octave.—When a body vibrates in several different modes, there are present in the note several tones of frequencies which are multiples of the frequency of the original or the *fundamental*, which is the tone of the lowest pitch. The other tones are alled *overtones*. When the frequencies of the overtones are exact multiples of that of the fundamental, they are, in particular, called *harmonics*.

The tone whose frequency is twice that of the fundamental is called an *octave* higher or the first harmonic of the fundamental. All tones of frequencies between any number n and $2n$ are said to be in *same octave*.

Harmonics of a Stretched String.—(i) When a string vibrates as a whole, it sounds its fundamental. This is the simplest mode of of vibration. In this manner of vibration, when the string is plucked at its centre there are *two nodes N_1, N_2 at the two fixed ends, and one antinode A_1* in the middle [Fig. 27 (1)], and the length of the string $l = \lambda/2$.

$\therefore \quad n = \frac{V}{\lambda} = \frac{V}{2l}$. But the string may vibrate in other ways also.

(ii) If the string be plucked at a point *one-fourth* the length of the wire from one end, and at the same time the middle point of the wire is lightly touched, it will vibrate in two segments. In this manner of vibration there are *three nodes* and *two antinodes*, [Fig. 27 (II)]. In this case $l = \lambda$.

$\therefore \ n_1 = V/l$; *or*, $n_1 = 2n$.

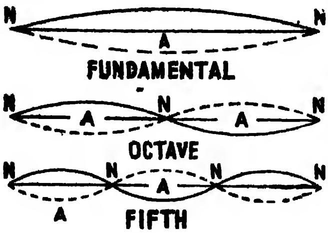


Fig. 27

This note is an *octave higher* and is called the *first harmonic of the fundamental note.*

(iii) In the next mode of vibration, if the string is held at one third of its length and if the middle of the shorter segment is bowed, the wire will vibrate in three segments, and in that case there will be *four nodes* and *three antinodes* [Fig. 27 (III)]. In this case $\lambda = \frac{2}{3}l$.

$\therefore \quad n_2 = \frac{3V}{2l}$. or $n_2 = 3n$.

This note is called the *second harmonic* of the fundamental note. In this way it will have all the *odd* and *even* harmonics. But if the bowing is at random such that the segments into which the string vibrates are not regular, tones will also be produced which are not exact multiples of the fundamental. These higher tones are called **Upper tones or Overtones or upper partials.**

Examples.—*1. An iron wire of $\frac{1}{8}$ inch diameter is stretched by a weight of 224 lbs. Find the velocity of the transverse wave along it.* (*Specific gravity of iron is 7·8*)

We have $V = \sqrt{\frac{T}{m}}$.

Here density of iron $= 62{\cdot}5 \times 7{\cdot}8$ per cu. ft.

Again $m = \pi \times (\frac{1}{2} \times \frac{1}{8} \times \frac{1}{12})^2 \times 62{\cdot}5 \times 7{\cdot}8$ lbs. per ft.

and $T = 224 \times 32$ poundals whence $V = 311{\cdot}6$ ft. per sec.

2. A tuning-fork is in unison with the fundamental note of a stretched string and the observed readings are the following—

(1) *Length of the string = 35 cms.* (2) *Mass of 1 metre of the string = 0·323 gms.*
(3) *Loads applied = 4 kilograms. Calculate the frequency.* (*Pat. 1924*)

$l = 35$ cms ; $m = \frac{0{\cdot}323}{100} = 0{\cdot}00323$ gms. per cm. ; $T = (4 \times 1000 \times 981)$ dynes.

We have, $n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 35}\sqrt{\frac{4000 \times 981}{0{\cdot}00323}}$ whence $n = 497$ vibrations per sec.

3. The length of a sonometer wire between two fixed ends is 100 cms. Where should two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio of 1 : 2 : 3. (*All. 1917*)

Let the segments be l_1, l_2, and l_3, and let n be the frequency of the note given by l_1 ; then the frequencies of the other two notes are $2n$ and $3n$ respectively. Each of the above segments represents the distance between two bridges, *i.e.* two fixed points, which are nodes. Hence for the fundamental note, $l_1 = \lambda_1/2$, $l_2 = \lambda_2/2$, $l_3 = \lambda_3/2$, when λ_1, λ_2 and λ_3 are corresponding wavelengths. Thus we have, $V = n\lambda_1 = 2nl_1$; $V = 2n\lambda_2 = 4nl_2$; and $V = 3n\lambda_3 = 6nl_3$,

when V is the velocity of sound along the stretched wire.

$\therefore \quad V = 2nl_1 = 4nl_2 = 6nl_3$; or $l_1 = 2l_2 = 3l_3$.

But $l_1 + l_2 + l_3 = 100$ or $3l_3 + \frac{3}{2}l_3 + l_3 = 100$; or $\frac{11}{2}l_3 = 100$

$\therefore \quad l_3 = \frac{200}{11}$ cm. ; $l_2 = \frac{3}{2} \times \frac{200}{11} = \frac{300}{11}$ cm. ; $l_1 = \frac{3 \times 200}{11} = \frac{600}{11}$ cm.

That is, the first bridge should be placed at a distance of $\frac{600}{11}$ cms. or 54·55 cms. from one end, and the second at a distance of $\frac{300}{11}$ cms. or 27·27 cms. from the first bridge or 81·82 cms. from the same end.

Otherwise thus.—Let the segments be l_1, l_2 and l_3, then their frequency ratios are 1 : 2 : 3. But as $l \propto 1/n$, we have $l_1 : l_2 : l_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = \frac{6}{6} : \frac{3}{6} : \frac{2}{6} =$ 6 : 3 : 2.

$\therefore \; l_1 = \frac{6}{11}$ of $100 = \frac{600}{11}$ cm. ; $l_2 = \frac{3}{11}$ of $100 = \frac{300}{11}$ cm. ; $l_3 = \frac{2}{11}$ of $100 = \frac{200}{11}$ cm.

4. A wire 50 cms. long and of mass 6·5 gm. is stretched so that it makes 80 vibrations per second. Find the stretching force in grams-weight.

How would you double the frequency, (i) by changing the length of the wire ; (ii) by changing the tension in the above case. (*All. 1925*)

The frequency of the note is given by $n = \frac{1}{l}\sqrt{\frac{T}{m}}$.

Here $l = 50$ cm., $n = 80$, m (mass per unit length) $= 6·5/60$, $T =$? On calculation,

$$T = \frac{6·5 \times 80 \times 80 \times 100 \times 100}{50} \text{ dynes.}$$

$\therefore$ The stretching force in grams-weight $= \frac{6·5 \times 80 \times 80 \times 100 \times 100}{50 \times 981} = 8481·14.$

(i) The frequency may be doubled by reducing the length of the vibrating wire to one-half of its former length ; (ii) $n \propto \sqrt{T}$, hence, frequency may be doubled by increasing the tension 4 times, *i.e.* by applying 4 times more weight.

5. A tuning-fork gives 15 beats per second when sounded with a sonometer wire of length 20 cms. and 20 beats with that of length 25 cms. Calculate the frequency of the fork. The tension and mass per unit length of the wire are 1·25 kg. and 0·025 gm. ($g = 980$ cm./sec.^{2}). (*All. 1930*)

The frequency of the fundamental note $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ in which $l = 20$ cms.

$T = 1·25 \times 1000$ gms. and $m = 0·025$ gm. in the first case whence $n_1 = 175$.

Since the tuning-fork gives 15 beats per second with the sound of frequency 175, the frequency of the fork is either $(175+15)=190$; or $(175-15)=160$.

In the second case, $l=25$ cms., T and m same whence $n_2=140$.

Hence the frequency of the fork giving 20 beats with the sound of frequency 140 is either $(140+20)=160$; or $(140-20)=120$.

The fork in both the cases being the same, its frequency is 160.

6. Two similar strings of a sonometer are tuned to unison. One is 36 inches long and stretched by 100 lbs. Find the weight on the other one which is 45 inches long. (C. U. 1941)

$$\text{Here } T'_2 = T_1 \times \left(\frac{l_2}{l_1}\right)^2 \text{ where,}$$

$T_1=(100\times 32)$ poundals ; $l_1=36$ in. ; $l_2=45$ in. ; $T'_2=$?

whence $T'_2=5{,}000$ poundals $=156\cdot25$ lbs. wt.

7. A bridge is placed under the string of a monochord at a point near the middle and it is found on plucking the two parts of the string that 3 beats per second are produced when the load stretching the string is 6 kilos. If the load be then increased to 12 kilos, determine the rate of beating of the two parts of the string. (Pat. 1936)

Here, $n_1=\frac{1}{2l_1}\sqrt{\frac{T_1}{m}}$; $n_2=\frac{1}{2l_2}\sqrt{\frac{T_1}{m}}$; $\therefore$ No. of beats $=3=n_1-n_2=\frac{\sqrt{T_1}}{2\sqrt{m}}\left(\frac{1}{l_1}-\frac{1}{l_2}\right)$. When the tension is changed to T_2, we have, no. of beats,

$x=n'_1-n'_2=\frac{\sqrt{T_2}}{2\sqrt{m}}\left(\frac{1}{l_1}-\frac{1}{l_2}\right)$; $\therefore \quad \frac{3}{x}=\frac{\sqrt{T_1}}{\sqrt{T_2}}$ whence $x=3\cdot07$.

8. Two tuning-forks when sounded together give 4 beats per second. One is in unison with a length of 128 cms. of a monochord string under constant tension and the other with 130 cms. of the same string. What are the frequencies of the forks? (C. U. 1939)

Let n_1 and n_2 be the vibration frequencies of the two forks. From Art. 46(1) we have,

$\frac{n_1}{n_2}=\frac{l_2}{l_1}=\frac{130}{128}=\frac{65}{64}$; or $n_1=\frac{65}{64}n_2$; $\therefore$ $n_1>n_2$. So we have (Art. 36), $n_1-4=n_2$; whence $n_1=260$.

Questions.

Arts. 46 & 47.

1. A sonometer is stretched with a force of 200 gms. weight.

(a) The force is increased to 800 grammes, (b) the length of the string is

halved. How is the pitch of the note emitted by the string affected in each case ? (C. U. 1912)

[*Ans* : (*a*) $n_2 = 2n_1$; (*b*) $n_2 = 2n_1$, *i.e.* the pitch is doubled in each case.]

2. The string of a monochord vibrates 100 times a second. Its length is doubled and its tension altered until it makes 150 vibrations a second. What is the relation of the new tension to the original ? (C. U. 1924)

[*Ans* : $T_2 : T_1 :: 9 : 1$]

3. What will be the frequency of the note emitted by a wire 50 cm. in length when stretched by a weight of 25 kilograms, if 2 meters of the wire are found to weigh 4·79 grams ? (C. U. 1934)

[*Ans* : 320 per sec.]

4. On what factors does the frequency of vibration of a stretched string depend ? (All. 1925 ; cf. C. U. 46)

When the wire of a sonometer is 73 cm. long, it is in tune with a tuning-fork. On shortening the wire by 5 mm. it makes 3 beats a second with the fork. What is the frequency of the fork ? (Pat. 1939)

[$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 73}\sqrt{\frac{T}{m}}$; $(n+3) = \frac{1}{2(72 \cdot 5)}\sqrt{\frac{T}{m}}$ $\therefore \frac{n}{n+3} = \frac{72 \cdot 5}{73}$; $n = 435$]

5. A wire 50 cms. long vibrates 100 times a second. If the length is shortened to 30 cms. and the stretching force quadrupled, what will be the frequency ? (All. 1927)

[*Ans* : 333·3]

6. Two strings of the same length and diameter are constructed of materials of densities 1·21 and 9 gm/c.c. respectively. Compare the tensions which must be applied to them in order that the note given by the second string may be an octave below that of the first. (L. M. B.)

[*Ans* : 4·84 : 9]

7. A stretched string 1 metre long is divided by two bridges into three parts so as to give notes of the common chord whose frequencies are in the ratio of 4 : 5 : 6. Find the distance between the bridges.

[*Ans* : 32·432 cms.]

8. A string 24 inches long weighs half an ounce and is stretched on a sonometer with a weight of 81 lbs. Find the frequency of the note emitted when struck. (Dac. 1934)

[*Ans* : 101·8]

9. State and explain the laws of vibration of a stretched string. Why are the strings of musical instruments mounted on hollow wooden boxes ?

A brass wire (density 8·4), 100 cm. long and 1·8 mm. in diameter is stretched by a weight of 20 kilograms. Calculate the number of vibrations which it makes per second when sounding its fundamental note. ($g = 980$ cm. per sec^2.) [*Ans* : 47·88 nearly.] (C. U. 1930)

10. State the laws of transverse vibration of a stretched string and describe experiments to verify them.

(C. U. 1925, '34, '36, '41 ; All. '27, '29, '45 ; Pat. '23, '27, '33, '37, '40, '42).

11. A stretched wire under tension of 1 kg. weight is in unison with a fork of frequency 320. What alteration in tension would make the wire vibrate in unison with a fork of frequency 256 ?

$$\left[\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}} ; \therefore \frac{320}{256} = \sqrt{\frac{1}{T_2}} \quad or \quad T_2 = \frac{16}{25} .\right.$$

So the tension should be reduced by $1 - \frac{16}{25}$ or $\frac{9}{25}$ kg.]

12. How would you verify with sonometer the law connecting the frequency of a stretched string with its tension ? If an additional weight 75 lb. raises the pitch an octave, what was the original tension ? (L. M.)

[*Ans* : 25 lb. wt.]

13. Given two tuning-forks, how would you determine the pitch of the note emitted by one of them if that of the other is known ?

(C. U. 1919 : Pat. 1930).

14. How would you verify the relation between the pitch of the note emitted by a stretched string and its tension ? (Pat. 1943)

15. Explain how you would find acoustically whether two wires are made of the same material or not.

16. Wires of brass and steel are stretched on a sonometer and are adjusted to emit the same fundamental note. The two wires are of equal length, but the tension of the brass wire is 5 kg. weight and of the iron 3 kg. weight. Assuming that the steel wire has a diameter of 0·8 mm., find that of the brass. (C. U. 1946)

$$\left\{ Ans : 0·8 \sqrt{\frac{5 \text{ density of iron}}{3 \text{ density of brass}}} \text{ mm.} \right\}$$

Art. 48.

17. Show how the frequency of a tuning-fork is determined with the help of a stretched string. (Pat. 1937 ; All. '45 ; C. U. '45)

CHAPTER VIII

Vibrations of Air Columns : Longitudal Vibrations of Rods (Dust-tube Experiment.)

51. Stationary Vibration of Air Column within Organ Pipes.—

The column of air enclosed in a pipe can be set into momentary vibration when any sudden disturbance is communicated to it, or the pressure at the mouth of the pipe is suddenly altered. For example, a sound is produced by suddenly withdrawing a cork from a tightly corked cylindrical bottle, because the sudden withdrawal of the cork disturbs the air-pressure at the mouth of the bottle which is the cause of the vibrations of air in the bottle. The whistling sound produced by blowing across the open end of the barrel of a key is also another example of vibration of air column. In various musical instruments such as flute, clarinet, etc., the musical sound is produced and maintained by vibrating the air column enclosed within the pipe. Air column in a pipe, closed or open, vibrates longitudinally when disturbed at the mouth.

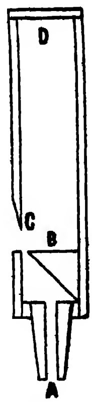


Fig. 28 Closed Organ Pipe

An organ pipe is the simplest form of a wind instrument. Fig. 28 shows a longitudinal section of an organ pipe. It consists of a hollow tube *BD* in which air can be blown through a pipe *A*. The air issues through a narrow slit *B*, and strikes against the sharp edge *C*, called the *lip*, of the mouthpiece. This sets up vibrations in the air column enclosed in the pipe. When the blast is directed into the pipe it produces compression, and when directed outwards it can, by suction, produce rarefaction at the lower end of the air column. **An organ pipe is called closed or open according as it is closed at one end *or* open at both ends.**

(a) **Closed Organ Pipe.**—As air is blown through the pipe (Fig. 28) it strikes the edge, and a slight upward deviation of the air-blast produces a compressed wave which travels to the closed end (which is a rigid wall), and so the air near the end is compressed to a pressure greater than the atmospheric pressure. This compressed air forces back the air behind it in order to return to atmospheric pressure, and in so doing it starts a compressed wave which returns along the pipe. Thus a compressed wave is reflected from the closed end as a compressed wave, and returns to the mouth. But the mouth being open, and the air free to expand, the pressure of the compressed wave pushes the sheet of air outside and so the layers of air relieve themselves from any strained state and as a result there is reversal of the type of the

wave and so a wave of rarefaction starts inside (Art. 29). This wave of rarefaction again comes back to the mouth as a rarefied wave after being reflected at the closed end. This is again reflected as a compressed wave at the mouth which is a free end and is also intensified by the compressed wave directed inwards, by the blast of the air outside. In this way, of the vibrations of various frequencies set up by the impact of the air blast with the lip C of the pipe, the air column inside the pipe takes up only those with which it can resound, and pulses pass up and down the length of the pipe, the result being the propagation of a musical note and the pipe is found to 'speak'.

The result of the reflected pulse meeting with the direct ones *is a stationary longitudinal wave set up inside the pipe where the nodes and antinodes occur at definite places. The air at the open end is free to move inwards or outwards with the maximum freedom and, therefore, it is the seat of an antinode. The closed end being a rigid wall, the air in contact with it has the least freedom of movement and so the closed end is always a node.*

(b) **Open Organ Pipe**—In an open pipe, when a compressed wave reaches the far end, the air at that point is for an instant at a pressure greater than ordinary atmospheric pressure, and the mouth of the tube being open, the air there can vibrate with the utmost freedom and so suddenly expands into the surrounding air. Thus the pressure diminishes so quickly that it falls somewhat below the pressure of the surrounding air, which causes a sudden rarefaction at the end of the pipe. This sets up a rarefied wave which passes back along the pipe. This rarefied wave is reflected back as a wave of compression at the other free end. Within the tube, the reflected pulses meet with the direct ones blasted into the mouth from outside and the result is the formation of a stationary longitudinal wave having nodes and antinodes at definite intervals. Both the open ends of the tube are seats of antinodes, the air there being most free to move either inwards or outwards. For the fundamental tone emitted by the tube, there is one node between these two antinodes.

52 Fundamentals of a Closed and of an Open Organ Pipe of the Same Length —

Closed Pipe.—In the simplest mode of vibration in the case of a *closed organ pipe*, there is a node at the closed end and an antinode at the open end. *In a stationary wave the distance between two consecutive nodes, or two consecutive antinodes, is equal to one-half the wave-length* : so in this case the length of the tube is one-fourth of the wave-length ; *i.e.*, the wave-length is *four-times* the length of the tube. This is the fundamental tone.

Let n_1 and λ_1 represent frequency and wave-length of the funda-

mental tone given by a closed pipe of length l. Hence $\lambda_1 = 4l$; and $V = n_1\lambda_1$, where V is the velocity of sound.

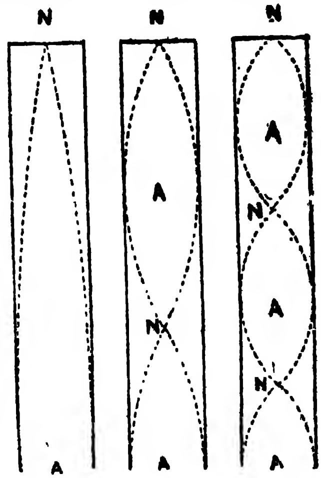
$$\therefore \; n_1 = \frac{V}{\lambda_1} = \frac{V}{4l}.$$

Open Pipe.—In the case of the fundamental of an *open pipe, i.e.* a pipe open at both ends, there is an antinode at each end of the pipe with a node in the middle. If n' and λ' be the frequency and wave-length of the fundamental tone for the open pipe, we have $\lambda' = 2l$. Again, $V = n'\lambda'$.

$$\therefore \; n' = \frac{V}{\lambda'} = \frac{V}{2l} = 2n_1.$$

Hence, *the pitch of the fundamental of an open organ pipe is twice, i.e. one octave higher than that of a closed organ pipe of the same length :*

N. B.—If an open pipe, while giving out a note, is suddenly closed, *the pitch of the note at once decreases* and the sound emitted becomes less sharp. If an organ pipe is closed at one end by a movable shutter, the pitch of the note emitted by the pipe is found to rise on slowly opening the shutter and to fall as the shutter is gradually closed.



Frequencies—
n_2 (*a*) $3n_1$ (*b*) $5n_1$ (*c*)

Fig. 29.

52 (*a*) Overtones (or Harmonics) of Organ pipes —Production of harmonics depends to some extent on the nature of excitation of the tube. If the air is blown more and more powerfully, the nature of the stationery waves remains the same no-doubt but the number of nodes and antinodes is increased *i.e.* higher and higher harmonics are produced.

(*i*) **Closed Pipe**—In the case of a closed pipe, the closed end is always a node and the open end always an antinode [Fig. 29 (a)]. The next possible mode of vibration after the fundamental is to have *one* intermediate node and *one* antinode [Fig. 29. (*b*)], or the length of the pipe l is three-fourth of the wave-ltngth λ_1, so, in this case, $\lambda_1 = \frac{4}{3}l$.

If n_2 be the frequency of the note, $n_2 = 3V/4l$. Hence, $n_2 = 3n_1$, where n_1 is the frequency of the fundamental.

For the *next* higher overtone, there will be *two* nodes and *two* antinodes alternately placed [Fig. 29 (*c*)].

In this case, $\lambda_2 = \frac{4}{5}l$; and the corresponding frequency $n_3 = 5V/4l$. Hence, $n_3 = 5n_1$, and so on. In the case of a closed pipe, therefore, only *harmonics proportional to the odd natural numbers are present, and this makes the quality of the note given out by a closed pipe lacking in fullness.*

Harmonics of a Closed Pipe.—

No.	Wave-length in air	Frequency of the note	Relation with the fundamental
1	$4l$	$n_1 = \frac{V}{4l}$	Fundamental
2	$\frac{3}{4}l$	$n_2 = \frac{3V}{4l}$	$n_2 = 3n_1$
3	$\frac{4}{5}l$	$n_3 = \frac{5V}{4l}$	$n_3 = 5n_1$
&c.	&c.	&c.	&c.

Therefore in a closed pipe the possible frequencies of vibration are in the ratio 1 : 3 : 5 : 7, etc.

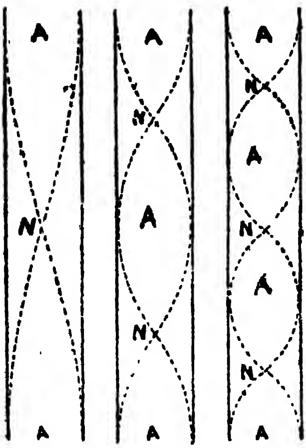
(*ii*) **Open Pipe.**—We have already seen that in the case of the fundamental of an open pipe, there is an antinode at each end, and a node in the middle [Fig. 30 (*a*)]. If n' be the frequency of the fundamental,

$$n' = V/2l.$$

For the *next overtone*, there will be *two* intermediate nodes and *one* intermediate antinode [Fig. 30 (*b*)]. In this case $\lambda'' = 2l/2 =$ the length of the pipe and the frequency $n'' = V/l = 2n'$, *i.e.* it stands an octave higher than the fundamental.

In the *next overtone*, there will be *three* intermediate nodes and *two* intermediate antinodes [Fig. 30 (*c*)]. In this case $\lambda''' = 2l/3$, and the frequency, $n''' = 3V/2l = 3n'$; and so on.

Hence, in the case of an open pipe, both *odd* and *even* harmonics are present.



Frequency—
n' $2n'$ $3n'$
(*a*) (*b*) (*c*)
Fig. 30

Harmonics of an Open Pipe.—

No.	Wave-length in air	Frequency of the note	Relation with the fundamental
1	$2l$	$n' = \frac{V}{2l}$	Fundamental
2	l	$n'' = \frac{V}{l}$	$n'' = 2n'$
3	$\frac{2}{3}l$	$n''' = \frac{3V}{2l}$	$n''' = 3n'$
&c.	&c.	&c.	&c.

Therefore, in an open pipe the frequencies of the fundamental and overtones are in the ratio of 1 : 2 : 3 : 4, etc.

It should be noted that the *note given out by an open pipe* is an *octave higher* than that given out by a closed pipe of the *same length*, and that owing to the presence of all the harmonics proportional to the natural numbers in an open pipe, *the quality of the note given out by an open pipe is richer and sweeter than that given out by a closed pipe.*

53. Effect of Temperature and Moisture on the Pitch of an Organ Pipe—The pitch of any sound (which depends upon the frequency of vibration) is given by the relation $V = n.\lambda$; so anything which changes the velocity V will also change the frequency or wave-length or both. In an organ pipe the length which determines the wave-length does not change appreciably with change of temperature. The velocity increases with temperature and so it follows that a rise of temperature of an organ pipe increases the frequency and so the pitch of the note emitted by it. The presence of moisture diminishes the density of air in the pipe and so it increases the velocity (Art. 18); and consequently the pitch of the note emitted also rises.

Example.—*If the frequency of the note emitted by an organ pipe is 260 in a room at a temperature of 0°C., what will be its frequency if the temperature rises by 27°C.?* *(C. L.)*

We know that, $\frac{V_0}{V_{27}} = \sqrt{\frac{273}{273+27}}$ (Art. 18)

If n_0 and n_{27} be the frequencies at $0°C.$, and $27°C.$ respectively, $V_0 = n_0\lambda$, $V_{27} = n_{27}\lambda$ and $n_0 = 260$.

$$\therefore \frac{V_0}{V_{27}} = \frac{n_0}{n_{27}} = \sqrt{\frac{273}{300}}\text{; whence } n_{27} = 272{\cdot}5.$$

54. Position of Nodes and Antinodes in an Open Organ Pipe.— The position of the nodes and antinodes in an open organ pipe can be demonstrated by the following experiments.

Experiments. (1) An open organ pipe constructed with one of its sides of glass is taken (Fig. 31). A small ring covered with a piece of stretched thin paper, and suspended by strings, like a scale pan, covered with some dry sand granules. This is gradually lowered into the pipe, while the pipe is gently blown to give out its fundamental tone. It will be seen that the particles remain still in the middle position of the pipe indicating a node *i.e.*, a place of minimum agitation of air-particles, and that at the top and bottom of the pipe, the sand particles dance vigorously indicating the positions of antinodes,—places of maximum agitation of air-particles.

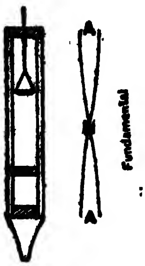


Fig. 31

Blowing the pipe more strongly in order to have other overtones and noticing the dancing of the sand granules, other positions of nodes and antinodes can be discovered.

N. B It should be noted that an antinode will occur wherever there is free communication between the inner and the outer air. Hence *by opening a hole* in the wall of an open pipe, an antinode is created there in addition to the two antinodes at the two ends of the pipe ; and the column of air will vibrate by satisfying the conditions already stated. Thus the note emitted by the pipe is at once changed. From this the reason of having different notes from ordinary bamboo or tin flutes, or from instruments such as clarinets, piccolos, etc., by opening and closing holes in the tube of the instrument, is clear.

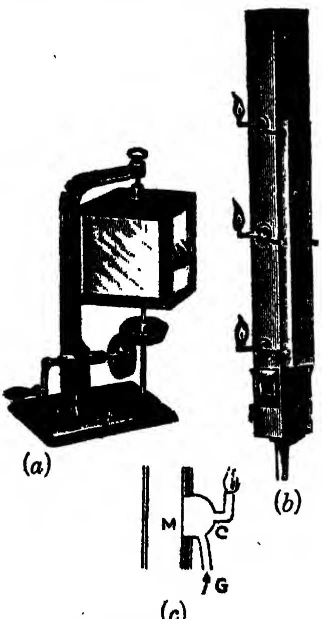


Fig. 32

2. **Manometric Flame Method** — Another method of studying the variations in pressure at the nodes and the antinodes in an organ pipe was devised by Koenig, a German, and is known as Manometric Flame (or Capsule) method.

A circular aperture is made at any desired point in the wall of an organ pipe and is then covered with a stretched diaphragm of thin rubber. A piece of

metal in the form of a capsule is fitted on the aperture so that the membrane constitutes one side of a small chamber fitted with two narrow pipes through one of which G (Fig 32c) coal gas is led which escapes through the other terminating in a pin-hole burner where the gas is burned. Any vibration of the air inside the pipe, which forms the other side of the chamber C, throws the membrane M in contact with it into similar state of vibration and which again causes corresponding vibration in the pressure of the gas in the chamber C, and thus a corresponding change takes place in the length of the flame. If the change in pressure be periodic, the length of the flame also varies periodically. But the change in pressure being very rapid the alterations in the length of the flame cannot be followed by the eye due to persistence of vision To render them distinct, the light is received on a cubical box having plane mirror on its four sides (Fig. 32a) which may be rotated rapidly about a vertical axis in front of the flame, and the successive steps of the flame are seen by looking at the reflection of the flame in the rotating mirror. When the flame (Fig. 32b) burns steadily a continuous band of light will appear on the rotating mirror. So when the manometric flame is at an *antinode*, where there is no variation of pressure of the vibrating air column (*see* p. 441 Art. 51), the membrane will not be agitated and so the flame is quite steady, and a long band of light will appear on the mirror. When, however, the flame is at a *node* where there is the maximum change of pressure, the flame jumps up and down with a frequency equal to that of the membrane and the reflection in the rotating mirror presents a broken-up-toothed-appearance.

Fig. 33

Fig. 33. represents the appearance of the flame in the revolving mirror produced by different tones. Fig. 33(a) represents that due to an organ pipe blown gently, and Fig. 33(b) that due to the pipe blown hard having double the frequency.

Comparison.—The manometric flame method is also applied in *comparing the frequencies of two organ pipes*. When a capsule is applied at a node in each pipe and the corresponding flames are examined side by side it will be found that n teeth in one image will occupy the same length as n' teeth in the other. So the frequencies of the two pipes are evidently in the ratio $n : n'$.

Examples.—*1. If the length of an open organ pipe sounding its fundamental note be one metre, what shall be the length of such a pipe in order that it may sound the fifth of the previous note?* [*Pat. 1926*]

If l_1 be the length of the pipe giving out its fundamental and l_2 the fifth of this note (see Art. 41), then, in the first case,

$V=2nl_1$, where n is the frequency of the fundamental note.

Now because a fifth corresponds to a ratio of $\frac{3}{2}$, the frequency in the second case is $\frac{3n}{2}$; hence, $V=2\times\frac{3n}{2}\times l_2=3nl_2$; $\therefore 3nl_2=2nl_1$ ($\because V$ is constant) $=2n\times 1$ ($\because l=1$ metre); $\therefore l_2=\frac{2}{3}$ metre.

Thus the length of the pipe sounding the fifth of the fundamental is $\frac{2}{3}$ metre or about 66·6 cms.

2. *The top of an organ pipe is suddenly closed. If it emits next above the fundamentals in both the cases and the difference in pitch be 256, what was the pitch of the note emitted originally by the open pipe?* [*Pat. 1938*]

Let V be the velocity of sound in air and n_1 the frequency of vibration of the open pipe next above the fundamental, then we have $n_1=V/l$, where l is the length of the pipe. When it is closed, it becomes a closed pipe having a frequency of vibration n_2, say. As the pipe now emits also the frequency next above the fundamental, we have, $n_2=3V/4l$; but $n_2=n_1-256$ (n_1 being greater than n_2).

$$\therefore n_1-256=\frac{3V}{4l}=\frac{3}{4}n_1\text{; whence } n_1=1024.$$

3. *Two open pipes are sounded together, each note consisting of the fundamental together with two upper harmonics One fundamental note has 256 vibrations per second, the other 170 Would there be any beats produced? If so, how many per second?* [*C. U. 1931*]

The vibration frequencies of the first pipe are 256, (256×2) or 512, and (256×3) or 768: and those of the other 170, 343 and 510. Of these notes two have got very nearly equal frequencies, viz., 512 and 510. So there will be beats, and the number of beats per second $=512-510=2$.

4. *Two organ pipes give 6 beats when sounded together in air at a temperature of 10°C. How many beats would be given when the temperature is 24°C. (Velocity of sound in air at 0°C. is 1088 ft. per sec.)* [*All. 1932*]

In the case of an open organ pipe the velocity of sound in it at 10°*C*., $V=2nl$, where l is the length of the pipe and n is the frequency of the note given out. For another pipe whose length is l', $V=2n'l'$, where n' is the frequency of the note. Number of beats $=n-n'=\frac{V}{2}\left(\frac{1}{l}-\frac{1}{l'}\right)=6$ ... (1)

Now if V' be the velocity of sound at 24°*C*., no. of beats $N=\frac{V'}{2}\left(\frac{1}{l'}-\frac{1}{l}\right)$. ... (2)

From (1) and (2), $N/6=V'/V$. But $V=V_0+2t$ ft. per sec., where V_0 is the velocity of sound at $0°C.=1088+20=1108$ ft. per sec., and $V=1088+2\times 24=1136$ ft. per sec.

$$\therefore \frac{N}{6}=\frac{1136}{1108}\text{; or } N=6·15. \quad \therefore \text{ Number of beats}=6·15.$$

5. *Two organ pipes, one closed at one end and the other open at both ends, are*

respectively 2·5 ft. and 5·2 ft. long. When sounded together, the number of beats heard was found to be 4 per second. Calculate the velocity of sound. [*Pat. 1941*]

Let n_1 and n_2 be the frequencies of the closed and open pipes respectively.

Then $n_1 = \frac{V}{4 \times 2 \cdot 5} = \frac{V}{10}$; and $n_2 = \frac{V}{2 \times 5 \cdot 2} = \frac{V}{10 \cdot 4}$; No. of beats $= 4 = n_1 - n_2$.

$= \frac{V}{10} - \frac{V}{10 \cdot 4}$; whence $V = 1049$ ft. per second.

55 Velocity of Sound by Resonance of Air Column —A vibrating tuning-fork F is held close to the upper end of a glass tube vertically placed in a long cylinder almost full of water (Fig.34) On gradually raising or lowering the tube, a particular length of air column in the tube will be found when the sound will be strongly reinforced. Thus it is an arrangement for a closed pipe of adjustable length. Adjust the position of the tube when the intensity of the sound becomes maximum. In that position the frequency of vibration of the air column agrees with that of the fork and, the fork and the air-length in the tube are then said to be in resonance. It should be noted that the pitch of the sound heard is independent of the diameter of the tube, and also of the material, whether glass or metal. The action may be explained as follows :—

Each movement of a prong of the fork towards the mouth of the tube compresses the air in front of it, and thus sends a compressed wave down the tube. The compressed wave on reaching the surface of water, which is denser medium, is reflected back as *compressed wave* (Art. 29). This reflected compressed wave on reaching the open end of the tube is relieved from the strained condition by moving sideways and it is again reflected, but, this time, as a *rarefied wave* which starts down the tube (see Art. 51). Now, if the prong has just at the same instant reached the extreme downward position and begins to move upwards, a wave of rarefaction will proceed downwards into the tube. The reflected rarefied wave will *coincide* with the rarefied wave started down the tube due to the backward motion of the fork, and so will be *intensified*. Again, the reinforced waves will be reflected back from the closed end (water surface) as rarefied wave, which reach the open end just when the prong begins to move down. So the wave of compression formed by the reflection of the rarefied wave at the open end is helped by the fresh compressed wave sent by the prong. This shows that the fork and the air column of the tube agree in motion, (*i.e.* their time-periods are the same). and so *resonance* is produced.

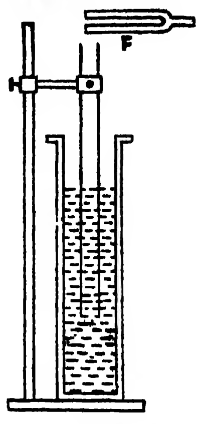


Fig. 34

Thus resonance is the intensification of sound due to the union of direct and reflected waves.

From the above it is evident that when resonance is produced, the wave travels over *twice* the length of the air column in the time taken by the prong to make half a vibration. Therefore, in a complete vibration of the prong, the wave travels over *four times* the length (l_1) of the air column AN (Fig. 35). We have, therefore, $l_1 = \lambda/4$, or $\lambda = 4l_1$ where λ is the wave-length, and l_1 the length of the air column. But, if V be the velocity of sound, and n the frequency of vibration of the fork, we have, $V = n\lambda$; $\therefore$ $V = 4l_1 n$.

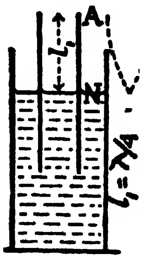


Fig. 35

In fact the antinode is not exactly at the mouth of the tube but is a little outside the tube, the distance depending on the diameter of the tube. Lord Raleigh has shown that this correction is 0·6r, and thus the effective length of the vibrating air column is $l_1 + 0{\cdot}6r$, where r is the radius of the tube, and 0·6r is called the *end correction.*

Hence, $$V = 4n(l_1 + 0{\cdot}6r).$$

Thus from the resonant air column the velocity of sound can be determined by knowing the frequency of the fork.

If the temperature of the air in the tube is t, the velocity of sound at $O°C.$ can be found from the relation

$V_t = V_o\sqrt{1 + \frac{t}{273}}$, or $V_t = V_o\sqrt{\frac{T}{273}}$, where T is the temperature upon the absolute scale corresponding to t°C.

The end correction can be avoided in the following way—In the first position of resonance l_1 (Fig. 39) $= \lambda/4$, but if the tube be sufficiently long, then by raising the tube further out of water a second position of resonance, of weaker intensity, may be obtained where the length of the resonant air column l_2 (Fig. 36) $= 3\lambda/4$ [see Art. 52(a), p. 443].

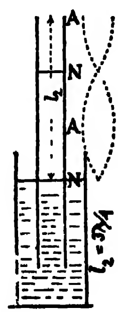


Fig. 36

since, in the first case $\frac{\lambda}{4} = l_1 + 0{\cdot}6r$, and

in the second case, $\frac{3\lambda}{4} = l_2 + 0{\cdot}6r$,

we have, $\frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} = l_2 - l_1$. $\therefore$ $V = n\lambda = 2n(l_2 - l_1)$

By this means the wave-length can be determined without considering the end-correction.

N. B. In order to obtain the velocity of sound in *dry air* the result corrected form temperature should also be corrected for moisture contained in the air by the formula of Art. 19.

Examples.—*1. You are provided with a vessel containing water, a glass tube about 40 cms. long, open at both ends, and a tuning-fork whose frequency is 256. What experimental result do you expect? (The velocity of sound in air is 33280 cms. per second nearly).* (*C. U. 1914*).

Let l be the length of the air column which emits the fundamental note. Then, wave-length $= 4l$. Velocity of sound = frequency × wave-length ;

or $33280 = 256 \times 4l$, whence $l = 32·5$ cms.
that is, $(40 - 32·5)$ or $7·5$ cms. of the glass tube should be dipped in water when resonance will be produced.

2. A tuning-fork is held above the mouth of a closed glass cylinder, whose capacity is 150 cubic inches and height 14 inches and water is poured slowly until the most perfect resonance is obtained. The volume of the water introduced was 20 cu. in. What was the vibration number of the tuning-fork? (Velocity of sound in air = 1120 ft. per sec.)

Volume of air in the tube for resonance $= 150 - 20 = 130$ cu. in.

Area of cross-section of cylinder $= \dfrac{150}{14}$ sq. in. ∴ Length of air column for perfect resonance. $l = 130 \div \dfrac{150}{14} = 12·133$ in. ;

Again $\dfrac{150}{14} = \pi r^2$, where r = radius of cylinder, whence $r = 1·85$

Hence end correction $= 0·6r = 0·6 \times 1·85 = 1·11$ in. We have, $V = 4n\ (l + 0·6r)$, where n is the required frequency.

$\therefore\ (1120 \times 12) = 4n \times 13·24$ ($\because\ V = 1120 \times 12$ in.) ; whence $n = 253·8$.

3. A certain tuning fork first produced resonance in a glass tube with an air column of 33 cm. and it could again produce resonance with a column of 100·5 cm. in the same tube. Calculate the end-correction. (*All. 1921*).

In the first case, if l_1 be the length of air column for resonance, the effective length of air column $= l_1 + x$, where x is the end-correction.

$\therefore\ l_1 + x = \lambda/4$, where λ is the wave-length. In the second case, if l_2 be the length of air column for resonance, the effective length $= l_2 + x$

$$l_2 + x = \frac{3\lambda}{4}\ ;\ \text{or}\ \frac{\lambda}{4} = \frac{l_2 + x}{3}\ ;\ \therefore\ l_1 + x = \frac{l_2 + x}{3}\ ;\ \text{or}\ 3(l_1 + x) = l_2 + x\ ;$$

$\because\ l_1 = 33$ and $l_2 = 100·5$, $x = 0·75$.

4. A closed pipe is filled with a gas whose density is 0·00126 gm. per c.c. If the length of the pipe is 50 cm., find the frequency of the note emitted. (The velocity of sound in air at 0°C. is 332 metres per second.)

As the density of air is 0·001293 gm. per c.c., and as the velocity of sound in any gas is inversely proportional to the square root of its density, the velocity of sound in the gas of the pipe. $V=33200\sqrt{\frac{0·001293}{0·001260}}$ cm. per sec.

But $V=4\,nl$; whence $n=\frac{V}{4l}=168$.

56. Longitudinal Vibration of Rods.—When a rod of wood or glass, firmly clamped at its middle point, is rubbed lengthwise with a piece of resined cloth, or a wet linen, it is set in longitudinal vibration, that is, in planes parallel to its axis, and it gives out a shrill note. The rod is alternately elongated and compressed in its course of movement and the vibration takes place exactly in the same manner as the stationary vibration of an open pipe sounding its fundamental.

The free ends of the rod being the parts of maximum vibration are *antinodes* whilst, for the simplest mode of vibration, there will be a *node* in the middle where it is clamped. Evidently the length of the rod is half the wave-length (distance between two consecutive antinodes).

The velocity of sound in the rod is given by, $V=\sqrt{\frac{E}{D}}$, where E is the Young's modulus of elasticity and D the density of the material of the rod. Again we have, $V=n\lambda$, where λ, the wave-length, is in this case, equal to twice the length l of the rod.

$$\therefore\quad V=2nl\,;\text{ or } n=\frac{V}{2l}\,;\qquad\text{or}\quad n=\frac{1}{2l}\sqrt{\frac{E}{D}}.$$

Thus knowing the velocity of sound in the rod, the frequency, or the pitch of the sound emitted, can be calculated. Again, if the pitch of the sound is determined by compression with a sonometer wire, the velocity of sound is known from the relation $V=2nl$. (Thus, this also provides a method of determining *the velocity of sound in a solid rod.*)

57. Kundt's Dust-tube Experiment.—The velocities of sound in different gases were determined by Kundt by using longitudinal vibration of rods. The velocity of sound in a rare gas is very conveniently determined in the laboratory by this method.

Expt.—The apparatus consists of a metal or glass rod which is clamped exactly at its middle point C and has a card-board disc D firmly fixed at its end within a long glass

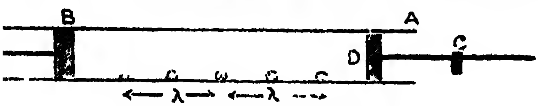


Fig. 87.—Kundt's Tube

tube AB in which it can move without touching its walls. The other end of the tube AB is closed by an adjustable stopper B (Fig. 37).

Before fixing the tube in position it is thoroughly dried by blowing hot air through it, and then some dry lycopodium powder is evenly spread along its sides. The rod is now stroked (rubbed lengthwise) with a resined cloth, if it be metal, or with a cloth moistened with methylated spirit, if it is of glass, causing it to vibrate longitudinally. Waves are emitted by the disc D which is moving backwards and forwards with the frequency of the note emitted by the rod and thus setting up vibrations in the air within the tube. These waves started from the disc D are reflected by the surface of the piston B, and thus stationary waves having fixed nodes and antinodes are set up in the tube. The position of the adjustable piston B is carefully adjusted until a resonance is produced, when the fundamental note emitted by the rod coincides with a harmonic of the enclosed air column within the tube.

When resonance is reached the fine lycopodium powder is seen to be thrown into violent agitation and the powder will be seen to fly away from the *loops* (antinodes), the places of maximum displacement of air particles, and will collect in heaps at the *nodes*, the places of minimum displacement of air-particles. In general, several nodes and loops will be formed within the tube as shown in the diagram. If l be the mean distance between two consecutive nodes, the wave-length λ of the longitudinal vibration of air is $2l$, and if n be the frequency of the note emitted by rod, it is also the frequency of vibration of the air in the tube, as the rod and the tube are in resonant vibration, and the velocity of the sound in air $V = n\lambda = n \times 2l$. Now, for the simplest mode of vibration of the sounding rod, a node is formed at the middle where it is clamped and two *loops* are formed at the two ends. So, if l' be the length of the rod, the wave-length λ' of the longitudinal vibration, within the rod is $2l'$, and if V' be the velocity of sound in the rod, $A' = n\lambda' = n \times 2l'$; so we have

$$\frac{V}{V'} = \frac{n \times 2l}{n \times 2l'} = \frac{l}{l'} = \frac{\text{length of a loop}}{\text{length of the rod}}.$$

The above relation provides a method of calculating V or V', when one of them is known ; and if frequency n be found by means of a sonometer and a standard fork, then *velocity of sound in air and also in the rod can both be determined.*

Velocity in Different Gases.—To compare velocities of sound in two gases, first fill the tube with one of the gases and find out the average distance l_1 between two nodes formed at resonance. Repeat the

experiment with the other gas and let the length of the loop in this case be l_2; then, if V_1 and V_2 are the respective velocities in the two gases, we have $\frac{V_1}{V_2} = \frac{n \times 2l_1}{n \times 2l_2} = \frac{l_1}{l_2}$.

Questions

Art. 51.

1. Describe in detail with a diagram an open organ pipe, and explain its mode of excitation. What effect is produced on the pitch and character of the note, if the open end is suddenly closed? (See also Art. 52).
(C. U. 1926 ; Pat. '28)

2(a). Give an account of nodes and antinodes in open and closed organ pipes. (All. 1918, '22 ; C. U. 1931, '32)

(b) How are stationary waves produced in (i) an open organ pipe. (ii) a closed organ pipe? (C. U. '47 ; cf. All. '45)

Art. 52.

3. What do you understand by pitch of a musical note? Two organ pipes of the same length are given, one open and the other closed. What should be the relation between the pitch of fundamental notes emitted by them? (C. U. 1924, '26 : Pat. 1928, '39)

4. What is the frequency of the fundamental notes of an open organ pipe 4 ft. long? (Velocity of sound in air = 1100 ft. per second).

What would be the effect of (*a*) covering its open end, (*b*) increasing the temperature, and (*c*) varying the nature of the gas enclosed in the tube?
(Pat. 1930)

[**Hints.**—$1100 = n - 4$; *or* $n = 137·5$; (*a*) when one end is closed, the pitch will be halved, *i.e.* will be lowered an octave; (*b*) velocity will be increased (*See* also Art. 53) with the increase of temperature, hence pitch will be increased, (*c*) pitch increases or decreases with the increase or decrease of velocity which again varies inversely as the square root of density of the gas.]

5. What is meant by resonance? Calculate approximately the length of the resonance box closed at one end on which a tuning-fork is to be mounted the pitch of which is 256, the velocity of air being 1120 ft. per sec. Would the same resonance box answer for a fork of another pitch? If so, of what pitch? (All. 1926)

[**Hints.**—The resonance box acts as a closed organ pipe ; so, $V = 4nl$; *or* $1120 = 4 \times 256 \times l$; or $l = \frac{35}{32}$ ft. The box will also speak for a fork whose frequency is 3 or 5 times the fundamental frequency.]

6. The velocity of sound in hydrogen is 1296·5 meters per second. What will be the length of a closed organ pipe, filled with hydrogen, which gives a note having a vibration frequency of 512 per second? (C. U. 1915 ; Dac. '33)

[*Ans.* : 63·3 cm. (approx.)]

7. What is the frequency of the note emitted by a siren having 32 holes and making 1575 revolutions per minute. A closed organ pipe sounding its fundamental is in unison with the above note. What is the length of the pipe ? (Velocity of sound in air = 1120 ft. per sec.)

[*Ans* : 840 ; $\frac{1}{3}$ ft.]

8. Calculate the shortest length of a pipe 4 cm. in diameter which will be set in resonant vibration by a tuning-fork making 256 vibrations per second. (Velocity of sound in air = 340 meters per second).

[*Ans.* : 33·21 cm.]

9. Two organ pipes, open at both ends, are sounded together and four beats per second are heard. The length of the short pipe is 80 in. Find the length of the other. (Vel. of sound = 1120 ft. per sec.) (C. U. 1935)

[*Ans.* : $80\frac{6}{11}$ in.]

10. What are the fundamental and harmonic notes of the organ pipes, open and closed ? (C. U. 1947)

11. What effect is produced on the frequency and quality of a note given by an organ pipe if the top is suddenly closed ? If the frequencies of the first overtones of the two notes so obtained differ by 440, what was the original frequency ? (All. 1924)

[*Ans* : 880.]

12. The pitch of the fundamental note of an open organ pipe 100 cm. long is the same as that of a sonometer wire 200 cm. long which has the mass one gram per centimetre. Find the tension of the wire. (Pat. 1937)

[*Ans* : $4{\cdot}356 \times 10^9$ dynes, taking $V = 330$ meters per sec.]

Art. 53.

13. Calculate the change of pitch of an open organ pipe 3 ft. long when the temperature changes from 10°*C*. to 15°*C*. (L. C.)

[*Ans* : 1·009]

14. The frequency of a note given by an organ pipe is 312 at 15°C. At what temperature will the frequency be 320 supposing the pipe to remain unchanged in length. (*See* Art. 18).

[**Hints.**—$V_{15} = 312\lambda$, and $V_t = 320\lambda$; $\frac{V_t}{V_{15}} = \frac{320}{312} = \frac{40}{39}$.

Again, $V_t = V_o\left(1 + \frac{1}{2}.\frac{t}{273}\right)$; and $V_{15} = V_o\left(1 + \frac{1}{2}.\frac{15}{273}\right)$

$\therefore \frac{V_t}{V_{15}} = \frac{546 + t}{561}$. So, $\frac{546 + t}{561} = \frac{40}{39}$, whence $t = 22{\cdot}4$°C.]

15. If an organ pipe gives a note of 256 when the temperature of air is 40°C., what will be the frequency of the note when the temperature falls to 20°C. ? (Pat. 1946)

[*Ans* : 247·2].

Art. 54.

16. How can the existence of nodes and antinodes in a sounding organ pipe be demonstrated ? (C. U. 1937)

Art. 55.

17. Suggest any experiment by which you can determine the wave-length of any note in air. (Pat. 1926)

18. How would you demonstrate that the best resonant length is one-fourth the wave-length in the case of a closed pipe and one-half the wave-length in the case of an open pipe. (Pat. 1929)

[**Hints.**—Describe the resonant column experiment (Art. 55). The tube is considered to be a closed pipe as one end of it is closed by water—a medium denser than air. After getting the first position of resonance (*i.e.* for $l=\lambda/4$) raise the tube still further until a second position of resonance is obtained. In this position $l=3\lambda/4$. Raising it still more, a third position for $l=5\lambda/4$ may be obtained. It will be observed that the sound is loudest in the first case, and gets fainter and fainter for the overtones.

In the second case, hold the same tuning-fork in front of an open pipe (both ends open), the length (say about 10 inches) which is made adjustable by slipping up and down over it a tightly fitting roll of ordinary writing paper. Adjusting the length and proceeding as above, it will be observed that sound is maximum for $l=\lambda/2$, and gets fainter and fainter for the overtones, *i.e.* for $l=2\lambda/2$ and $3\lambda/2$, etc.]

19. A vibrating tuning-fork is placed at the mouth of an open jar, and water is poured into the jar gradually. Explain what will happen.

Explain how would you determine the velocity of sound in air by an experiment of this kind. (C. U. 1915, '18, '26, '29, '31, '47 ; Pat. 1918, '21, '23, '25, '28, '36, '41 ; Dac. '33, '34.)

20. Describe fully a method of determining the vibration frequency of a tuning-fork in the laboratory. (C. U. 1928 ; cf. Pat. 1941 ; All. '44)

21. Describe an experiment to find out the velocity of sound in carbon dioxide. (Pat. 1939 ; cf. '18 ; All. '22 ; cf. Dac. '31)

22. What is meant by resonance ? Show how the phenomenon of resonance may be used to measure the velocity of sound in a gas. (C. U. 194

A cylindrical tube 100 cm. long, closed at one end and of one cm. internal radius, is placed upright and filled with water, and a tuning-fork of frequency 510 is sounded continuously over its open end. Assuming the velocity of sound in air to be 340 metres per sec., describe exactly what you would expect to observe if the tube were gradually emptied. (Pat. 1936)

[*Ans* : The tube will speak when the length of the air column is 16, 48, 80 cm.]

23. A tuning-fork, whose frequency is 410, produces resonance in a glass tube of diameter 2 cms., when lowered vertically in water; on lowering the tube further down another point of resonance is found. Find the lengths of the air column producing resonance. ($V = 340$ metres per second).

[*Ans* : $l_1 = 61{\cdot}59$ cms. ; $l_2 = 20{\cdot}18$ cms.]

CHAPTER IX

Musical Instruments : Physiological Acoustics

58. Musical Instruments.—The musical instruments can be divided mainly into two classes :—(a) *Wind instruments* ; (b) *Stringed instruments* ; (c) *Percussion instruments* :

(*a*) **Wind Instruments.**—The working of these instruments depends upon the vibration of air column. These again can be divided into two classes.—(i) Instruments without reeds such as the flute, piccolo, etc. ; (ii) Instruments with reeds such as clarinet, harmonium, etc. The most familiar example of the wind instruments is the organ pipe, which may be of the above two types, (a) one without reeds, known as the flue pipe, and (b) the other with reeds known as the reed pipe.

It is already known that only a column of air of right length may be made to respond to a particular note. But, in the case of

column of air contained in a pipe, resonance can be produced by making a flutter in the air at one end of the pipe. The pipe selects from the flutter (which is merely a combination of pulses of various wave-lengths) that particular pulse with which it can resound in order to produce a musical note. This is the *principle of various musical instruments*, in nearly all of which the sounding part is a column of air.

The Flue Pipe.—The simplest form of this type is an ordinary organ pipe the working of which has been described in Art. 51. The note emitted by this pipe depends primarily upon the length of the pipe. The fundamental note is given out at a certain minimum blowing pressure by increasing which higher harmonics are given out.

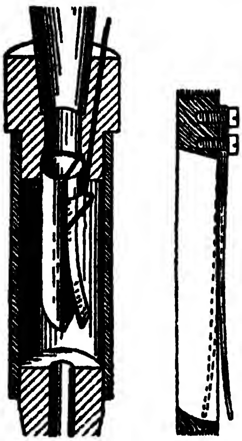


Fig. 38.—Reed pipe.

In the **Organ**, there is a set of pipes of fixed pitch and the instrument is provided with a fixed keyboard as in harmoniums.

The Reed Pipe.—In this form the air blast impinges on a flexible metal strip (Fig. 38), called the *reed*, which controls the amount of air passing to the pipe by wholly or nearly covering the aperture through which the air passes. The reed which completely closes the aperture of the pipe is called a *beating reed*, which behaves as a stopped end of the pipe, and the other by which the aperture is nearly, but not fully, closed is called a *free reed*. Free reeds are used in harmoniums and American organs, where the wind is forced into a rectangular air-chamber at one side of which the reed is attached. The air presses against the reed and causes it to vibrate. A single beating reed made of cane is used at the mouthpiece of a *clarinet*.

(*b*) **Stringed Instruments.**—In this class the note is produced by the vibration of strings kept under tension, such as the harp, piano, violin, esraj, setar, etc.

(*c*) **Percussion Instruments.**—These are tuned to a fixed pitch, such as the kettle-drum, tambourine, etc., in which the vibration of air is produced by striking with a hammer a stretched membrane or metal plate.

59 The Phonograph.—Long before the invention of phonograph, Thomas Young, an English scientist, succeeded in recording sound vibrations on a rotating drum, but it was Thomas Alva Edison, an American, who in 1877 invented the phonograph which was capable of both recording and reproducing sound vibrations.

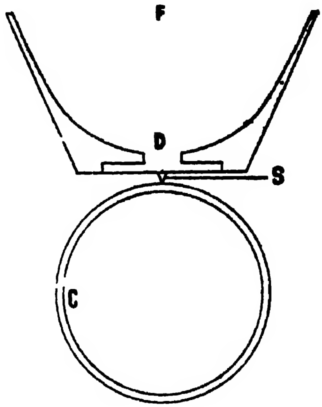


Fig. 39.

The *Phonograph* consists of a funnel F, which is closed at the lower end by a thin glass or mica diaphragm D (Fig. 39) When sound vibrations are directed into the funnel, they set the diaphragm into vibration, and with it a pointed steel, or a chisel-shaped sapphire crystal S, attached at its centre, also vibrates. The chisel is in contact with a cylinder C of paraffin wax, and, at the time of vibrating, cuts a groove of *varying depth* on the cylinder which is rotated, and at the same time moved lengthwise, by clock-work. The depth of the groove is not uniform but corresponds to the strength and complexity of the vibrations communicated to D. *The cylinder is thus a faithful record of the sound vibrations directed at F.*

To *reproduce* the sound, a smooth sapphire point, attached to a similar diaphragm fitted in a frame, called the *sound-box*, is placed at the beginning of the groove of the cylinder which is rotated and shifted sideways at the same speed as before. The sapphire point rises and falls in accordance with the height and depth of the groove, and thus the diaphragm of the sound-box reproduces exactly the movements of the disphragm D of the records. These movements communicated to the air produce the same sound which was originally directed into the funnel P.

The materials with which records are prepared being very soft, the records do not last long and so the reproduction is not very faithful.

The Gramophone.—It is a machine for the recording and reproducing of sound, usually in the form of vocal or instrumental music, speech, etc. It is an improvement upon the phonograph. Here the records are made in the form of flat discs in which spiral grooves representing sound tracks run from the rim to the centre. The grooves are of varying width and not of varying depth, as a result of which the resistance to the movement of the needle along the furrow is much less than in the phonograph and so the reproduction of sound is more faithful. Moreover the discs are made of a matrix (comp

of shellac, tripole powder and other ingredients) which is much harder than the wax used in the phonograph and so do not deteriorate with use so early.

Recording of sound.—The modern method of recording is electrical. The source of sound is put in front of a microphone the current passing through which is thereby modified. This fluctuated current is amplified to a required extent by the use of wireless valves. The amplified fluctuating current is used to actuate the cutting chisel upon a disc of wax, through electromagnetic action. This record of wax is called the negative. An electro-plate of it is made on a copper disc by electrolysis. This electro-plate is called the 'mother shell' or the "parent record" or the positive. Two 'working matrices' of two different musics are made from two mother shells and are fixed to the top and bottom plates of a hydraulic press with their recorded surfaces facing each other. The recording material (the disc) previously warmed a little, is placed in between the two working matrices and the two records are stamped on the two faces of the recording material by pressure.

Reproduction of sound.—This is done through the mechanism of a "*Sound box*" which has a needle, with rounded point, rigidly screwed

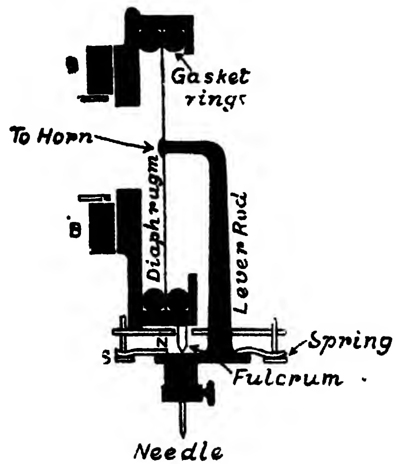


Fig 40.—Sound box

to the shorter arm of a lever system (Fig. 40). The needle slides [illegible] the spiral grooves of the record, the record being made to rotate

at a uniform speed with the help of an adjustable *governor*, under the action of the energy of a wound spring The end of the longer arm of the lever is fixed to the centre of a circular mica diaphragm. The diaphragm is mounted between rubber rings called *gaskets* and forms the front of a cylindrical metal box, called the *sound box* This is connected to a metallic conical pipe called the *tone arm* which is capable of moving freely about a vertical axis The tone arm with the sound box gradually moves to the centre of the record as the needle slides on it The sound from the tone arm is finally magnified through a horn which is usually housed within the cabinet The vibration of the needle running on the furrows sets the diaphragm to motion and the sound is reproduced The lever system is balanced on a knife edge forming the fulcrum The vibration of the lever is controlled by the two springs

In the **Radio Gramophone** the mechanical sound box is replaced by an electric ' pick up which periodically modulates a feeble current as the needle slides on the grooves of the record This feeble modulated current, after suitable amplification through a combination of wireless valves, is led through a **loud speaker** by which a voluminous sound commanding a large assembly of audience is reproduced

Physiological Acoustics

60. **The Ear**—Human ear (Fig 41) consists of three parts, (*a*) *the external ear* (or **pinna**) by which the sound wave is collected (*b*) the *middle ear* (or **drum**) in which the vibrations are transmitted from the external ear to (*c*) the *internal ear* (or **labyrinth**)

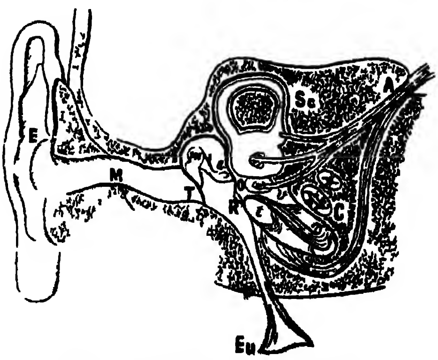


Fig 41 —Section through the Human Ear

(*a*) **External Ear**—From the outside, there is at first the *external ear E* (the part external to the head) from which extends the ear passage called the external **auditory meatus** *M* down which air vibrations travel This is closed at its end by a stretched membrane, called the **membrana tympani** *T*, beyond which lies the cavity called the *ear drum* or tym middle ear

(b) **Middle Ear.**—This cavity is bounded upon its outside by the tympanic membrane and its inner side by bony walls except at two places, the **fenestra ovalis** O and the **fenestra rotunda** R where membranes are stretched. A combination of three little bones or **ossicles**, the first of which is the **malleus** M or the hammer bone extends from the inside of the tympanum. This bone communicates with the internal ear through two other bones, the *anvil* (or **incus**) and the *stirrup* S (or **stapes**), the base of which is joined to the *fenestra ovalis*, which separates the middle ear from one part of the inner ear. The middle ear is connected to the throat by an **eustachian tube** Eu. This tube is usually closed but the action of swallowing opens a valve in this tube and serves to keep the air pressure inside the middle ear equal to that of the atmosphere. Ear-ache is often caused when the valve does not work and due to which the outside pressure becomes greater than that inside so that the bones are pressed hard causing painful results.

(c) **Labyrinth.**—It is a complicated structure having a set of cavities. The cavities have bony walls, called the **osseous labyrinth** and internal membranes, known as **membranous labyrinth.** The osseous labyrinth consists of the following—(1) *Vestibule* v in the outer wall on which lies the *fenestra ovalis*. Through the inner wall of the vestibule the divisions of the auditory nerve A enter into the internal ear. (2) **Cochlea** t C at the entrance to which lies the *fenestra rotunda* It is a spiral canal like the form of a snail shell. It contains a fluid which receives and transmits vibrations to the auditory nerve. In this canal there is a membranous partition, called the **basilar membrane**. which plays an important part in the act of hearing. (3) The semicircular canals Sc serve to maintain equilibrium and do not take part in the hearing.

The *membraneous labyrinth* contains a fluid, *endolymph* and between it and the *osseous labyrinth* is another fluid, *perilymph*.

61. How we hear.—The waves generated in the air by the sounding body are collected by the *pinna* and passing through the *auditory meatus* strike the *tympanic membrane* which is forced to execute corresponding vibrations. These vibrations are transmitted through the three little bones in succession, the *malleus*, the *incus*, and the *stapes*, to the membrane of the *fenestra ovalis* of the inner ear. The vibrations of the fenestra ovalis starts waves which reach *cochlea* where [illegible] vibrations are handed on by the fluid to the basilar [illegible] brane. The vibrations thus generated affect the auditory nerve [illegible] brain. and give rise to the sensation of sound.